StaRAI: From a Probabilistic Propositional Model to a Highly Compressed Probabilistic Relational Model

MARCEL GEHRKE[1], MALTE LUTTERMANN[2]

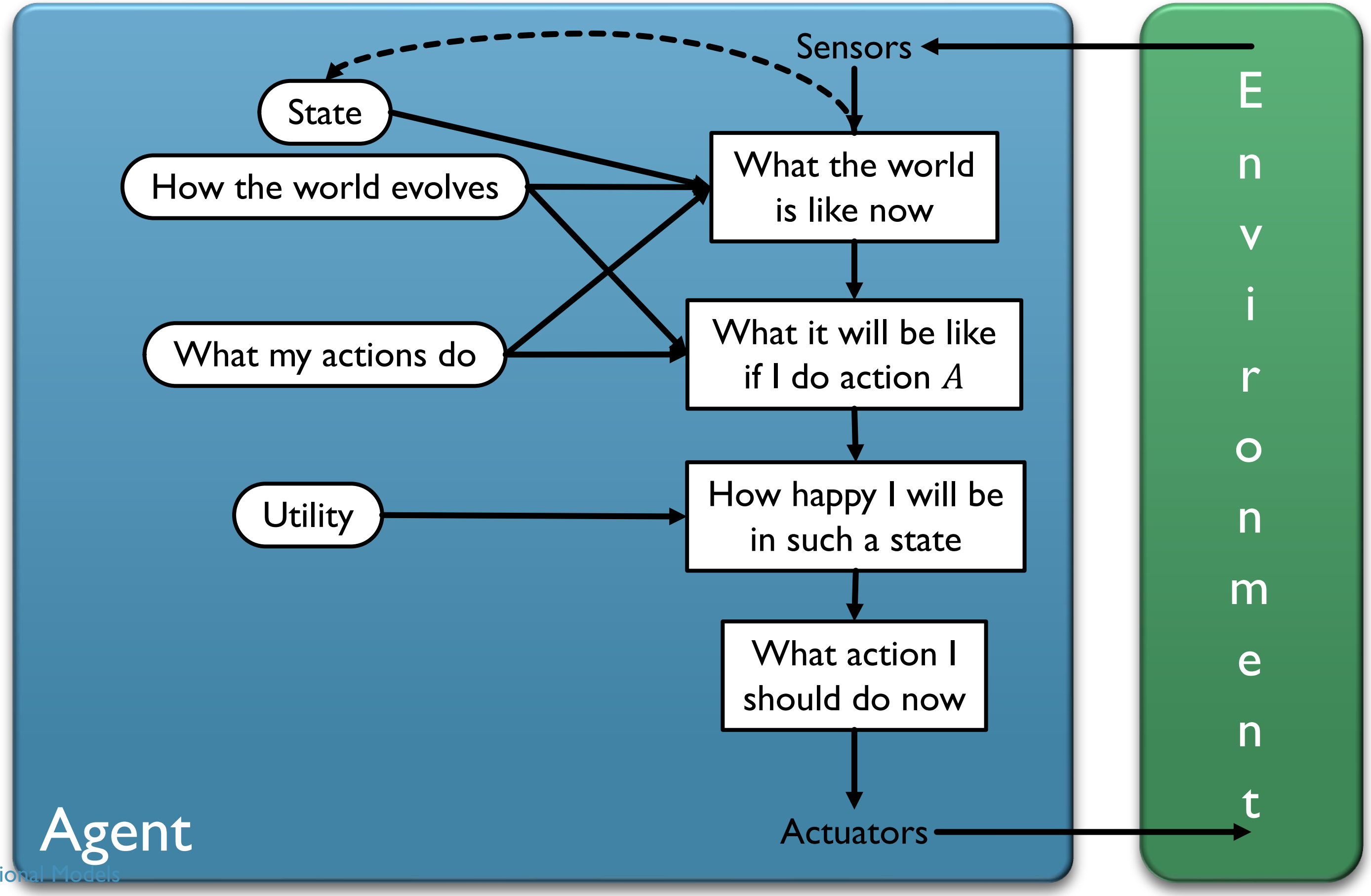
[1]Institute of Humanities-Centered Artificial Intelligence, University of Hamburg
[2]German Research Center for Artificial Intelligence (DFKI)

AGENDA

1. Introduction to relational models [Marcel]
   - Relational models under uncertainty
   - Obtaining a compressed representation
2. Compressing probabilistic relational models [Malte]
3. Application: Lifted causal inference [Malte]
4. Summary [Marcel]

Especially thanks to Tanya Braun for providing many of the slide

GENERAL AGENT SETTING



AIMA, Russell/Norvig

RELATIONAL MODELS UNDER UNCERTAINTY

INTRODUCTION

STATISTICAL RELATIONAL ARTIFICIAL INTELLIGENCE (STARAI)

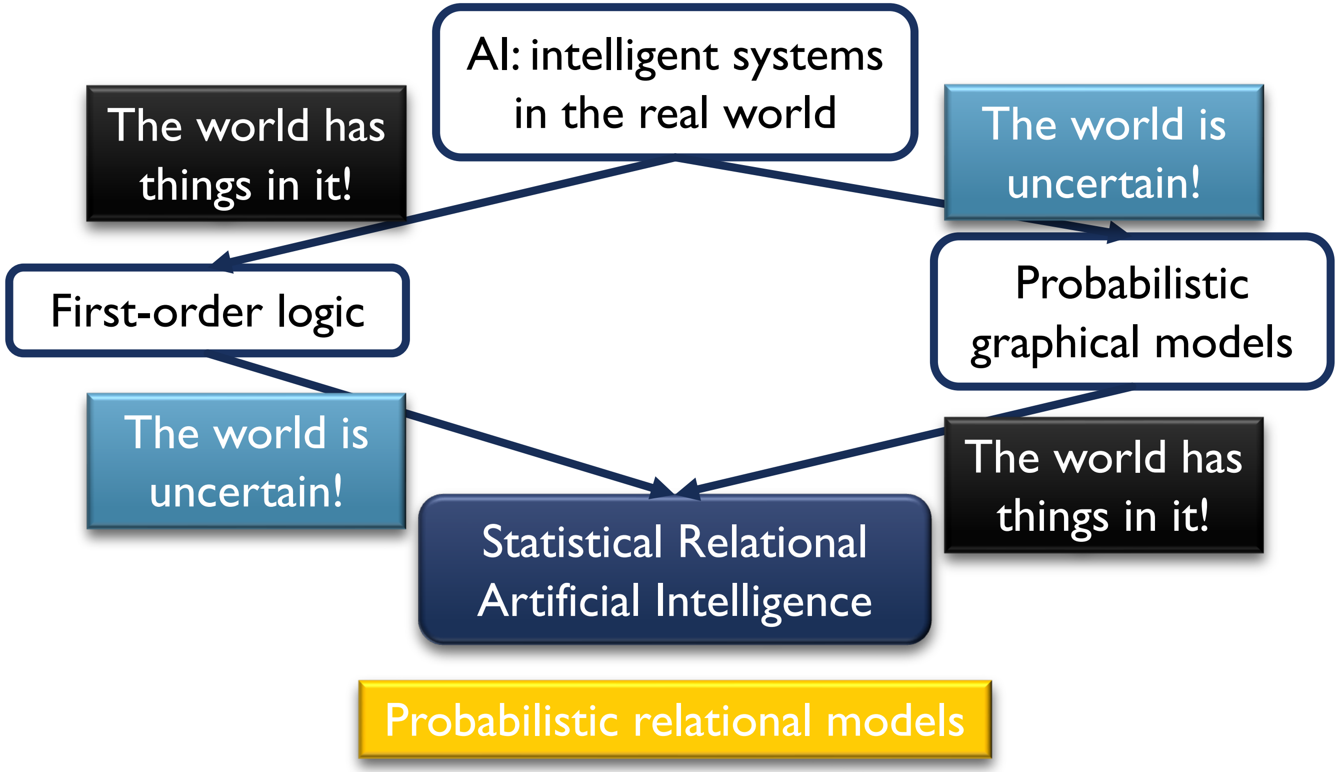


Figure based on a figure by Stuart Russell

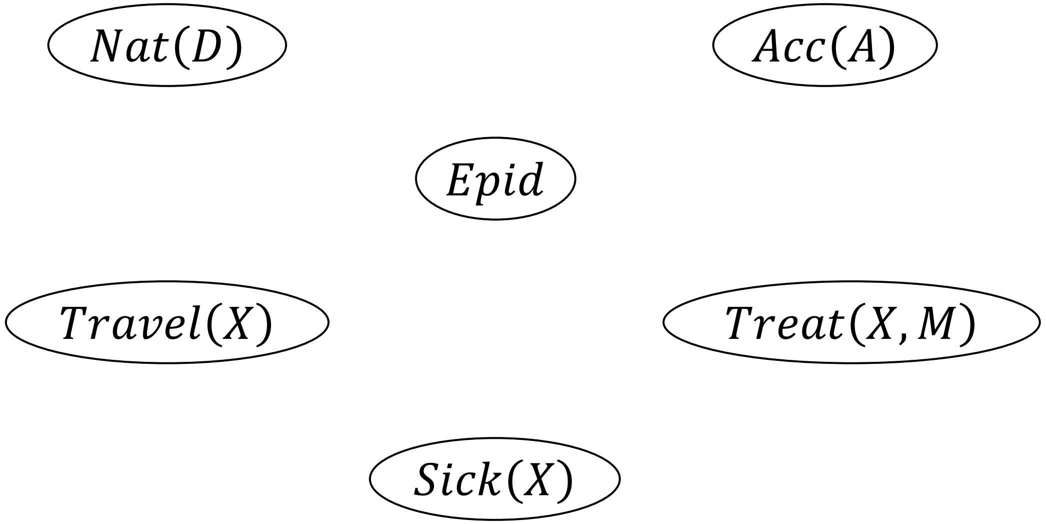
LOGICAL VARIABLES IN RANDOM VARIABLES

- Atoms: Parameterised random variables = PRVs
  - With logical variables
    - E.g., X, M
    - Possible values (domain):

$$dom(X) = \{alice, eve, bob\}$$
$$dom(M) = \{injection, tablet\}$$

  - With range
    - E.g., Boolean, but any discrete, finite set possible
    - $ran(Travel(X)) = \{true, false\}$
  - Represent sets of *indistinguishable* random variables

$$Nat(D) = natural\ disaster\ D$$
$$Acc(A) = accident\ A$$



Poole 03

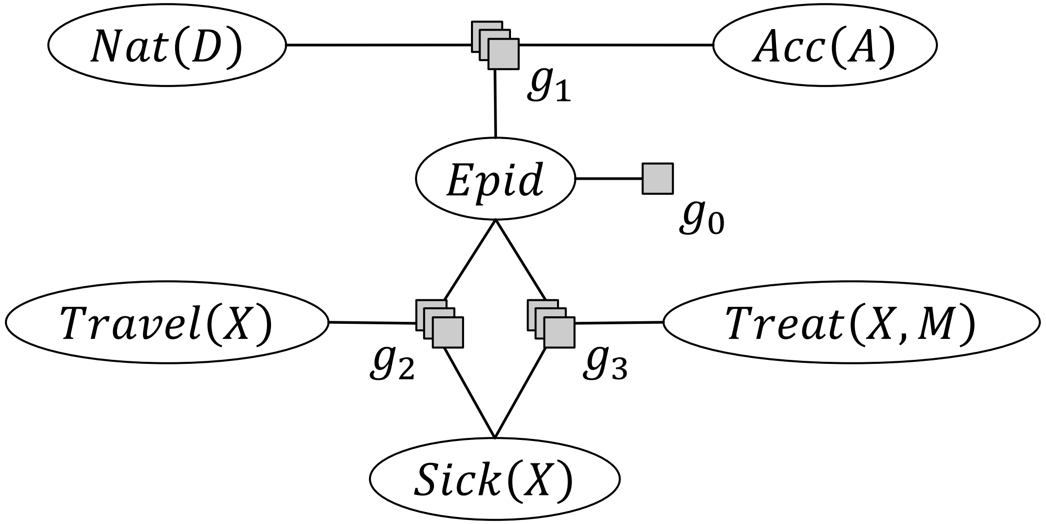
PARFACTORS

- Factors with PRVs = parfactors

$Travel(X)$	$Epid$	$Sick(X)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$false$	$true$	6
$true$	$true$	$false$	2
$true$	$true$	$true$	9

