

*Proceedings of the
16th International Conference on
Technology in Mathematics Teaching*

ICTMT 16



National and Kapodistrian
UNIVERSITY OF ATHENS

ICTMT in the Era of Societal and Digital Transformations

Department of Secondary Education, School of Philosophy,
National & Kapodistrian University of Athens
Athens, Greece
June 7–10, 2023
Editors
Dimitris Diamantidis
Myrto Karavakou
Marianthi Grizioti
Chronis Kynigos

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ABOUT THESE PROCEEDINGS

Editors

Dimitris Diamantidis, Myrto Karavakou, Marianthi Grizioti, & Chronis Kynigos

Title

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Introduction

The ‘International Conference ICTMT 16, was organized and held between 7th and 10th of June, 2023, under the responsibility, of the National and Kapodistrian University of Athens, in the Central building of the National Kapodistrian University of Athens and the Maraslio Didaskalio Venue, in Athens.

The School’s of Philosophy, Educational Studies’ Department, with the Educational Technology Lab in the foreground took charge of the organization, staffing the local organizing committee (LOC) of the conference.

For the conference theme, the conference international organizing committee adopted a **transformational** approach to Mathematics Education, welcoming perspectives trying to make sense of mathematics education in an era of intense, unpredictable, complex, ill-defined societal and digital transformation. How can digital technologies become an integral tool in the ways rationality and mathematical thinking is cultivated in schools transforming from an academic to a citizenship paradigm? How can we use digital media to instill rationality and mathematical understanding in individual and collective citizenship? What is the value of mathematics in the so-called 21st century skills movement in education focusing on trans-disciplinarity and the grappling with socio-scientific issues? What can STEM and STEAM bring to mathematics education in this context? How can programming, computational thinking and modeling bring added educational value to the teaching and learning of Mathematics? What kind of educational practices and uses of digital media can we expect to value in the digital transformation era?

Under this frame, the conference was organized in the following topics:

1. Designing technology for a transforming society

for mathematical learning and its assessment in the Digital Transformation Era – a focus on theoretical or actual ‘designs’ with contributions from researchers, industry and teachers.

2. Making sense of educational practice in a transforming society

with and through technology – a focus on the work of teachers and lecturers, where the classroom might be geographically located or mobile. What kind of practice can we envisage having value in the near future? Again, contributions could be both theoretical and practical.

3. Fostering mathematical collaboration and communication

with and through technology – a focus on the communications’ aspect of technology, including assessment strategies. What kinds of communication may promote mathematical thinking and rationality in the Transformation Era.

4. Understanding the value of Analytics and AI in Mathematics Education

how can mathematical understandings support the understanding of analytics and algorithms used in popular media? What kind of user information could be capture-worthy for mathematics education? How can analytics support rather than restrict creativity and agency in meaning-making?

The connection between theory and practice around using digital media for teaching, investigating, and learning mathematics, as a fertile ground to discuss the challenges to come and possible pathways for the next years in mathematics education. In a social environment that changes in multiple directions, what does academic research (theory) gain from the study of practice in domains like the role of teachers and students, the teaching profession, the curriculum design, the learning outcomes concerning competencies, assessment of the learning and teaching procedure, functionalities of novel digital tools, etc.? And does practice benefit from new theoretical approaches, or classical ones implemented innovatively, and how?

The structure of ICTMT16 proceedings follow these four topics.

We owe special thanks to the invited speakers and to all the participants who honored us with their presence and strengthened the scientific works and the products of the conference. We hope this edition lives up to the high level of presentations and scientific discussion that took place during the conference.

Chronis Kynigos

Chair

International Program Committee

Invited Speakers.

Plenary Sessions and Round tables

Nathalie Sinclair, Simon Fraser University.

Title: Aesthetising mathematics education research.

Abstract: A discussion of the historical, colonial roots of the dismissal of aesthetics in current mathematics education research, as well as examples of what an aestheticising of mathematics education might look like, drawing on Nathalie's Sinclair own research with multitouch digital technologies.

Watch the lecture: <https://delos.uoa.gr/opendelos/player?rid=2bce6ca6>

Osama Swidan, Ben Gurion University of the Negev.

Title: Augmented reality: just a big name or a technology that really does promote mathematical thinking?

Abstract: Since its initial development in the 1990s, the use of augmented reality technology has become widespread over a variety of fields and disciplines. The effectiveness of augmented reality technology has been shown in fields such as medicine, engineering, and industry; on the other hand, its effectiveness in mathematics education is still under discussion among researchers. In my talk, I will first review the development of the use of augmented reality technology in teaching and learning mathematics. Afterward, in order to demonstrate the potential of augmented reality in mathematics, I will present some of the latest developments in this field and new augmented reality prototypes. Finally, some augmented reality studies in which I have been involved will be discussed from diverse theoretical perspectives. This will show that augmented reality may not only have practical importance but theoretical importance as well.

Watch the lecture: <https://delos.uoa.gr/opendelos/player?rid=21ff9679>

Title: Designing digital environments in the context of mathematics education.
A Panel Discussion

Chair: Alison Clark Wilson, UCL-Knowledge Lab.

Panelists:

Nick Jakiw, Simon Fraser University.

Chronis Kynigos, National Kapodistrian University of Athens.

Yang Yang, University College London.

Abstract: All digital environments embed aspects of design. All designers of digital environments bring their own perspectives and "principles" to what they do. In mathematics education, this results in very different looking digital environments that can have very different goals and purposes. Our panelists will share their ideas on their design process, alongside providing some insights and reflections on their resulting digital tools, resources and associated tasks.

Watch the panel discussion: <https://delos.uoa.gr/opendelos/player?rid=d2f15207>

Title: The role of Artificial Intelligence and Learning Analytics in transformative mathematics education.

A Panel Discussion

Chair: Manolis Mavrikis, University College London.

The hype of Artificial Intelligence (AI) and Learning Analytics (LA) is increasingly having an influence on how education could be carried out and improved. For example, around the world, there are several multi-million-dollar-funded corporations claiming to be using AI to ‘transform’ education. Despite these efforts, there is not always enough evidence to support these claims. From an academic perspective, most of the research in AI and mathematics education in the past witnessed the emergence of intelligent tutoring systems and intelligent assessment, at times simply digitising poor pedagogical practices. This panel will look into the promises and pitfalls of AI and LA investigating the relevance of this set of emerging technologies with a focus on the theme of the conference, namely – transforming mathematics education. The panelists will present and discuss their research and visions for opening new avenues of research and innovation but also adopting a balanced critical view of the current state-of-the-art. The audience will have the opportunity to contribute in defining priority areas and all together we will try to reach a vision of how AI and LA can be used for transforming mathematics learning and teaching.

Panel members:

Celia Hoyles, University College London.

Richard Noss, University College London.

Marcelo Milrad, Linnaeus University.

Sonia Abrantes Garcêz Palha, HvA.

Christian Bokhove, Southampton Univeristy.

Watch the panel discussion: <https://delos.uoa.gr/opendelos/player?rid=9e8f4a40>

“Teachers’ day”

Title: Scaling up in-service professional development for teachers: a 20-year intervention towards the use of digital media in classroom.

Charalampos Zagouras, CTI Diophantus.

Dimitra Egarchou, CTI Diophantus.

Chronis Kynigos, National Kapodistrian University of Athens.

Watch the lecture: <https://delos.uoa.gr/opendelos/player?rid=486069b>

Research Papers

Designing technology for a transforming society

FEELING THE SLOPE: LEARNING THE DERIVATIVE CONCEPT WITH AUGMENTED REALITY

Sara Bagossi, Osama Swidan, and Omar Abu Asbe

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The affordances of augmented reality technology allow the design of learning environments that can foster the embodied learning of calculus concepts. The augmented reality design presented in this contribution allows students to follow a function graph by hand, so as to simultaneously create the related derivative graph. This contribution explores how the students' bodily interaction with the function graph may help them become aware of the function-derivative relationship. Grounding our work in the notion of meaning-making as a process of objectification, we qualitatively analyzed the video recordings and transcript of the learning experiment involving a tenth-grader student. The results present three focuses of mathematical awareness the student achieved through the mediation of his body.

Keywords: augmented reality, function, derivative, meaning-making, objectification

INTRODUCTION

Meaning-making of mathematical concepts is one of the themes that has drawn the attention of mathematics education researchers for decades, and a well-consolidated direction of research concerns the meaning-making of calculus concepts (e.g., Thompson, 2013; Swidan et al., 2020). The idea that cognitive development in mathematical thinking grows from perceptions and actions to formal productions (Tall, 2009) and the recent focus on embodied cognition (Wilson, 2002) have motivated researchers to explore how students make meaning of calculus concepts when they use different kinds of digital tools. Dynamic and multiple representations tools were extensively used to explore how students make meaning by them (e.g., Noss et al., 1997; Swidan, 2022); embodied artifacts have been used recently to examine the meaning-making of the rate of change (Swanson & Trininc, 2021), and the augmented reality sandbox tool was adopted to help undergraduate students to make meaning of the gradient concept (Bos et al., 2022).

The affordances of augmented reality (AR) technology allow the design of digital tools that can foster the embodied learning of calculus concepts. This use of augmented reality has not been extensively explored, and its potential for advanced meaning-making of calculus concepts is still blurred. This study explores how a specific augmented reality design can help students make meaning of the derivative concept. The AR design presented here allows students to trace a function graph by hand, and the derivative graph simultaneously appears in front of the students. In particular, the aim of this contribution is to explore how the students' bodily interaction with the function graph may help them become aware of the function-derivative relationship.

THEORETICAL FRAMEWORK

In this study, meaning-making is considered to be a process of objectification (Radford, 2003). In other words, meaning-making is a matter of actively endowing with meaning the conceptual objects made available by the AR artifact used in the study. Meaning-making as objectification requires making use of different semiotic means such as words, symbols, actions with the artifact, rhythmic speech, and gestures available in the universe of the discourse (Radford, 2003). Radford referred to this process as a "semiotic means of objectification." From a pragmatic view, Radford suggests a semiotic tool to analyze educational mathematical activities in the classroom. The basic components of the semiotic tool are the *students' attention* and *awareness* of the mathematical object. A variety

of semiotic means of objectification that have a representational function attract the students' attention to mathematical objects. Furthermore, the artifact's properties can help students attend to the mathematical objects related to the activity under consideration. Students become aware of the attributes of mathematical objects within that phenomenon by paying attention to the necessary aspects of the mathematical phenomenon and using various semiotic means of objectification including gestures and bodily interactions with the virtual graphs. By being aware, students attain objectification of the mathematical objects, which then become apparent to them through various devices and signs.

METHOD

The learning experiment

The learning experiment described here is conducted in a lab condition with the support of an AR headset called Magic Leap: it is a pair of glasses through which the user can see the reality and the augmented objects. In the default interface of the [application](#) designed for the study purpose, the student can choose among seven elementary types of functions (upper part of Figure 1a). In the lower part of the interface, the gestures useful to interact with the technology are recalled jointly with their role (Figure 1a). When selecting one of the functions, a Cartesian system with numbered axes appears showing the selected function in blue color (Figure 1b). The learner is asked to move his hand along the graph: simultaneously with the hand's movement, a green tangent line appears, and the derivative curve is sketched point by point in the same color (Figure 1b). Moreover, a yellow number denoting the value of the slope of the tangent line is displayed. The student should be close enough to the function graph so that the tangent line is displayed. Otherwise, the line becomes red instead of green, and the derivative graph is not created (Figure 1c).

The design principles underlying this software were inspired by Alberto et al. (2019) work in which students can create the graph of a function by coordinating their hands' movement along a specific constraint. The a priori hypotheses behind such a learning environment are mainly two. First, since the environment juxtaposes the function graph, its derivative function graph, and the hand movement, students should come up with conjectures about the mathematical relationship between the two graphs. Second, body involvement should help the students to find that relationship by 'feeling' the slope behavior as they move and control their hands.