Potentials

- In parfactors, just like in factors, no probability distribution as factors required



Poole 03

FACTORS

- Grounding

$Travel(X)$	$Epid$	$Sick(X)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$false$	$true$	6
$true$	$true$	$false$	2
$true$	$true$	$true$	9

$Travel(eve)$	$Epid$	$Sick(eve)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$fals$		
$true$	tru		
$true$	tru		

$Travel(bob)$	$Epid$	$Sick(bob)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
		$true$	6
		$false$	4
		$true$	6
		$false$	2
		$true$	9

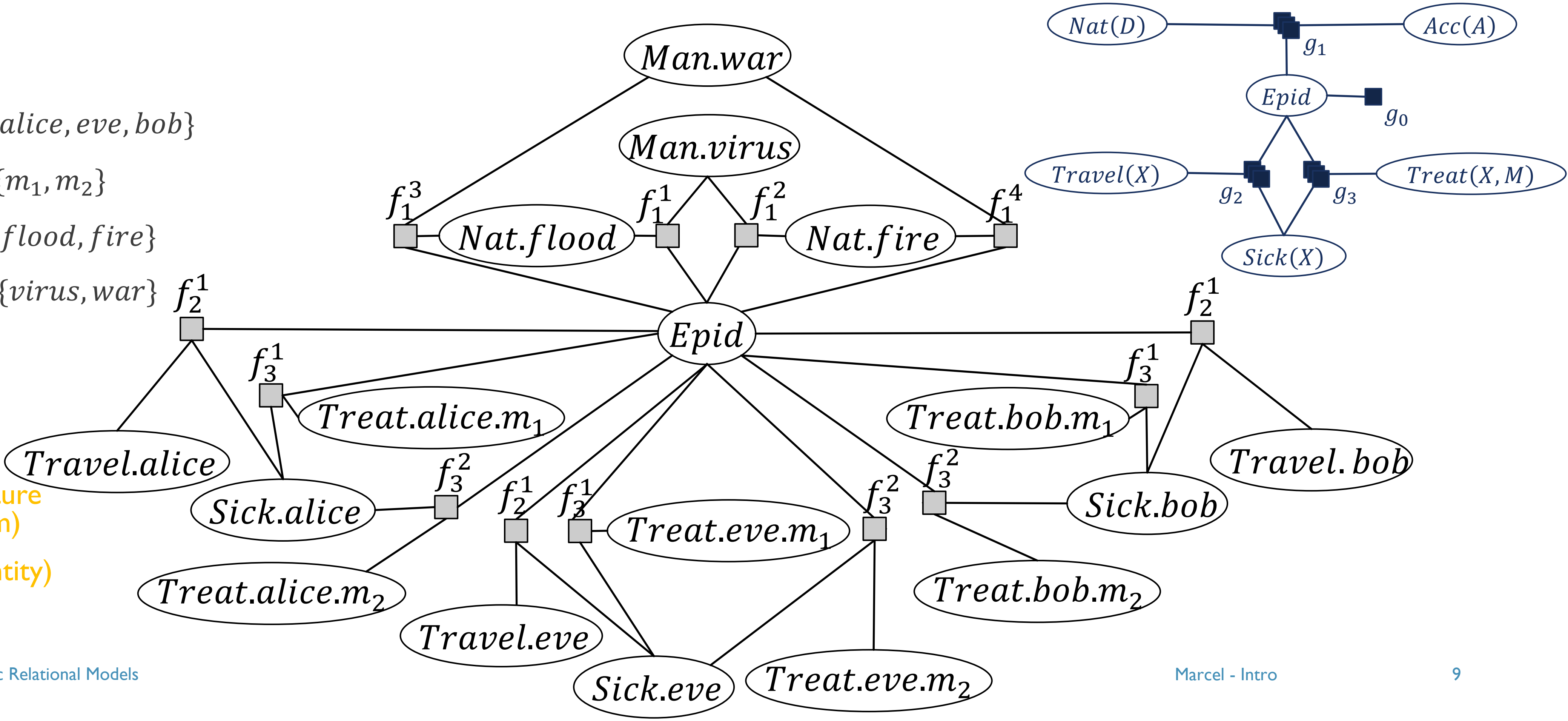
$Travel(alice)$	$Epid$	$Sick(alice)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$false$	$true$	6
$true$	$true$	$false$	2
$true$	$true$	$true$	9

$reat(X, M)$

Poole 03

GROUNDED MODEL

- Given domains
  - $dom(X) = \{alice, eve, bob\}$
  - $dom(M) = \{m_1, m_2\}$
  - $dom(D) = \{flood, fire\}$
  - $dom(W) = \{virus, war\}$
- Symmetry in
  - Graph structure (isomorphism)
  - Factors (identity)



Poole 03

MARKOV LOGIC NETWORKS (MLNS)

- Weighted logical formulas to soften otherwise hard constraints [Richardson & Domingos 06]
  - Implicitly connected via conjunction
    - I.e., set of formulas ψ_i = knowledge base/theory
  - Worlds that violate constraint become less likely but not impossible
    - As w_i increases, so does the strength of ψ_i
    - Infinite weight: Hard constraint = pure logic formula
      - Probabilities of worlds that do not satisfy hard constraint set

Hard constraint

$\infty \; Presents(X, P, C) \Rightarrow Attends(X, C)$

Soft constraint, weight = $\exp(3.75)$

$3.75 \; Publishes(X, C) \wedge FarAway(C) \Rightarrow Attends(X, C)$

GROUNDING

- Each (w_i, ψ_i) represents a set of *propositional* sentences, each sentence with weight w_i
  - One sentence for each possible substitution of the free variables $free(\psi_i)$ in ψ_i given a finite domain (or a constraint set) D over $free(\psi_i)$
    - $\theta_D = \bigcup_{\boldsymbol{d} \in D}\{\bigcup_{t \in \boldsymbol{d}}\{X_d \rightarrow t\}\}$
  - Example: MLN $\Psi = \{(w_i, \psi_i)\}_{i=1}^{2}$
    - Domains
      - $dom(X) = \{alice\}$
      - $dom(P) = \{p_1, p_2\}$
      - $dom(C) = \{ijcai, kr\}$
    - Groundings on the right

$$(10, Presents(alice, p_1, ijcai) \Rightarrow Attends(alice, ijcai))$$
$$(10, Presents(alice, p_1, kr) \Rightarrow Attends(alice, kr))$$
$$(10, Presents(alice, p_2, ijcai) \Rightarrow Attends(alice, ijcai))$$
$$(10, Presents(alice, p_2, kr) \Rightarrow Attends(alice, kr))$$
$$(3.75, Publishes(alice, ijcai) \wedge FarAway(ijcai) \Rightarrow Attends(alice, ijcai))$$
$$(3.75, Publishes(alice, kr) \wedge FarAway(kr) \Rightarrow Attends(alice, kr))$$

$$10\ Presents(X, P, C) \Rightarrow Attends(X, C)$$

$$3.75\ Publishes(X, C) \wedge FarAway(C) \Rightarrow Attends(X, C)$$

MLNS: SEMANTICS

- MLN $\Psi = \{(w_i, \psi_i)\}_{i=1}^{n}$, with $w_i \in \mathbb{R}$, induces a *probability distribution* over all possible interpretations ω (world) of the grounded atoms in Ψ

$$\omega \in \{true, false\}^N$$

  - N = the number of ground atoms in the grounded Ψ
  - Probability of one interpretation ω

$$P(\omega) = \frac{1}{Z} \prod_{i=1}^{n} \exp\left(w_i \cdot n_i(\omega)\right) = \frac{1}{Z} \exp\left(\sum_{i=1}^{n} w_i n_i(\omega)\right)$$

    - $n_i(\omega)$ = number of propositional sentences of ψ_i that evaluate to $true$ given the assignments of ω

MLN: GRAPHICAL REPRESENTATION?

$Presents(X,P,C)$	$Attends(X,C)$	ϕ_1
$false$	$false$	$\exp 10$
$false$	$true$	$\exp 10$
$true$	$false$	$\exp 0$
$true$	$true$	$\exp 10$

- Usually not depicted by a graph but by the logical formulas with their weights to the left
- Since the name invokes Markov networks, which is a graphical model, let us build an analogue:
  - Logical atoms as nodes
  - Edges between atoms whenever atoms occur together in a formula
    - Each ψ_i forms clique in graph
    - Potential function ϕ_i for each clique from weights using $\exp w_i$ for each model and $\exp 0$ otherwise

$10\ Presents(X,P,C) \Rightarrow Attends(X,C)$

$3.75\ Publishes(X,C) \land FarAway(C) \Rightarrow Attends(X,C)$

FROM WEIGHTED FORMULAS TO PARFACTORS

$Presents(X,P,C)$	$Attends(X,C)$	ϕ_1
$false$	$false$	$\exp 10$
$false$	$true$	$\exp 10$
$true$	$false$	$\exp 0$
$true$	$true$	$\exp 10$

- MLNs with their logical formulas have the same value w_i for each interpretation the satisfies ψ_i
- Allowing for different values for each interpretation, i.e., arbitrary distributions in potential functions

→ Set of parfactors

  - Parfactor: Factor (potential function) whose arguments are parameterised with logical variables
  - An MLN can be translated into a set of parfactors and vice versa [Van den Broeck 13]