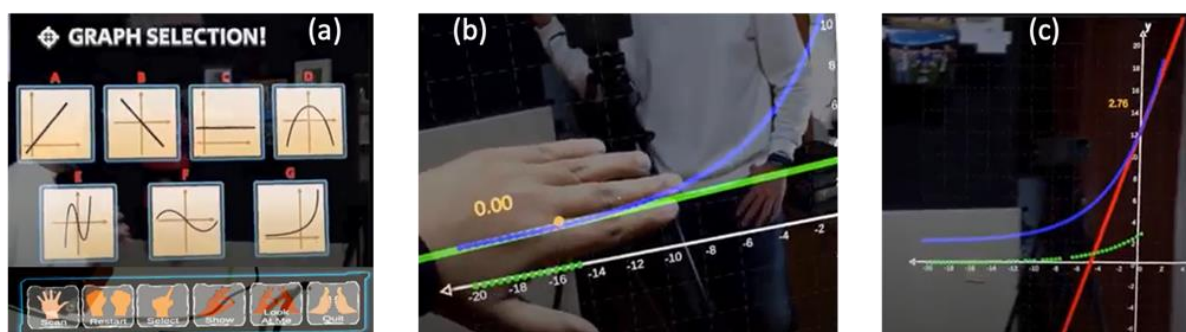


Figure 1. a) The software interface; b) A green tangent line and the derivative graph are created simultaneously with the hand's movement; c) If the student is not close enough to the graph, the tangent line becomes red

The learner and the researcher

The learner involved in this experiment is Karim, a 10th-grade Israeli student, who participated on a voluntary basis. He is a high achiever student and learns mathematics according to the high level of the Israeli curriculum. By the time of the experiment, he had already learned the linear function, the quadratic function, and reading graphs in the Cartesian system. The student was familiar with

the slope concept but not yet familiar with the derivative concept. The third author, Omar Abu Asbe, who also designed the software and so knew well how to manage it, was present throughout the duration of the experiment. The task, delivered orally by the researcher, requested Karim to produce some conjectures about the relationship between the blue (the function) and the green (the derivative) graph and to elaborate on the meaning of the yellow number (the slope). Karim was free to choose which of the provided functions to focus on in order to elaborate on a general relationship. The researcher and Karim were engaged in the learning experiment for a full hour. The researcher generally did not intervene during the experiment: he just provided technical support when needed.

Data collection and data analysis

All the session was video recorded with a camera focused on Karim. The researcher could always see what Karim was seeing through his headset due to a camera integrated into the headset and projected on a wider screen. This data was also recorded and analyzed. The two video clips of the session, the one focused on Karim and the one showing what he could see through his headset, were synchronized so that they could be watched simultaneously. The joint video was then transcribed and translated from Arabic into English. Each segment of the video and related lines of the transcript were analyzed according to the processes of noticing and focuses of awareness as mentioned in Swidan and Fried (2021). A focus of awareness should be intended as “a node in the learning process, a point at which students’ attention becomes drawn to a component of the mathematical idea via the digital technology and they begin to seek to enlarge their awareness of that component’s meaning and its place in the general decomposition of the idea” (p. 5). This gradual process of becoming aware happens through a “creative process of finding or noticing something” (Radford, 2008, p. 225). Hence, the preliminary analysis here presented consisted of a two-step coding: first, describing the mathematical elements Karim noticed during the learning experiment; then, based on these elements, focuses of mathematical awareness were elaborated by identifying the mathematical knowledge related to the function-derivative relationship Karim became aware of by the mediation of the elements he noticed. At the end of the coding phase, the video clips were divided into episodes illustrating Karim’s attempts to generate conjectures about the function-derivative relationship. The episodes were organized into categories based on the elements he noticed. For example, episodes concerned with Karim’s meaning-making of the yellow number were grouped into the category “the objectification of the yellow number.”

RESULTS

In this section, we present several short episodes showing three different focuses of awareness. Karim’s sentences were often interspersed with long pauses. To make the best use of the space available, we will merge his statements using the notation [$\#n\#$] to indicate a pause of n seconds between sentences.

Focus a): The objectification of the green points

At the beginning of the learning experiment, Karim selected the increasing line (Figure 1a, option A) and moved his hand slowly along the straight blue line, which signifies the function graph (Figure 2a). Once Karim moved his hand along the function graph, green dots, which signify the derivative function graph, appeared simultaneously with his hand movement (Figure 2b and 2c). While Karim was entirely focused on his endeavor, the researcher requested him to describe what he saw.

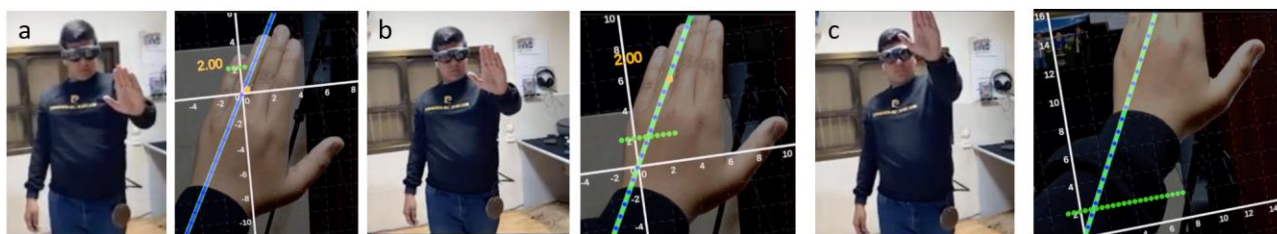


Figure 2. a) Karim moved his hand along the blue line; b) and c) His hand movement produced the appearance of the green points

- 1 Researcher: Karim, go back and tell me what you see
- 2 Karim: The entire (blue) function [#3#] the line is straight (gesture with his hand a straight line as the function graph, Figure 3a) [#2#] The points I was going through (on the blue line) [#4#] Each point I went through appeared [#2#] Appeared on this straight line (gesture with his hand a horizontal line, Figure 3b) [#8#] There is a number 2.00