$10\ Presents(X,P,C) \Rightarrow Attends(X,C)$

$3.75\ Publishes(X,C) \land FarAway(C) \Rightarrow Attends(X,C)$

SEMANTICS

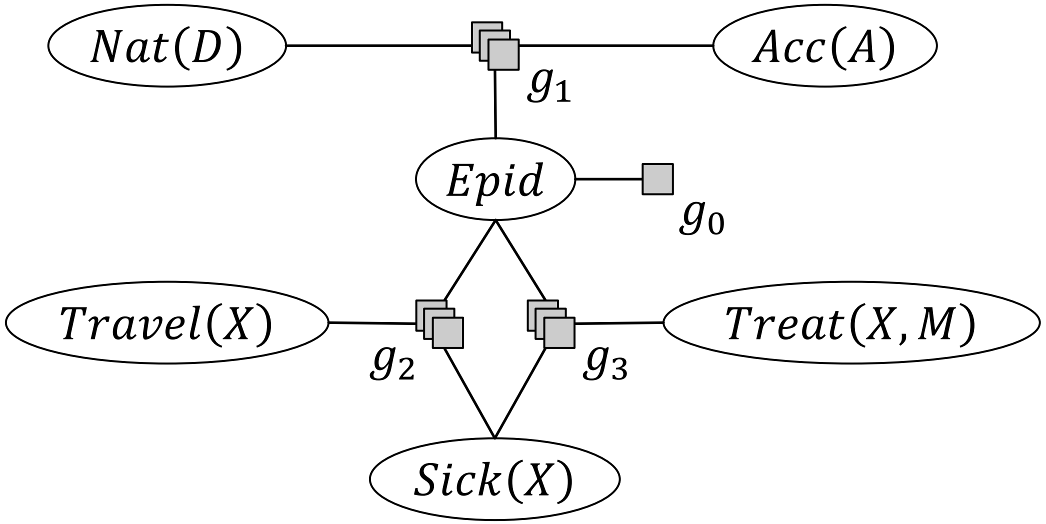
- Distribution semantics (aka grounding or Herbrand semantics) [Sato 95]
    - Completely define discrete joint distribution by factorisation
  - Probabilistic extensions to Datalog [Fuhr 95]
  - Relational Bayesian networks [Jaeger 97]
  - Bayesian Logic Programming [Milch et al. 05], ProbLog [De Raedt et al. 07]
  - Parfactor models [Poole 03, Taghipour et al. 13, Braun & Möller 18, G et al. 19]
  - Markov logic networks (MLNs) [Richardson & Domingos 06]
- Probabilistic Soft Logic (PSL) [Bach et al. 17]
  - Define density function using log-linear model
- Maximum entropy semantics [Thimm et al. 10]
  - Partial specification of discrete joint with "uniform completion"

INFERENCE PROBLEMS WITH AND WITHOUT EVIDENCE

$10\ Presents(X, P, C) \Rightarrow Attends(X, C)$

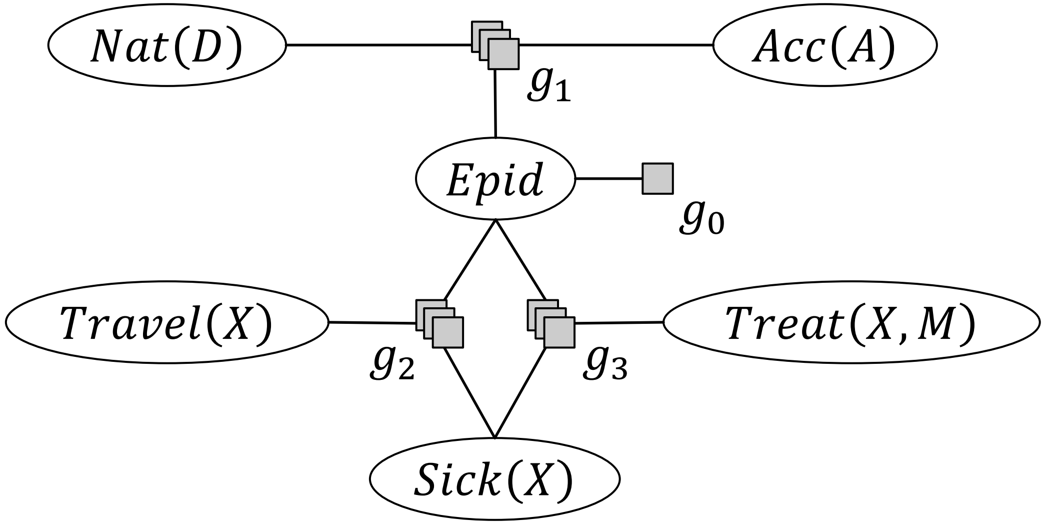
$3.75\ Publishes(X, C) \wedge FarAway(C) \Rightarrow Attends(X, C)$

- Query answering problem given a model:
  - Probability of events
    - E.g., $P(Att(eve, kr) = true), P(Epid = true)$
  - Conditional (marginal) probability distributions
    - E.g., $P(Att(ev, kr) | FarAway(kr)), P(Epid | sick(alice), sick(eve))$
  - Assignment queries:
    - Most probable states of random variables
    - Most-probable explanation (MPE), Maximum a posteriori (MAP)
- Lifted inference:
  Work with representatives for exchangeable random variables [Niepert & van den Broeck 14]
  - Avoid grounding for as long as possible



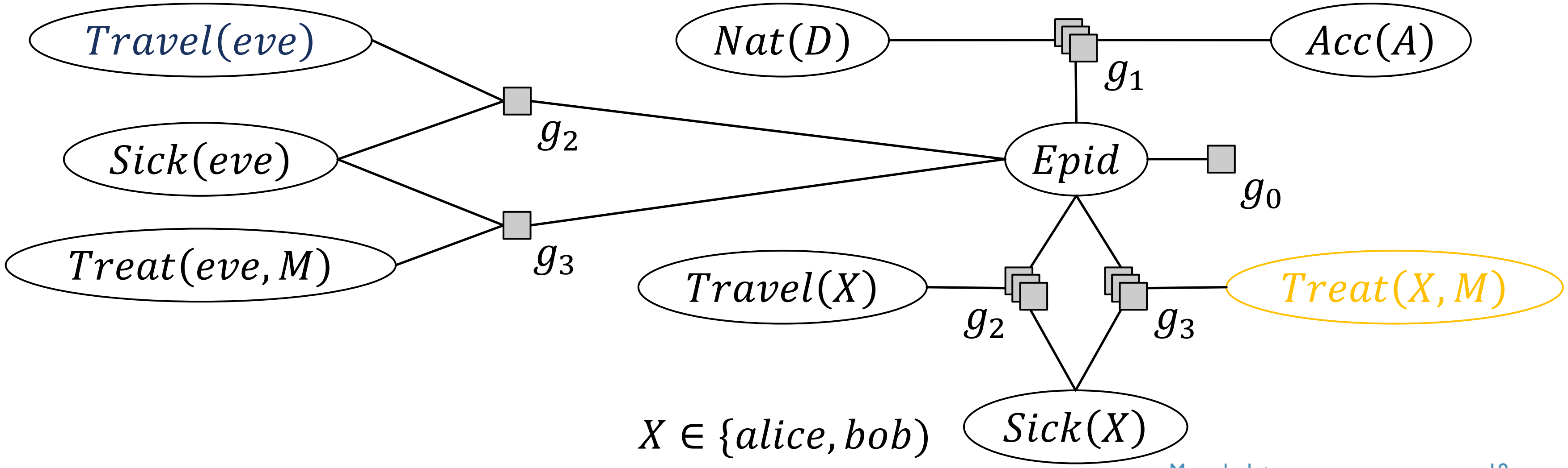
QA IN PARFACTOR MODELS: LIFTED VARIABLE ELIMINATION (LVE)

- Eliminate all variables not appearing in query
  - [Poole 03, de Salvo Braz et al. 05, 06, Milch et al. 08, Taghipour et al. 13, 13a, Braun & Möller 18]
- Lifted summing out
  - Sum out *representative* instance as in propositional variable elimination
  - Exponentiate result for exchangeable instances
- Correctness: Equivalent ground operation
  - Each instance is summed out
  - Result: factor f that is identical for all instance
  - Multiplying indistinguishable results
    → exponentiation of one representative f



QA: LVE IN DETAIL

- Eliminate $Treat(X, M)$
  - Appears in only one g: g_3
  - Contains all logical variables of g_3: X, M
  - For each X constant: the same number of M constants
  - ✓ Preconditions of lifted summing out fulfilled, lifted summing out possible

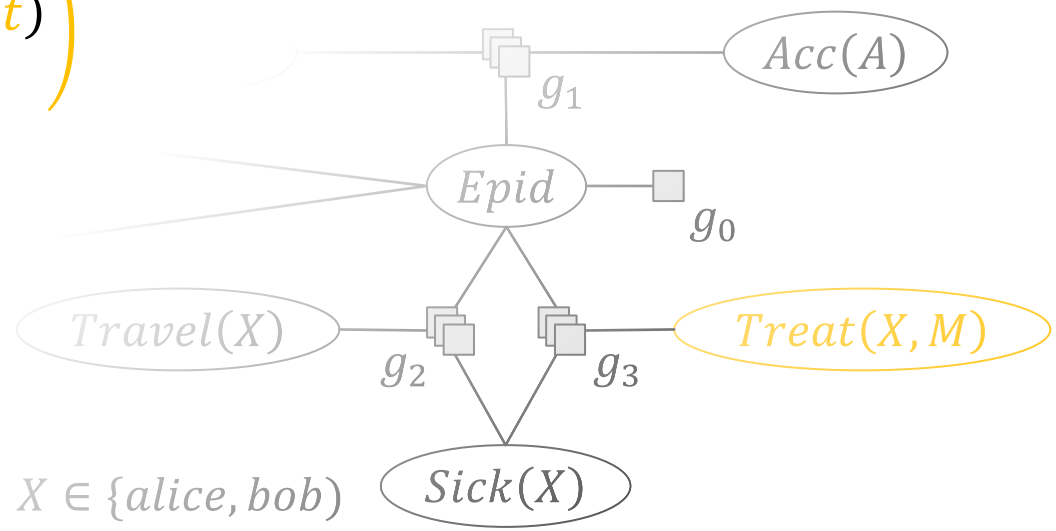


Poole 03

LVE IN DETAIL: LIFTED SUMMING OUT

- Eliminate $Treat(X, M)$ by lifted summing out
  1. Sum out representative

$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X,M) = t) \right)^{\#M|X}$$



Poole 03

LVE IN DETAIL: LIFTED SUMMING OUT

$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X,M) = t) \right)^{\#M|X}$$

$Epid$	$Sick(X)$	$Treat(X,M)$	g_3
$false$	$false$	$false$	9
$false$	$false$	$true$	1
$false$	$true$	$false$	6
$false$	$true$	$true$	3
$true$	$false$	$false$	7
$true$	$false$	$true$	5
$true$	$true$	$false$	4
$true$	$true$	$true$	8

+

$Epid$	$Sick(X)$	Σ
$false$	$false$	10
$false$	$true$	9
$true$	$false$	12
$true$	$true$	12

^

$Epid$	$Sick(X)$	^
$false$	$false$	10^2
$false$	$true$	9^2
$true$	$false$	12^2
$true$	$true$	12^2

Poole 03

SYMMETRIES WITHIN

- Assume four epidemics with identical characteristics
  - $Epid_1, Epid_2, Epid_3, Epid_4$
  - Reasonable to model the epidemics such that it does not matter which $Epid$ variables specifically are $true$ or $false$, i.e., they are interchangeable
  - CRV: $\#_E[Epid(E)]$
    - Range values

      [0,4], [1,3], [2,2], [3,1], [4,0]

      1 4 6 4 1

      how many assignments encoded
  - $g' = \phi(\#_E[Epid(E)])$

$\#_{true}$ $\#_{false}$

$\#_E[Epid(E)]$	ϕ'
[0,4]	8
[1,3]	6
[2,2]	4
[3,1]	2
[4,0]	0

$Epid_1$	$Epid_2$	$Epid_3$	$Epid_4$	ϕ
$false$	$false$	$false$	$false$	8
$false$	$false$	$false$	$true$	6
$false$	$false$	$true$	$false$	6
$false$	$false$	$true$	$true$	4
$false$	$true$	$false$	$false$	6
$false$	$true$	$false$	$true$	4
$false$	$true$	$true$	$false$	4
$false$	$true$	$true$	$true$	2
$true$	$false$	$false$	$false$	6
$true$	$false$	$false$	$true$	4
$true$	$false$	$true$	$false$	4
$true$	$false$	$true$	$true$	2
$true$	$true$	$false$	$false$	4
$true$	$true$	$false$	$true$	2
$true$	$true$	$true$	$false$	2
$true$	$true$	$true$	$true$	0

Milch 08

CRVS CONTINUED

- (P)CRV $\#_X[A_{|C}]$ with
  - $m = |ran(A)|$ (number of buckets)
  - $n = \sum_{i=1}^{m} n_i = |gr(A_{|\pi_X(C)})|$ (number of instances to distribute into buckets)
- Instead of m^n mappings in the ground factor, the counted factor has

$$\binom{n+m-1}{n-1}$$

mappings
  - Upper bound of range size of a CRV:

$$\binom{n+m-1}{n-1} \leq n^m$$

- Range of a (P)CRV = space of histograms fulfilling the conditions on the histograms
  - (All possible ways of distributing n interchangeable instances into m buckets)
- Single histogram encodes several interchangeable assignments at once
  - Given by multinomial coefficient $Mul(h)$

$$Mul(h) = \frac{n!}{\prod_{i=1}^{m} n_i!}$$

    - If $m = 2$, binomial coefficient:

$$\binom{n}{n_1} = \frac{n!}{(n-n_1)!\,n_1!} = \frac{n!}{n_2!\,n_1!}$$

Milch 08

SELECTED INFERENCE ALGORITHMS FOR MLNS AND PARFACTOR MODELS

Static

- Exact:
  - Lifted Variable Elimination [Poole 03]
  - Lifted Junction Tree [Braun & Möller 18]
  - First-order Knowledge Compilation [Van den Broeck et al. 11]
  - Probabilistic Theorem Proving [Gogate & Domingos 11]
  - CRANE [Dilkas & Belle 23]
  - FAST WFOMC [van Bremen & Kuželka 20]
- Approximative
  - Lifted Belief Propagation [Ahmadi et al. 13]
  - Weighted First-order Model Sampling [Wang et al. 24]
  - Lifted MCMC [Niepert 12]
  - Lifted Importance Sampling [Gogate et al. 11]

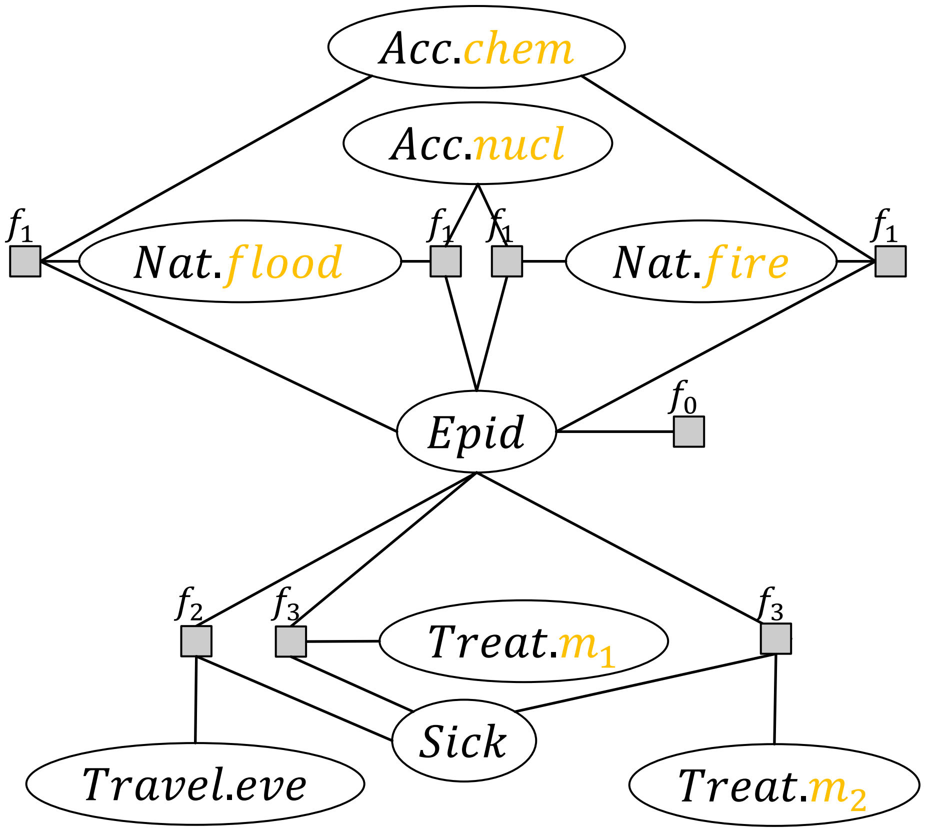
Temporal

- Exact
  - Lifted Dynamic Junction Tree Algorithm [G et al. 19]
- Approximate
  - Lifted Factored Frontier Algorithm [Ahmadi et al. 13]
  - Online Inference Algorithms [Geier & Biundo 11, Papai et al. 12]

OBTAINING A COMPRESSED REPRESENTATION

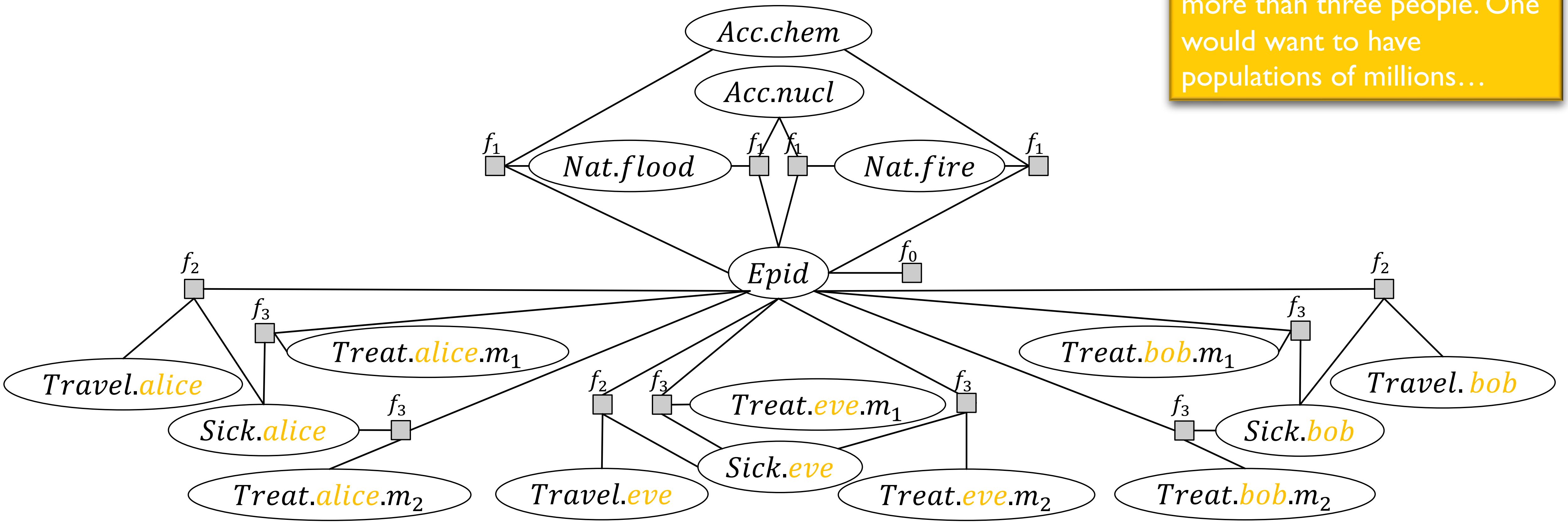
INTRODUCTION

PROBLEM: ADDING OBJECTS AND RELATIONS IN PROPOSITIONAL FORMALISMS CLUNKY…



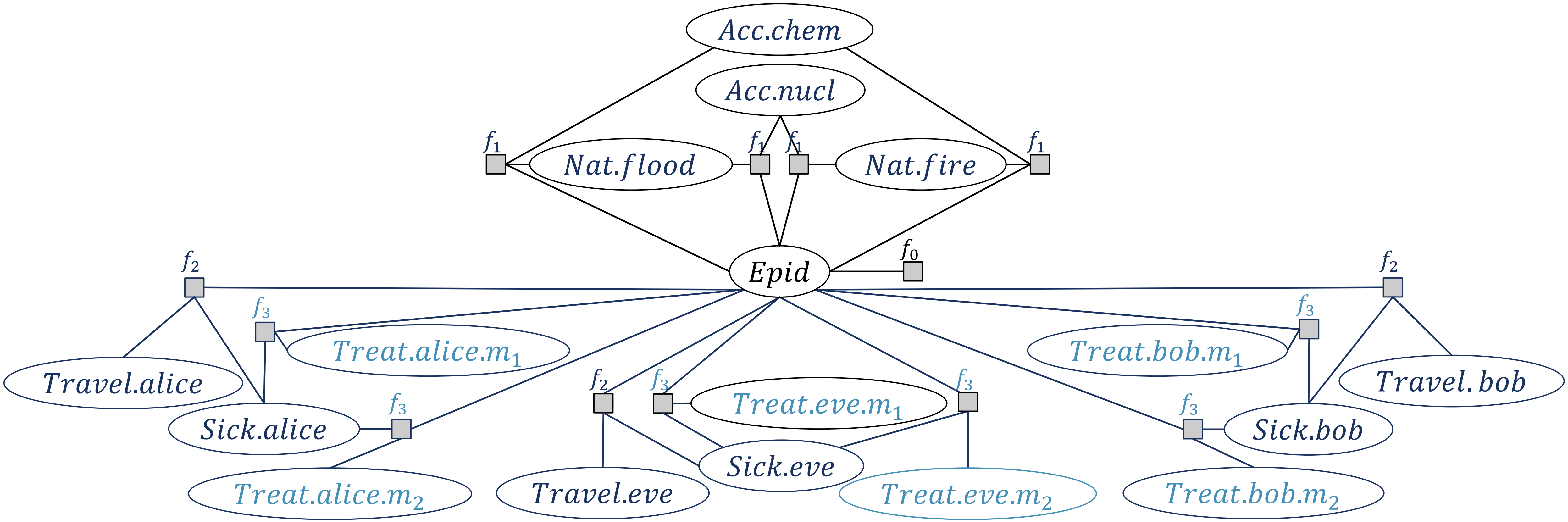
PROBLEM: … AND MODELS EXPLODE WITH THEM

And an epidemic depends on more than three people. One would want to have populations of millions…