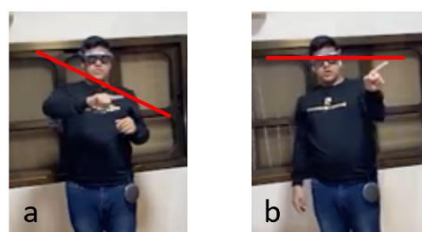


Figure 3. a) Karim gestures with his finger a straight line as the function graph; b) Karim gestures with his hand a horizontal line formed by the green points

In [this episode](#), Karim noticed, based on perceptual actions, that the green dots, which signify the derivative function graph, appeared due to his hand movement on the function graph. First, he noticed that the function graph is a straight line (linear function). In doing so, he used the words “entire function” simultaneously with gesturing the linear function (Figure 3a). Then Karim focused his attention on the blue points on the function graph. He used the verb “going through” to express his awareness of the blue dots on the function graph. It seems that the blue points, the perceptual action “Each point I went through,” and the green dots’ dynamicity helped Karim notice a relationship between the function graph and the derivative function graph (each point on the function graph appeared on the derivative function graph). Mathematically speaking, it seems that Karim noticed that the two functions share the same x-value.

Focus b): The objectification of the yellow number

In the [following episodes](#), Karim raised a conjecture about the displayed yellow number, which signifies the inclination of his hand. First, he elaborated on this meaning when he explored the constant function (Figure 1a, option C) and then selected other functions to check the conjecture.

- 3 Karim: The number is 0.00 [#5#] I expect this number is [#2#] The inclination of my hand (he positions his hand horizontally, Figure 4a) [#5#] The line I go along with my hand (moving his hand along the blue function, Figure 4b) [#2#] When I go along this line with my hand in this way [#6#] The number was 0000 [#3#] The inclination of my hand was constant [#6#] My hand didn’t tilt like this or that (tilting his hand, Figure 4c)

Karim noticed that the yellow number was 0 and endowed it with the meaning of the inclination of his hand, which was horizontal. Karim gestured the inclination simultaneously when he articulated the word “inclination” (Figure 4a) to connect the number and the inclination of his hand. He also connected it to “the line” he went along with his hand reproducing the same movement he did previously while he was interacting with the graph (Figure 4b). Then he expressed his awareness of

the connection between the inclination and the constant graph “when I go along this line in this way with my hand” and then also referred to the yellow number “The number was 0000”. Furthermore, he explained what he meant by constant inclination with a gestural action signifying no tilting of the hand (Figure 4c). It seems that the perceptual action “the line I go along with my hand” and the way of positioning his hand helped Karim to notice a relationship between the hand inclination and the yellow number. Mathematically speaking, it seems that by the mediation of Karim’s hand, he objectified the relationship between the value of the slope and the inclination of the tangent line.

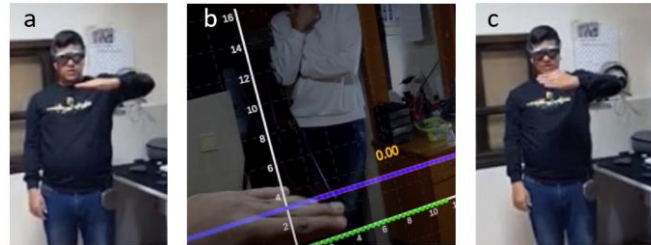


Figure 4. a) Karim positioned his hand horizontally; b) Karim moved his hand along the blue function; c) Karim tilted his hand

After the previous episode, Karim selected the increasing linear function (Figure 1a, option A) and kept on checking his conjecture.

4 Karim: When the function is increasing (moving his hand along the function, Figure 5a)

5 Researcher: Okay

6 Karim: And I make my hand this way (Figure 5a) [#3#] In this movement upward on the function (with his finger he reproduces the trend of the movement, Figure 5b) [#3#] On the curve [#1#] It displays 2.00 (pointing at the number, Figure 5c) [#3#] I expect that the inclination of my hand was two (positioning his hand inclined, Figure 5d)

In this except, Karim selected the increasing linear function and moved along it (Figure 5a). While he was moving his hand he noticed both the inclination of his hand and the displayed number. Indeed, immediately after, he reproduced with his finger the “movement upward” of his hand (Figure 5b), and then he pointed at the number 2 shown on the curve (Figure 5c). Eventually, he concluded that he assumed the inclination of his hand was 2 while simultaneously tilting his hand (Figure 5d). It seems that to check his conjecture, Karim noticed the same elements of the previous episode, hand inclination and number, not only in words but also through perceptual actions, hand and finger gestures. The way of positioning his hand helped Karim consolidate his mathematical awareness of the relationship between the value of the slope and the inclination of the tangent line, but as Karim himself remarked when saying “I expect”, it is still a conjecture to be validated.

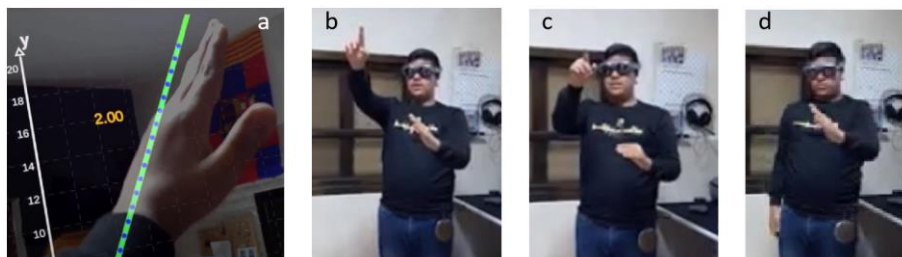


Figure 5. a) Karim moved his hand along the function; b) Karim reproduced with his finger the upward trend of the line he moved along; c) Karim pointed at the yellow number; d) Karim positioned his hand inclined

In this third episode, Karim switched from working with the increasing linear function to the decreasing one (Figure 1a, option B), with the purpose of verifying the relationship between the yellow number and the inclination of his hand.