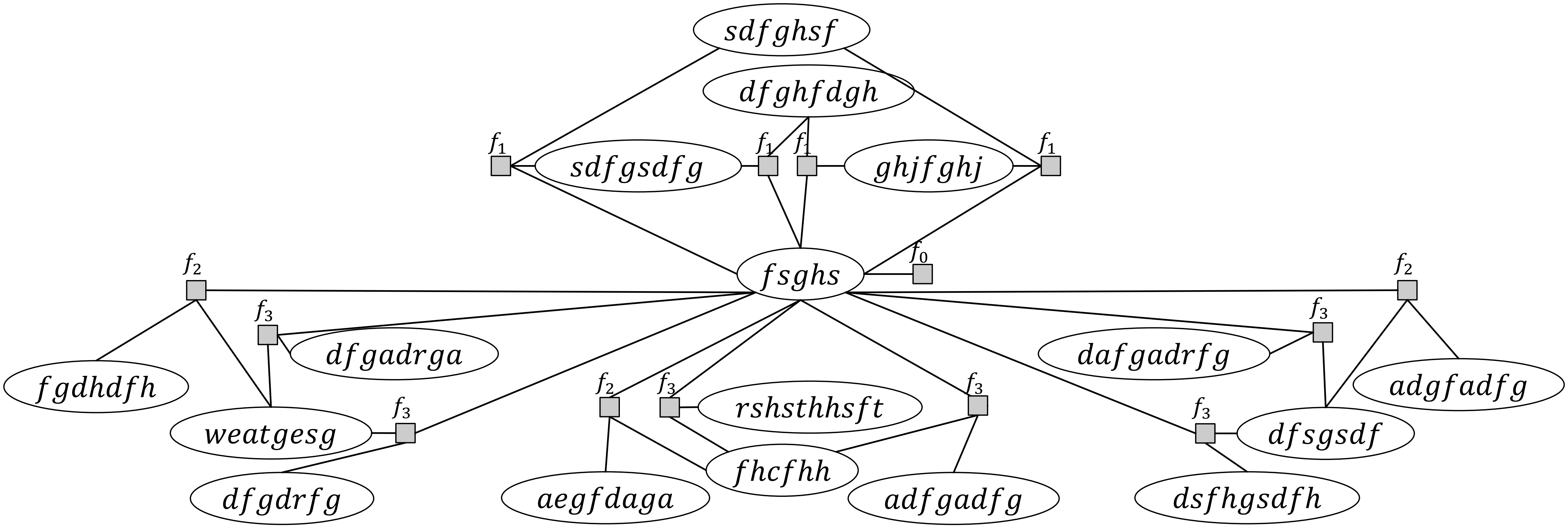
$13 \cdot 2^3 + 2 = 104$ **entries in 14 factors, 17 variables**

PROPOSITIONAL → FIRST-ORDER VIEW



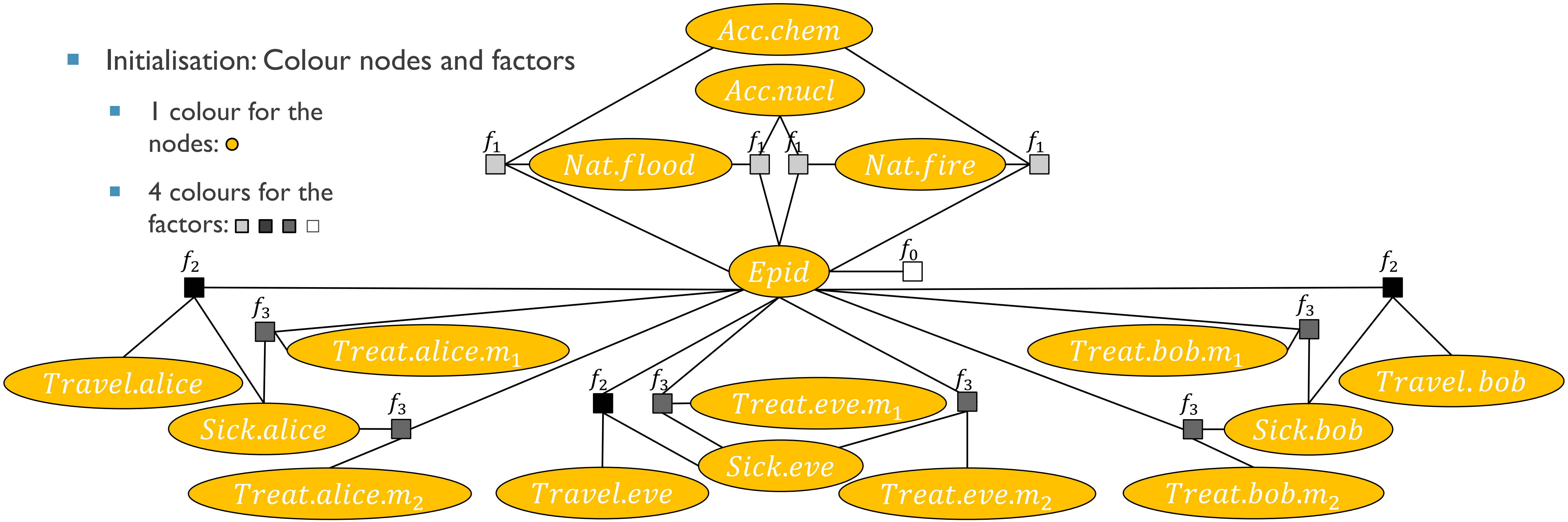
Symmetries in graph / relations in scenario

PROBLEM: … LEARNED FROM DATA MORE LIKE THIS



COMPRESSION

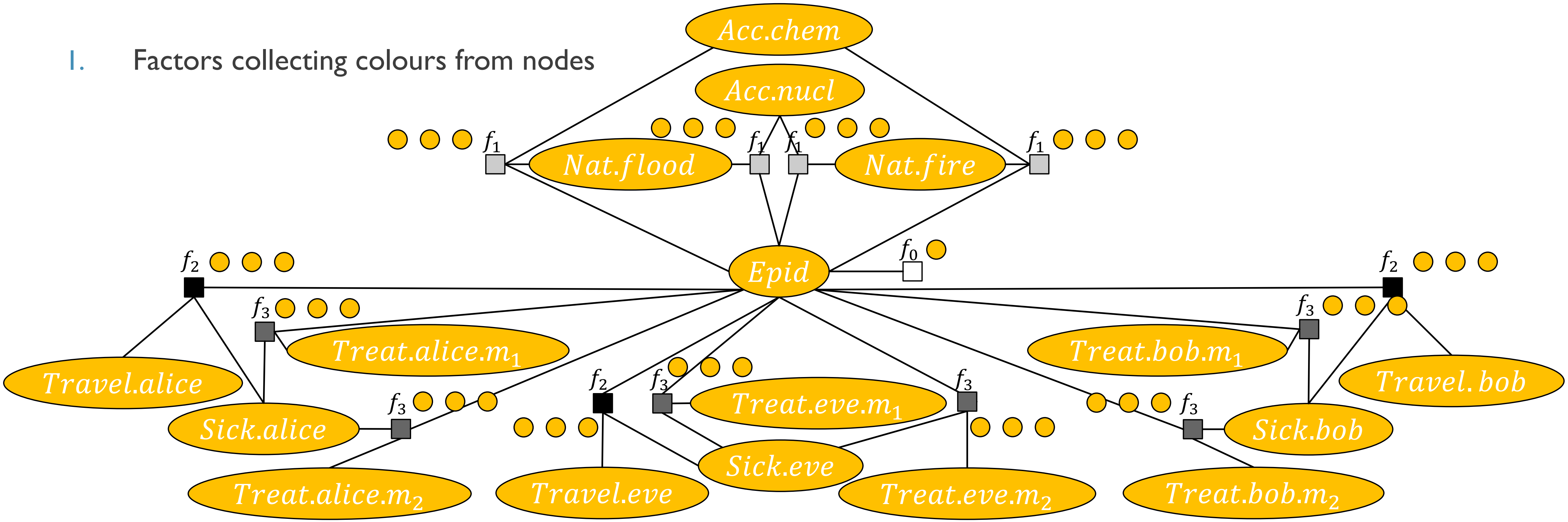
- Initialisation: Colour nodes and factors
  - 1 colour for the nodes: ●
  - 4 colours for the factors: ■ ■ ■ □



Ahmadi et al. 13

COMPRESSION

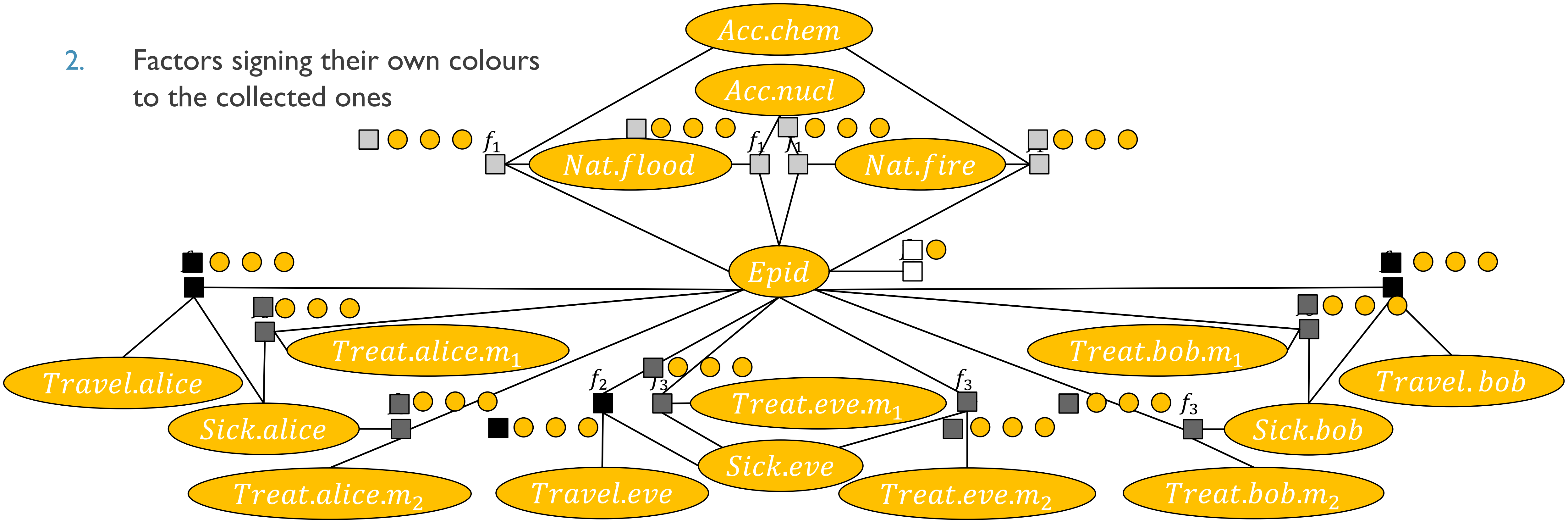
1. Factors collecting colours from nodes



Ahmadi et al. 13

COMPRESSION

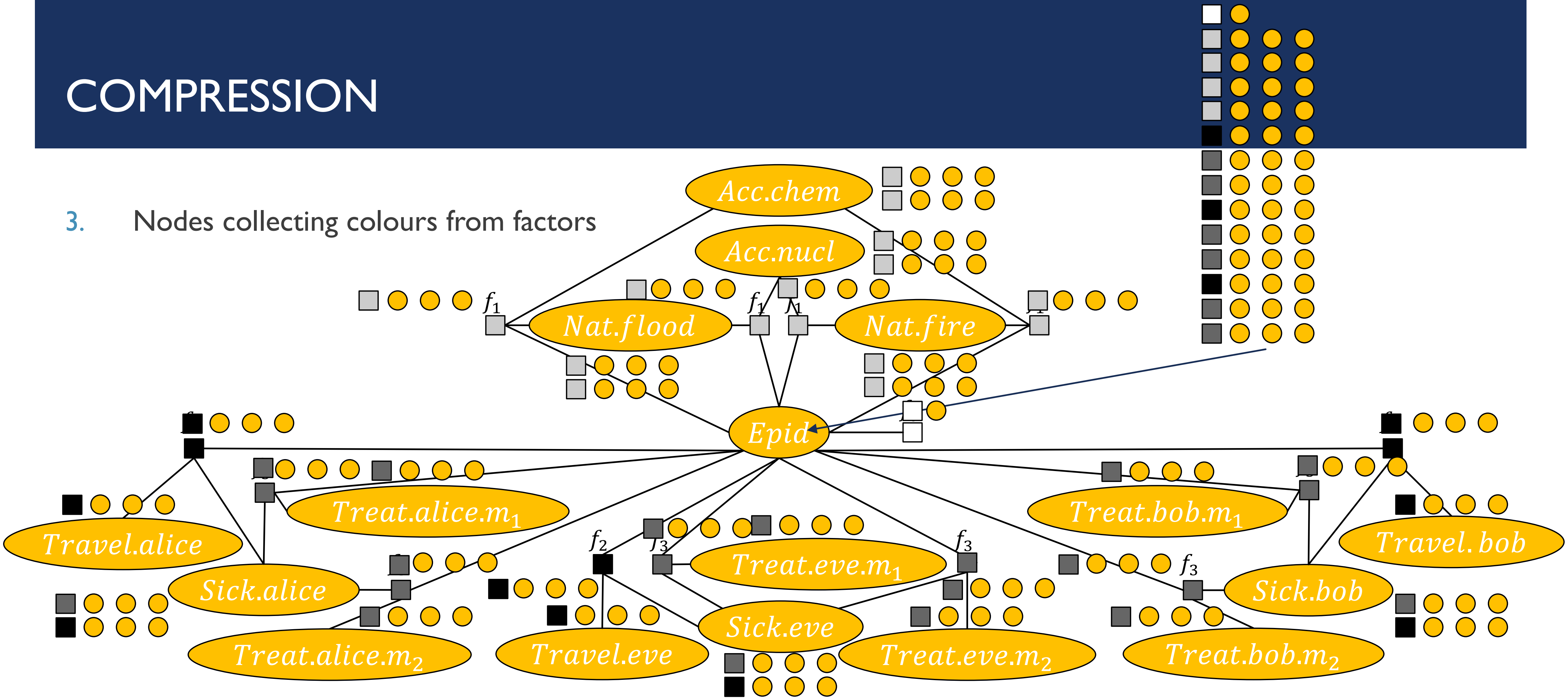
2. Factors signing their own colours to the collected ones



Ahmadi et al. 13

COMPRESSION

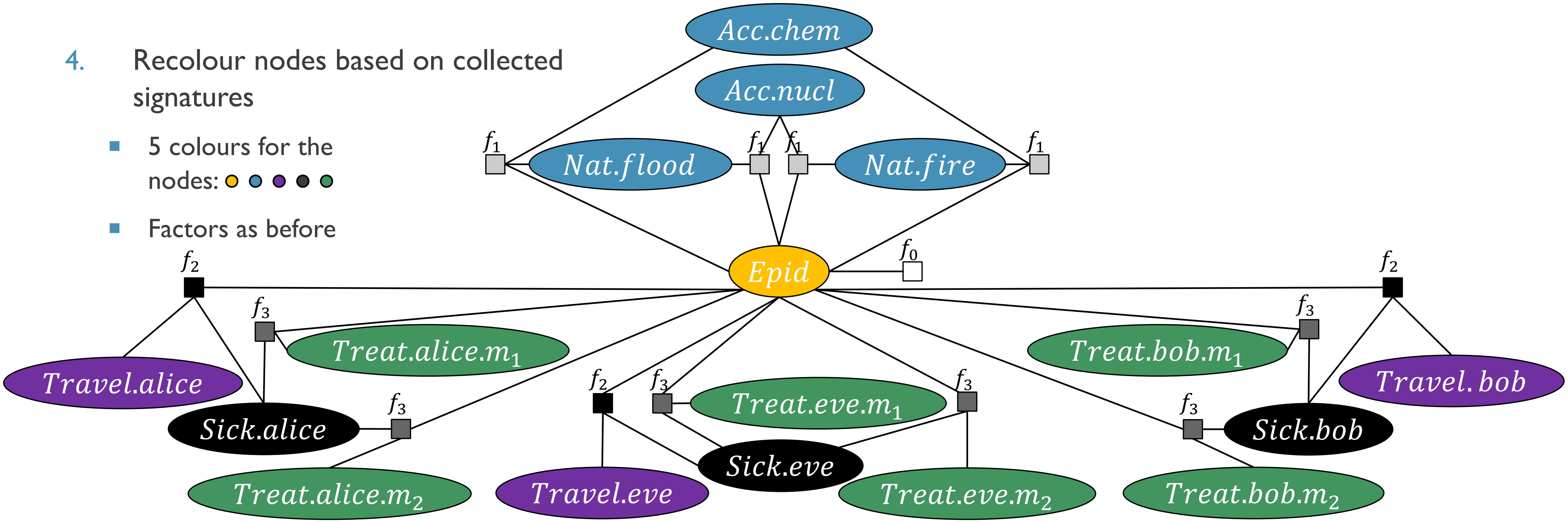
3. Nodes collecting colours from factors



Ahmadi et al. 13

COMPRESSION

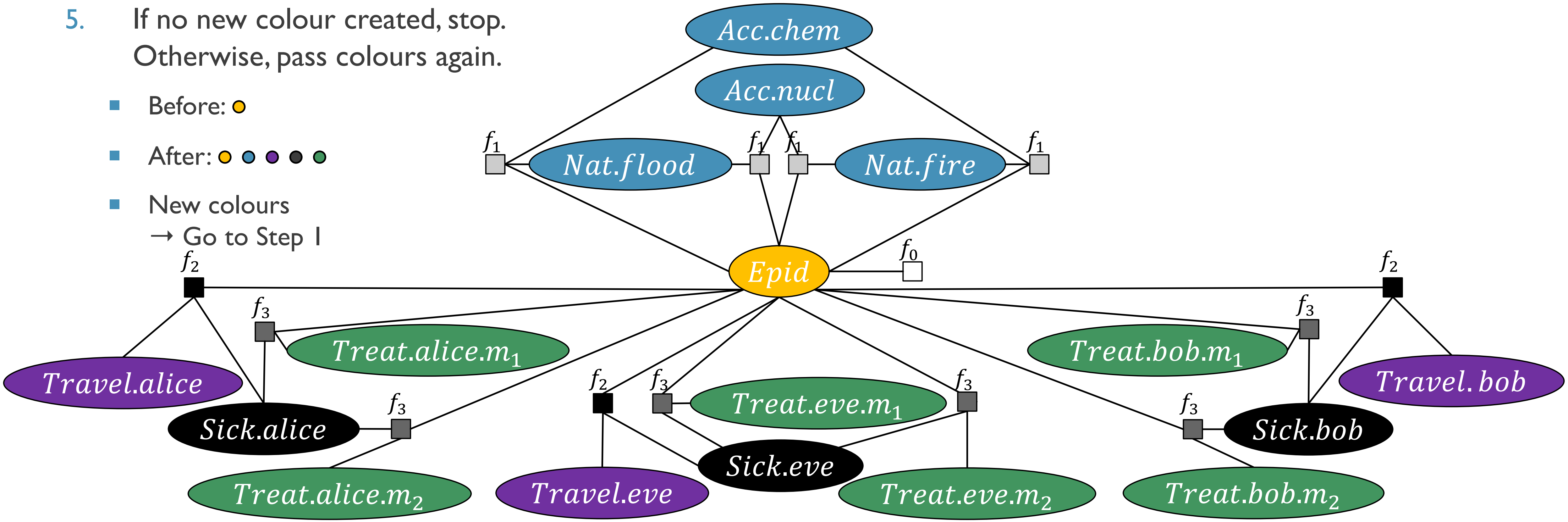
4. Recolour nodes based on collected signatures
   - 5 colours for the nodes: 🟡 🔵 🟣 ⚫ 🟢
   - Factors as before



Ahmadi et al. 13

COMPRESSION

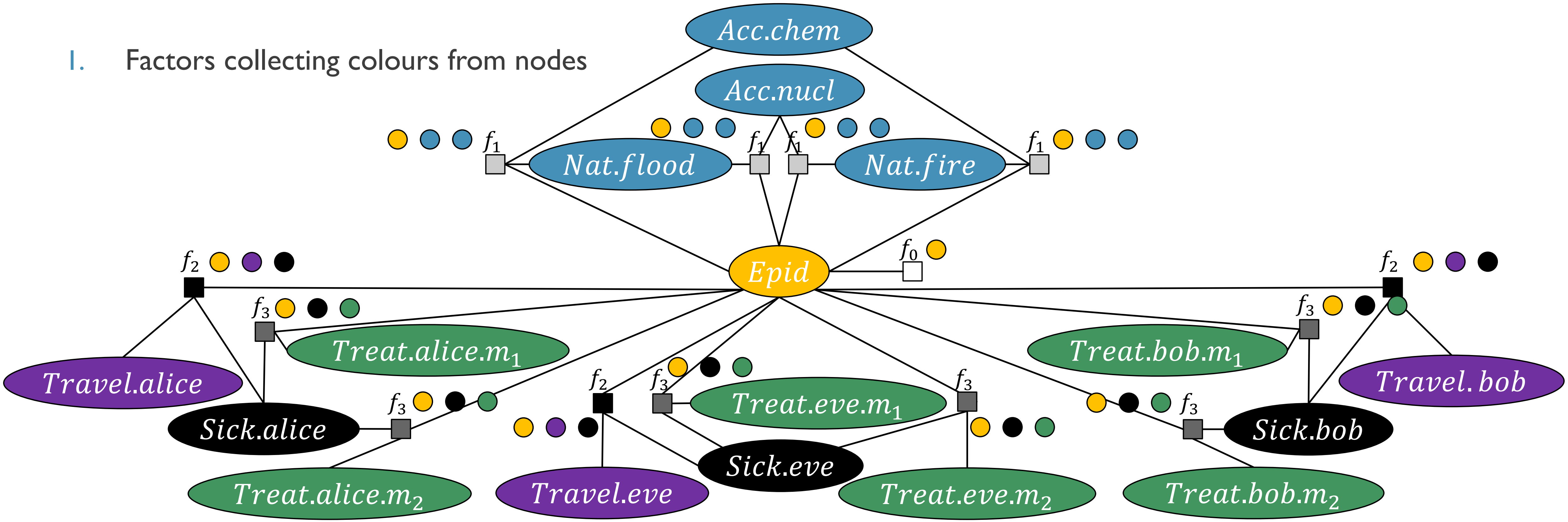
5. If no new colour created, stop. Otherwise, pass colours again.
   - Before:
   - After:
   - New colours → Go to Step 1