7 Karim: Now when I go along it (Figure 6a), the number becomes -2 (pointing at the number, Figure 6b) [#12#] Because the inclination of my hand is downward in this way (Figure 6c) [#5#] I expect that this is the reason why the number is negative and 2

Also here, Karim noticed that the yellow number, which signifies the slope value, became -2 when he selected the decreasing linear graph. Karim noticed that the yellow number became negative because he tilted his hand downward in a certain way (Figure 6c). Karim also remembered that in the previous case, the number displayed was 2. Indeed, Karim connected all the elements previously noticed. He argued that “the number is negative” because “the inclination of his hand is downward” and the number is 2 because the hand is tilted “in this way”. Hence, Karim’s awareness of the relationship between the number displayed and the inclination of his hand increased: the sign (negative or positive) of the number depends on the direction of the hand (upward/downward) while the value of the number depends on the inclination.

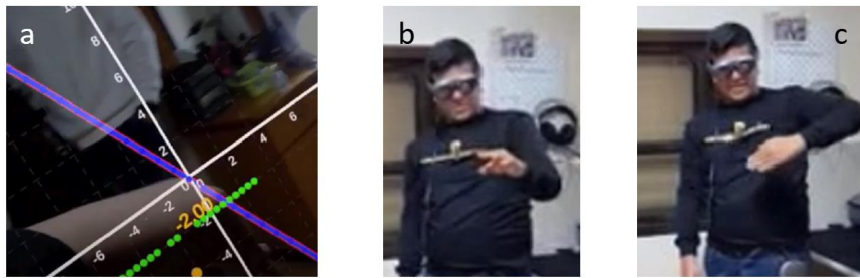


Figure 6. a) Karim moved his hand along the decreasing function; b) Karim pointed at the yellow number; c) Karim tilted his hand downward

Focus c): The objectification of the change in the slope value

Immediately after the previous episode, in which Karim explored functions that have a constant slope, he chose the cubic function graph (Figure 1a, option E). He moved his hand along the function until reaching the maximum point. Then he selected the parabola (Figure 1a, option D). He was moving along the function with his hand while he connected the yellow number and the inclination of his hand to the points on the function.

8 Karim: Once I reach this point (pointing gesture, Figure 7a), which is straight, it becomes 0.00

[Karim selects the parabola and moves along its graph]

9 Karim: In this point (pointing gesture, Figure 7b) the number is 4.00 [#6#] I expect the inclination of my hand is 4.00 [#9#] As I move in this direction the number is decreasing.

10 Researcher: Why?

11 Karim: Because the inclination of my hand (inclined hand held by the other hand, Figure 7c) decreased relative to this position [#14#] Here when I start to go downward [#4#] The number becomes negative [#2#] It is -2.80 [#5#] It starts [#7#] It starts to increase [#2#] Unlike here

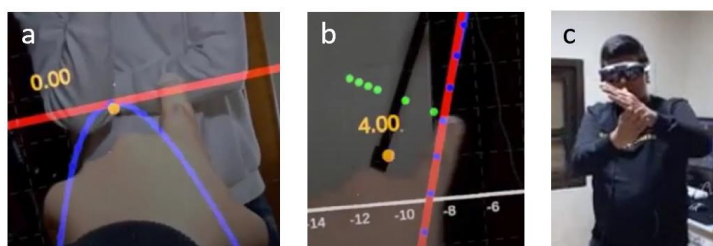


Figure 7. a) Karim pointed at the yellow number; b) Karim positioned his hand horizontally; c) Karim inclined his hand and held it with the other hand

In [this episode](#), Karim noticed the value of the number and the inclination of his hand in relation to the maximum point of the selected function. At this moment, it seems that Karim became fully aware of the relationship between the inclination of the hand, the number, and the points on the function. When selecting the parabola, he shifted his attention from focusing on the relationship between his hand inclination and the slope value in specific points of the function toward focusing on this relationship through a domain of the function “As I move to this direction the number is decreasing.” Here Karim used the verb ‘move’ to describe his hand movement and used the verb ‘decrease’ to objectify the change in the slope values. Then, Karim connected his hand inclination with the numerical values and the direction of his hand movement by the term negative.

DISCUSSION

The preliminary data analysis described in this contribution concerns three main focuses of awareness. Focus a) shows how Karim became aware that the green points are connected to the function graph. This happened due to the hand movement along the function which produced the simultaneous creation of the green points. Throughout focus b), Karim became aware of the yellow number, and its connection to the inclination of his hand. Perhaps, the fixation of the yellow number drew his attention, or the continuous movement of his hand in the same direction. Indeed, he chose various linear functions: first constant, then increasing and decreasing. The change in the direction of his hand helped him to notice the sign of the slope endowing it with mathematical meaning (negative or positive). It seems that the freedom in tilting the student’s hand in the space together with the appearance of the number that measures the hand inclination, allowed Karim to evaluate his hand inclination, which eventually helped him to objectify the relationship between his hand movement and the slope value. In focus c) Karim noticed the change in the slope values in different points of the function and then in domains of the function. Feeling the change of slope moving along the same graph helped him elaborate on his previous mathematical awareness. Further steps of analysis will address all the focuses of awareness throughout the entire learning experiment and investigate how a given focus of awareness involved the use of others.

ACKNOWLEDGMENTS

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Mathematical LITERACY DEMONSTRATED in PROGRAMMING-based MATHEMATICAL PROBLEM SOLVING: THE CASE OF COMPOUND INTEREST

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In this paper, we explore students' mathematical literacy in a problem-based digital making course. We report on a task-based interview in which pairs of students were using a block-based visual program "Scratch" to solve mathematical problems related to compound interest. Two pairs of students (one grade 9 group and one grade 11 group) were invited to join the course. Their code files, screen captures of computer work were analysed in terms of their developed mathematical reasoning from the perspective of mathematical literacy.