Ahmadi et al. 13

COMPRESSION

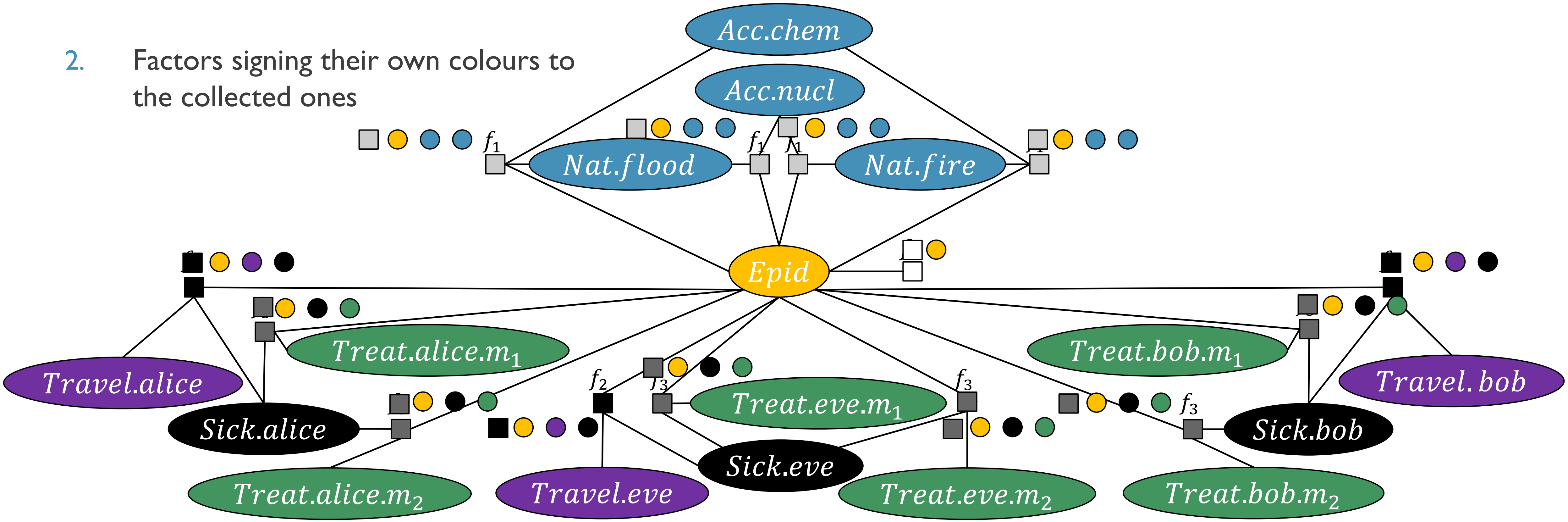
1. Factors collecting colours from nodes



Ahmadi et al. 13

COMPRESSION

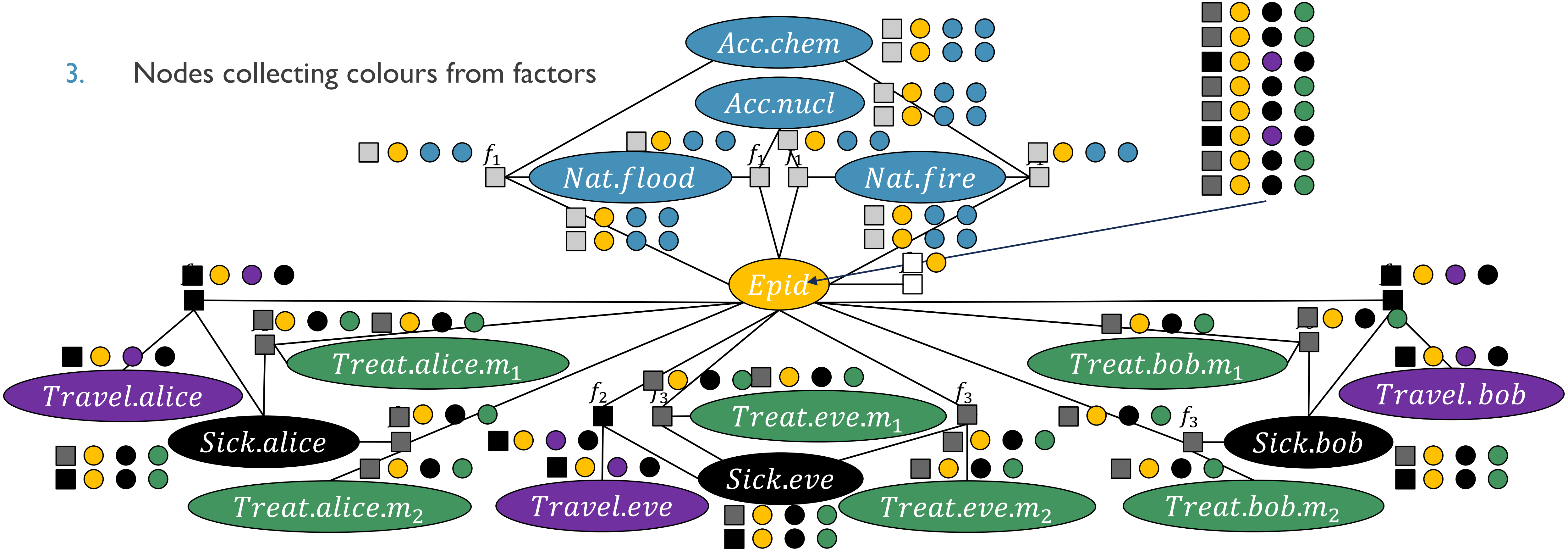
2. Factors signing their own colours to the collected ones



Ahmadi et al. 13

COMPRESSION

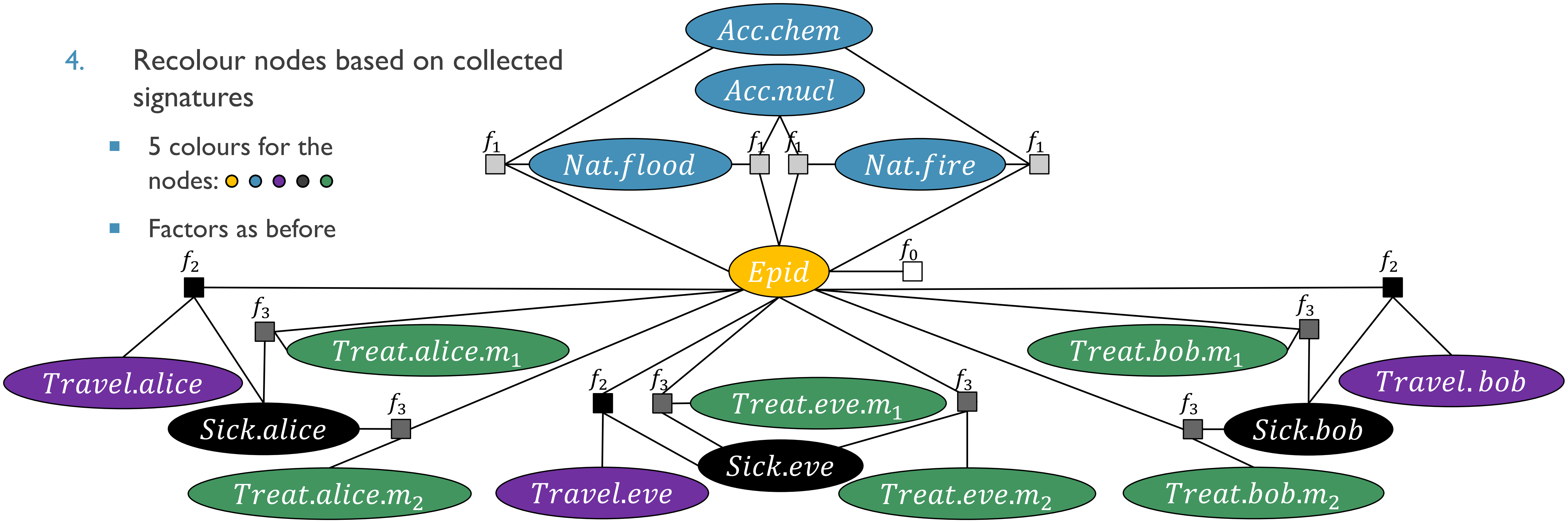
3. Nodes collecting colours from factors



Ahmadi et al. 13

COMPRESSION

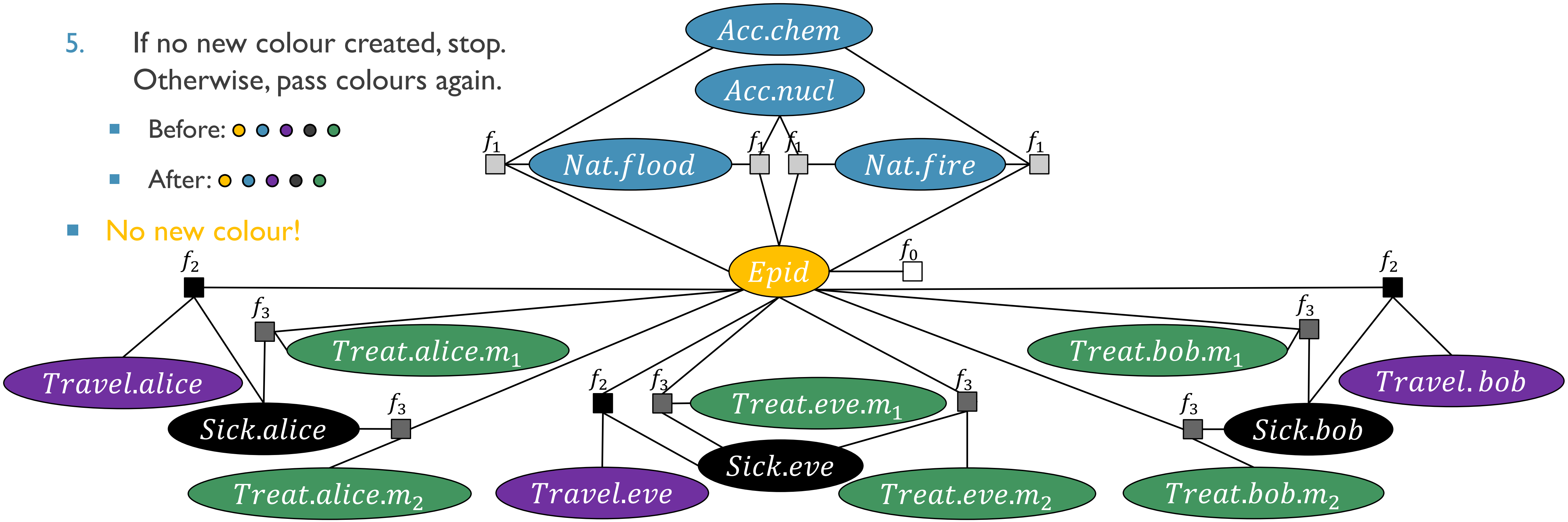
4. Recolour nodes based on collected signatures
   - 5 colours for the nodes: ● ● ● ● ●
   - Factors as before