Keywords: Scratch, programming, problem solving, mathematical literacy

INTRODUCTION

Mathematical literacy in the 21st century includes some aspects of computational thinking. It includes the use of mathematical tools such as a variety of digital and physical equipment, software, and calculation devices. Computational thinking (CT) tools offer students a context in which they can reify abstract constructs by exploring and engaging with math concepts in a dynamic way (OECD, 2018). CT supports the development of mathematical thinking (Ng and Cui, 2021). The combination of mathematical and computational thinking allows students to view realistically how mathematics is practised and used in real world and prepares them for pursuing careers in related fields (OECD, 2018). The Computer Science Teachers Association (CSTA) stated that computational thinking enables students to conceptualize, analyse and solve complex problems better by selecting and applying suitable tools and strategies in virtual and real world. (David et al., 2016). The nature of computational thinking is defining and elaborating mathematical knowledge which can be expressed by programming, allowing students to model mathematical concepts and relationships dynamically (OECD, 2018).

Researchers argued that writing computer program provides a big canvas on which students can sketch half-understood ideas and construct semi-concrete image of mathematical structures that students are building intellectually on the screen (Laura et al., 2017). The connection between abstract mathematical concepts and concrete experiences allows students to use mathematics in a meaningful way. For example, programming at school has the potential of developing higher order of mathematical thinking in relation to numbers linked to mathematical abstraction, including algebraic thinking and problem-solving abilities (Ng & Cui, 2021; Ng et al., 2021)

In this paper, students participated in a programming-based mathematical activity of using Scratch to solve mathematical problem. Scratch is a free online community and programming language which students can create their own stories, animations and games by coding. Scratch has been proved of providing opportunities for learning important computational and mathematical concepts and offering space for systematic reasoning, creative thinking and collaborative work (Laura et al., 2017). It is easily readable, composite and browsable alongside its visual, interactivity and dynamic outcomes. Our research questions are as follows: (1) How do secondary students develop and demonstrate mathematical literacy during programming-based mathematical problem-solving? (2) What opportunities and hinderance for learning do the programming context have on students' mathematical problem-solving?

RESEARCH BACKGROUND

Constructivist learning with programming-based mathematical problem solving

During the past two decades, the introduction of digital technology in schools has given rise to new ways of doing and representing mathematics, with dynamic geometry being a prime example of technology affording an environment in which mathematics is represented and learned in transformative ways. More recently, there has been increased focus on the impact of making, pointing towards new opportunities for engaging learners in constructionist practices with digital technology. Scholars have generally defined Making as the hands-on production of artefacts that are technologically enhanced (Ng & Cui, 2021). Stemming from the constructivist view of learning as “building knowledge structures”, constructionist learning underpins the context whereby the learner is consciously constructing a public entity (Papert & Harel, 1990). Of interests to this study is a subset of constructivist learning with programming-based mathematical problem solving which affords the active creation of both programming-based artefacts during the course of mathematical problem solving.

In recent years, research into integration of programming and mathematics in K-12 education has provided evidence that programming-based mathematical activities supports students’ development of critical thinking, creativity and communication and collaboration (Weng et al., 2022). It also develops students’ computational thinking and problem-solving competences (Ng, Liu & Cui, 2021). According to Feurzeig, Papert and Lawler (2011), programming fosters an experimental approach towards problem solving. It provides students opportunity to discuss mathematics through natural framework, standard vocabulary and personal experiences. Moreover, solving mathematical problem by computational thinking tools can encourage students to practice prediction, reflection and debugging skills (OECD, 2018).

Mathematical literacy

In this paper, we define mathematical literacy as not just doing arithmetic calculation nor solving algebra problems, but as according to PISA 2022 Mathematics Framework, i.e. students’ capacity to reason mathematically, and to employ, formulate and interpret mathematics to solve real life problems. It includes concepts, facts, procedures, and tools to describe, explain and predict phenomena. Mathematical literacy can help students understand the role of mathematics in the world and make judgements and decisions needed by engaged, constructive and reflective 21st Century citizens. It comprises two aspects: mathematical reasoning and problem solving. Mathematical reasoning involves situation evaluation, strategies selection, logical conclusion drawn, solution development and description, and solution application recognition. For the purpose of studying students’ mathematical reasoning and problem-solving competence in the context of programming-based mathematical problem solving, we illustrate the following shortened list of practices addressing how mathematical reasoning can be demonstrated in PISA 2021 Mathematics Framework (OECD, 2018):

1. Drawing simple conclusion
2. Selecting appropriate justification
3. Explaining the reason that a mathematical conclusion or result does or does not make sense with the problem context
4. Utilising and understanding rules, definition and formal systems as well as employing computational thinking and algorithms
5. Explaining, defending and providing a justification for the devised and identified representation of a real-world situation

6. Explaining or defending and providing a justification for the procedures or simulation and processes used to determine a mathematical solution or result
7. Identifying and critiquing the model limits used for solving a problem
8. Reflecting on mathematical arguments, justifying and explaining the mathematical result
9. Interpreting mathematical result back into real life situation in order to explaining the meaning of results
10. Explaining the relationships between context-specific language of a problem and the formal and symbolic language needed to represent it mathematically.
11. Reflecting on mathematical arguments, explaining and justifying mathematical result
12. Reflecting on mathematical solutions and creating arguments and explanations that support, qualify and refute a mathematical solution to a contextualised problem
13. Analysing similarities and differences between a computational model and the mathematical problem that is modelling
14. Explaining how a simple algorithm works, detecting and correcting errors in programs and algorithms

METHODOLOGY

The task-based interview

In this report, four students from Grade 9 and Grade 11, divided into two pairs respectively, attended a task-based interview in which they were tasked with making computer programs in Scratch while solving a mathematical problem related to simple and compound interest. The choice of two different grade levels was to examine the chosen research questions with participants having different levels of prior knowledge in mathematics; unlike the Grade 11 students, the Grade 9 students have not yet learned the formula of determining compound interest, $A = P(1+r)^n$ (A: amount, P: principle, r: interest rate, n: number of deposit period). The interview (first author) posed the students with two problems, one on simple interests and another on compound interests, with several sub-problems in three-hour session. As the students had minimal experience with Scratch prior to the study, they were initially guided to learn some basic functions, such as creating blocks and running the program in Scratch, and were then scaffolded to solve the very first problem posed. After that, they were given ample time to explore the solutions to the subsequent problems with their partners, and explain their thinking processes to the interviewer, who would provide adequate guidance when needed. Students' code files, screen captures of computer work and dialogue between teacher and students were analysed in terms of their developed mathematical literacy. An interview will also be carried out after the lessons.

The design of task

The students were required to compare two different saving plans using Scratch. To make the problems more relevant to real-life, we design scenarios in which different banks use different method to accumulate interests (simple interest and compound interest) and different interest rate to attract investors. To compare interest, return from banks, students need to design an interest calculation system using Scratch. The design of calculation system allows students to learn the setting of variables and conditional blocks by understanding the relationship between amount, principal, interest rate per period and number of deposit periods. Besides understanding the meaning

of different variables, students also need to understand the quantitative relationship of different deposit methods.

Alejandra, Noemi and Susanna (2019) divided digital making into four stages: introductory stage, design stage, production stage and final evaluation and diffusion stage. During the introductory stage, it is important to introduce the key programming concepts and not focus on learning the programming environment rather than learning through it (Laura et al., 2017). Furthermore, they suggest that the design of Scratch researchieral should be relevant and consistent in order to ensure that intervention is aligned with mathematical curriculum. Researchers should advocate a set of learning guides, best practices and curriculum models to help students encounter the richness of Scratch ideas (Laura et al., 2016). In the design stage, whole-class discussion should be led by teachers through inclusion of reflective questioning. Harel and Papert (1990) mentioned that generating verbal explanations can help clarify ideas. Moreover, the balance between direction (researchieral) and discovery (students' own exploration) is important. Finally, the design of tasks should provide students opportunities to investigate ideas by trying things and debugging errors (Laura et al., 2017). In line with these pedagogical considerations, we design the lesson flow, lesson objectives and content as shown in Table 1.

Prior knowledge	The four basic arithmetic operations Linear equations Inequality	
Learning objectives	1. Solve problem about numbers in Scratch 2. Use iterations and conditionals to model different deposit situations 3. Work with several variables at the same time	
Mathematics and Programming Outcomes	1. Mathematics: quantitative relationship, simple and compound interest 2. 2. Programming skills: variables, conditionals, operator, sequences	
Four stages of digital making	<i>Introductory stage</i>	Students learn the setting of variables and conditional blocks by designing a simple interest calculation system. The content is shown below:
	<i>Design stage</i>	Students are divided into two groups. They need to set up an interest calculation system to calculate compound interest offered by two banks and do the comparison.
	<i>Production stage</i>	Students demonstrate their model to other students and invite others to play the Scratch.
	<i>Final evaluation and diffusion stage</i>	Students complete reflection on the learning process by interview

Table 1. The design of the task

Analytical Method

We developed an analytical framework and used deductive coding to analyze students' discourse and programming actions during the task-based interview. This allowed us to observe how students develop their mathematical literacy when solving the task. Specifically, our data analysis took on the following steps. First, we synthesized various practices addressing mathematical reasoning as listed in PISA 2021 Mathematics Framework. Second, we used open coding to identify how students demonstrate these practices mathematical reasoning according to the listed practices. These steps allow us to look for evidence in how mathematical literacy and computational thinking can be meaningfully synergized into mathematics learning and problem-solving situations in a programming context. Finally,

we focus our attention on the learning opportunities and hinderance that programming had on students' problem solving by delving into the way the students interacted with the Scratch.

RESULTS

We detail students' practices of mathematical reasoning in the following three episodes. We use italics to highlight these practices analyzed.

Episode 1. When setting the simple interest calculation system, students first drew a timeline for creating the function (Figure 1).

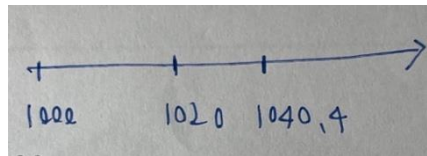


Figure 1. Students' drawing during Episode 1

Researcher:	Let's look at the timeline, the interests in each year are different which does not match the question.
Student A:	Maybe we should not add the index.
Researcher:	Could you write the equation for calculating the simple interest?
Student A and B:	
Researcher:	Let's look at the timeline, what is the interest after 1 year?
Student B:	\$20
Researcher:	After 2 years?
Student B:	$\$20 + 20$
Researcher:	After 3 years.
Student B:	$\$40 + 20$
Researcher:	After n years.
Student B:	$n + 20$
Student A:	No.
Student B:	After 5 years, the interest is $80 + 20 = 100$. 5 times 20 is also equal to \$100.
Researcher:	What is the interest after n years?
Student B:	$20n$.

From Episode 1, we observe that the students were able to explain their calculation by drawing a timeline and deduce interest equation. In doing so, they demonstrate *explaining, defending and providing a justification for the devised and identified representation of a real-world situation*. At first, the students applied the compound interest equation to solve the current problem, which is related to simple interests. They were able to identify variables but found it difficult to derive an algebraic equation for simple interest. Later, upon being guided by the researcher, they observed that the interests increase by \$20 every year and successfully set up the algebraic expression, $20n$, for the amount of interests earned after n years. This demonstrates that the students reflected on *mathematical arguments, explained and justifies mathematical result*.

Episode 2. Students needed to think about inequality prior to using “if...then...else” block for setting requirement of data entered.

Researcher:	What if the data is negative? How to present positive number?
Student C:	The principle is larger than 0.
Student D:	The principle is larger than interest.
Researcher:	Why?
Student D:	No.

Researcher: You may look at the data here. What do you notice about the data?
 Student C: Not negative.
 Researcher: Can the data be zero?
 Student C: Zero is not a positive number.
 Researcher: Zero is not positive but can the principle be zero?
 Student D: Yes. Can have no interest.
 Researcher: What is the meaning when the principle is zero?
 Student C and D: No money.

In Episode 2, the students explained *how a simple algorithm work* in the sense of presenting an inequality for the variables, which demonstrates their mathematical reasoning. Through solving the problem in a programming context, the need for prompting for input for different variables is observed. In particular, they noticed that the principle, interest rate and number of deposit periods must be non-negative number. As a form of mathematical reasoning, they *explained the relationship between context-specific language of a problem* “principle should be larger or equal than zero” *and the formal mathematical language* (i.e. not negative). The students also *interpreted the mathematical result* of “zero principle” *into real life situation*, stating that no money would be deposited in this situation. When setting up the “if...else...then” and “repeat until” situations, the students *checked their codes with their calculation results*. They *detected and corrected errors in programs* and successfully obtained the desirable result at the end (Figure 2).