Ahmadi et al. 13

COMPRESSION

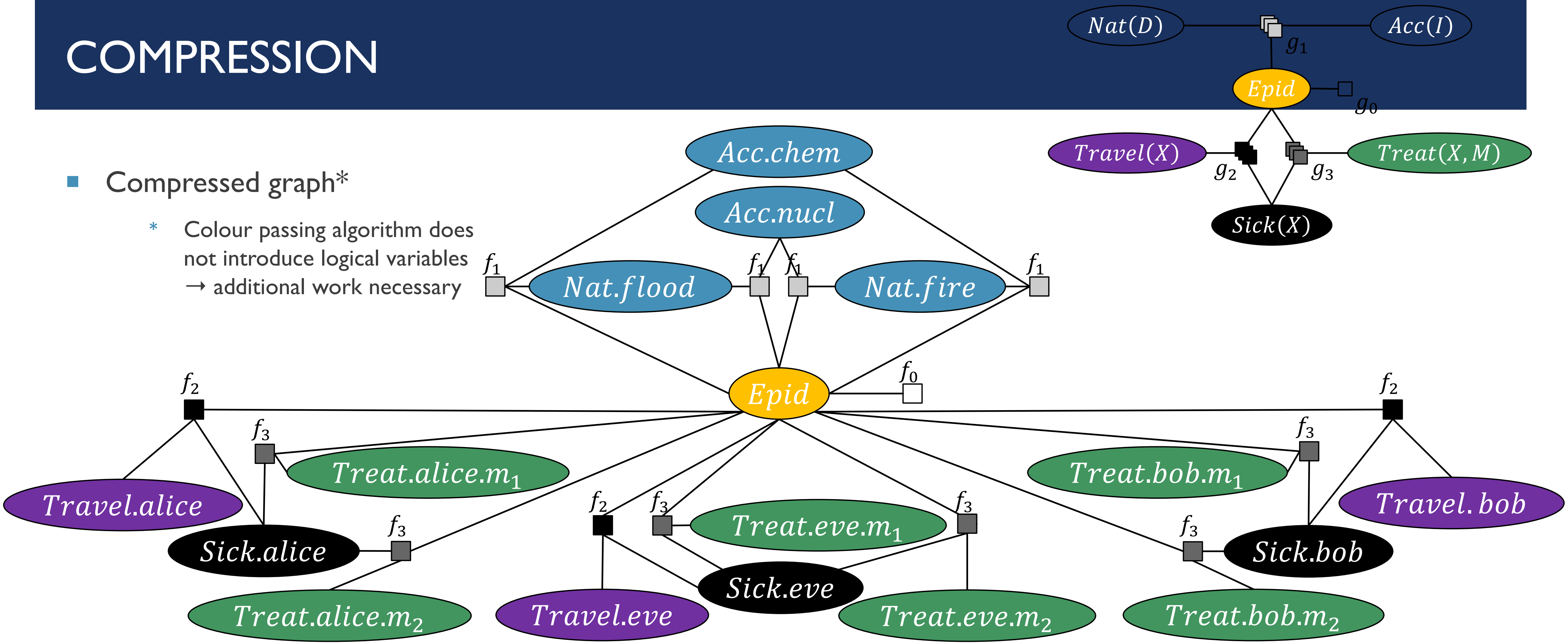
5. If no new colour created, stop. Otherwise, pass colours again.
   - Before: ● ● ● ● ●
   - After: ● ● ● ● ●
- No new colour!



Ahmadi et al. 13

COMPRESSION

- Compressed graph*
  - * Colour passing algorithm does not introduce logical variables → additional work necessary



Ahmadi et al. 13

AGENDA

1. Introduction to relational Models [Marcel]
2. Compressing probabilistic relational models [Malte]
   - Advancing the state of the art to obtain an exact compressed representation
   - Approximating a compressed representation with known error bounds
   - Handling unknown factors
3. Application: Lifted causal inference [Malte]
4. Summary [Marcel]

BIBLIOGRAPHY

ORDERED ALPHABETICALLY

BIBLIOGRAPHY

- Ahmadi et al. 13
  Babak Ahmadi, Kristian Kersting, Martin Mladenov, and Sriraam Natarajan. Exploiting Symmetries for Scaling Loopy Belief Propagation and Relational Training. In Machine Learning. 92(1):91-132, 2013.
- AMAI, Russel/Norvig
  Russell & Norvig: Artificial Intelligence: A Modern Approach. 2020
- Bach et al. (2017)
  Stephen H. Bach, Matthias Broecheler, Bert Huang, and Lise Getoor: Hinge-Loss Markov Random Fields and Probabilistic Soft Logic. In Machine Learning Journal, 2017.
- Braun & Möller (2018)
  Tanya Braun and Ralf Möller: Parameterised Queries and Lifted Query Answering. In IJCAI-18 Proceedings of the 27th International Joint Conference on Artificial Intelligence, 2018.
- Braun (2020)
  Tanya Braun: Rescued from a Sea of Queries: Exact Inference in Probabilistic Relational Models. PhD thesis, 2020.
- De Raedt et al. (2007)
  Luc De Raedt, Angelika Kimmig, and Hannu Toivonen: ProbLog – A Probabilistic Prolog and its Application in Link Discovery, In IJCAI-07 Proceedings of the 20th International Joint Conference on Artificial Intelligence, 2007.
- De Salvo Braz et al. (2005)
  Rodrigo de Salvo Braz, Eyal Amir, and Dan Roth: Lifted First-order Probabilistic Inference. In IJCAI-05 Proceedings of the 19th International Joint Conference on Artificial Intelligence, 2005.

BIBLIOGRAPHY

- De Salvo Braz et al. (2006)
  Rodrigo de Salvo Braz, Eyal Amir, and Dan Roth: MPE and Partial Inversion in Lifted Probabilistic Variable Elimination. In AAAI-06 Proceedings of the 21st Conference on Artificial Intelligence, 2006.
- Dilkas & Belle (2023)
  Paulius Dilkas and Vaishak Belle: Synthesising Recursive Functions for First-Order Model Counting: Challenges, Progress, and Conjectures. In KR-23 Proceedings of the 20th International Conference on Principles of Knowledge Representation and Reasoning, 2023.
- Fierens et al. (2015)
  Daan Fierens, Guy Van den Broeck, Joris Renkens, Dimitar Shterionov, Bernd Gutmann, Ingo Thon, Gerda Janssens, and Luc De Raedt: Inference and learning in probabilistic logic programs using weighted Boolean formulas. In Theory and Practice of Logic Programming, 2015
- Fuhr (1995)
  Norbert Fuhr: Probabilistic Datalog - A Logic for Powerful Retrieval Methods. In SIGIR-95 Proceedings of the 18th Annual International ACM SIGIR Conference on Research and Development in Information Retrieval, 1995.
- G et al. (2019)
  Marcel Gehrke, Tanya Braun, and Ralf Möller. Relational Forward Backward Algorithm for Multiple Queries. In FLAIRS-19 Proceedings of the 32nd International Florida Artificial Intelligence Research Society Conference, 2019.
- Geier & Biundo (2011)
  Thomas Geier and Susanne Biundo: Approximate Online Inference for Dynamic Markov Logic Networks. In ICTAI-11 Proceedings of the IEEE 23rd International Conference on Tools with Artificial Intelligence, 2011.

BIBLIOGRAPHY

- Gogate & Domingos (2011)
  Vibhav Gogate and Pedro Domingos: Probabilistic Theorem Proving. In UAI-11 Proceedings of the 27th Conference on Uncertainty in Artificial Intelligence, 2011.
- Jaeger (1997)
  Manfred Jaeger: Relational Bayesian Networks. In UAI-97 Proceedings of the 13th Conference on Uncertainty in Artificial Intelligence, 1997.
- Milch et al. (2005)
  Brian Milch, Bhaskara Marthi, Stuart Russell, David Sontag, Daniel L. Long, and Andrey Kolobov: BLOG: Probabilistic Models with Unknown Objects. In IJCAI-05 Proceedings of the 19th International Joint Conference on Artificial Intelligence, 2005.
- Milch et al. (2008)
  Brian Milch, Luke S. Zettelmoyer, Kristian Kersting, Michael Haimes, and Leslie Pack Kaelbling: Lifted Probabilistic Inference with Counting Formulas. In AAAI-08 Proceedings of the 23rd AAAI Conference on Artificial Intelligence, 2008.
- Niepert (2012)
  Mathias Niepert: Markov Chains on Orbits of Permutation Groups. In UAI-12 Proceedings of the 28th Conference on Uncertainty in Artificial Intelligence, 2012

BIBLIOGRAPHY

- Niepert & Van den Broeck (2014)
  Mathias Niepert and Guy Van den Broeck. Tractability through Exchangeability: A New Perspective on Efficient Probabilistic Inference. In AAAI-14 Proceedings of the 28th AAAI Conference on Artificial Intelligence, 2014.
- Papai et al. (2012)
  Tivadar Papai, Henry Kautz, and Daniel Stefankovic: Slice Normalized Dynamic Markov Logic Networks. In NeurIPS-12 Proceedings of the Advances in Neural Information Processing Systems 25, 2012.
- Poole (2003)
  David Poole: First-order Probabilistic Inference. In IJCAI-03 Proceedings of the 18th International Joint Conference on Artificial Intelligence, 2003.
- Richardson & Domingos (2006)
  Matthew Richardson and Pedro Domingos. Markov Logic Networks. In Machine Learning Journal, 2006.
- Sato (1995)
  Taisuke Sato: A Statistical Learning Method for Logic Programs with Distribution Semantics. In Proceedings of the 12th International Conference on Logic Programming, 1995.

BIBLIOGRAPHY

*PRMs are a true backbone of AI, and this tutorial emphasized only some central topics. We definitely did not cite all publications relevant to the whole field of PRMs here. We would like to thank all our colleagues for making their slides available (see some of the references to papers for respective credits). Slides or parts of it are almost always modified.

- Taghipour et al. (2013)
  Nima Taghipour, Daan Fierens, Guy Van den Broeck, Jesse Davis, and Hendrik Blockeel. Completeness Results for Lifted Variable Elimination. In AISTATS-13 Proceedings of the 16th International Conference on Artificial Intelligence and Statistics, 2013.
- Taghipour et al. (2013a)
  Nima Taghipour, Daan Fierens, Jesse Davis, and Hendrik Blockeel. Lifted Variable Elimination: Decoupling the Operators from the Constraint Language. Journal of Artificial Intelligence Research, 47(1):393–439, 2013.
- Thimm et al. (2010)
  Matthias Thimm, Marc Finthammer, Sebastian Loh, Gebriele Kern-Isberner, and Christoph Beierle: A System for Relational Probabilistic Reasoning on Maximum Entropy. In FLAIRS-23 Proceedings of the 23rd International Florida Artificial Intelligence Research Society Conference, 2010.
- van Bremen & Kuželka (2020)
  Timothy van Bremen and Ondřej Kuželka: Approximate weighted first-order model counting: Exploiting fast approximate model counters and symmetry. In IJCAI-20 Proceedings of the 29th International Joint Conference on Artificial Intelligence, 2020.
- Van den Broeck et al. (2011)
  Guy Van den Broeck, Nima Taghipour, Wannes Meert, Jesse Davis, and Luc De Raedt: Lifted Probabilistic Inference by First-order Knowledge Compilation. In IJCAI-11 Proceedings of the 22nd International Joint Conference on Artificial Intelligence, 2011.
- Wang et al. (2024)
  Yuanhong Wang, Juhua Pu, Yuyi Wang, Ondřej Kuželka: Lifted algorithms for symmetric weighted first-order model sampling. In AI, 2011.