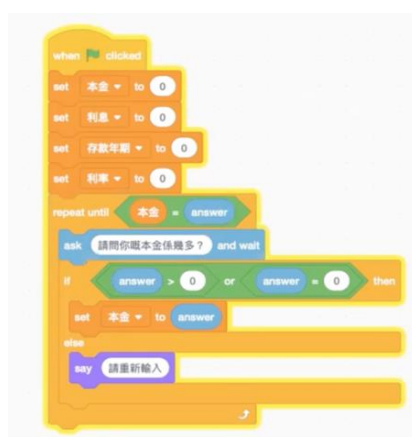


Figure 2. Students' codes of setting situations

Episode 3. Before setting the compound interest calculation system in Scratch, students calculated the amount of deposit in the first five years (Figure 3a).

Researcher: If we would like to design a calculation system, what should we enter?
 Students D: Principle, deposit period, interest rate
 Researcher: What is the equation?
 Student D: All variables should be multiplied together.
 Student C: Principle x Interest rate x Deposit Period. “Interest rate x Deposit period” keeps on repeating. Let’s add a bracket to “Interest rate x Deposit period”. In 1st year, the index should be 1. In 2nd year, the index should be 2 and so on. So the index is 5. The equation is Principle × (interest rate × deposit period)⁵.
 Student D: But when we substitute the numbers (Figure 3), the result is wrong.
 Student C: We have the index be the “deposit period” so we should not add deposit period inside the bracket. It should be Principle × (1 + interest rate)^{deposit period}.
 Researcher: The equation is correct, but it is difficult to set up index in Scratch.
 Student D: So we may set to multiply “1+interest rate” by 5 times.

Researcher: Yes you are right.

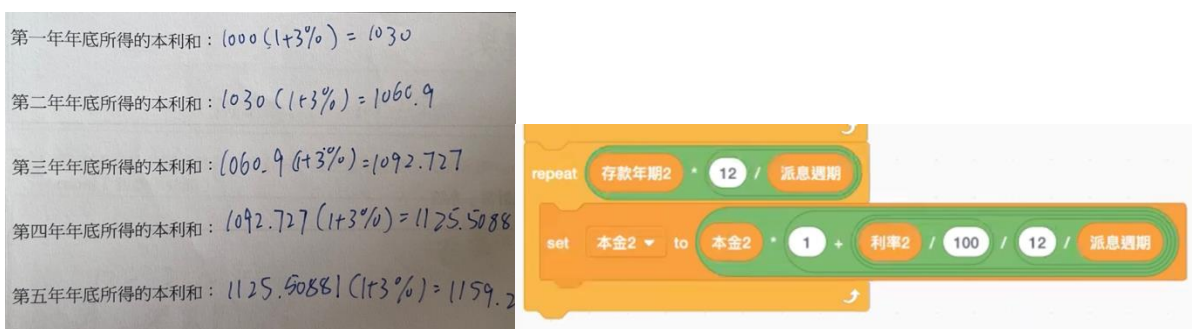


Figure 3(a) Students' calculation of compound interest. (b) Students' codes of calcuting amount under compound interest calculation

In Episodes 3, the students were able to *explain how a simple algorithm works*, and to *detect and correct errors in algorithms* by setting up codes that perform the calculation of compound interests. They could identify the index of deposit period as repeatedly multiplying $(1+\text{interest rate})$. Also, they *reflected on mathematical arguments, justified, and explained the mathematical result* of the compound interest rate equation.

DISCUSSION

This investigation examined how two pairs of secondary school students engaged in mathematical reasoning related to programming in a programming-based mathematical problem-solving environment. Results suggest that programming in Scratch promoted various practices of mathematical reasoning. They analyzed the situation and used symbols and representations to make sense of their programming actions, which is contextualized in a real-life scenario. It can be said that Scratch can allow students to make and test conjectures, thereby facilitating mathematical reasoning. When setting the conditions for data entered, students encoded by changing the representation of blocks in a script. They were able to detect and correct errors in program. Also, concept of variables is important in programming. When solving the task, students can perform their CT by setting up variables to calculate the amount of interest. During the process of deducing an algebraic expression for interest earned, the students explained their thinking and justified the mathematical result. They also gave an explanation that zero principle means no money deposited. Finally, they detected and correct errors of equation by comparing with previous calculation. When setting the formula for calculating the amount at the end of deposit period, they used graphical method like timeline to explain how the formula works and deduced equation.

The programming context provided opportunities and hinderance for students' learning on mathematical problem-solving. In conventional paper-and-pencil lessons, teachers would commonly introduce formula for computing compound and simple interests for the students. Using programming to set up equation can help students better understand the equation since they were required to think about the relationship between different variables. It is important to understand the problem-solving process or strategy used rather than just focusing on solution (Laura et al., 2017). However, some students explained that they had difficulties in programming since they needed to consider many conditions.

CONCLUSION

This study supported that students experienced mathematical reasoning in programming-based mathematical problem-solving. Though some students faced in-moment challenges when solving mathematical problem in programming, they learned to see the problem from the perspective of CT

and commented that programming could help them understand mathematical context. As stated in PISA 2021 Mathematics Framework, mathematical literacy includes mathematical reasoning and problem solving. Further study can be carried out to examine students' problem solving in programming-based mathematical problem-solving. For example, the effectiveness of programming in helping them understand mathematical concepts are worthy of future studies.

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EXPLORING STUDENTS' PERSPECTIVES ON THE SUPPORT PROVIDED BY DIGITAL META-SCAFFOLDING

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In this paper we present the results of a study aimed at investigating university students' perspectives on the support provided by specific digital meta-scaffolding elements that characterize the design of a digital resource aimed at fostering students' metacognitive activities when facing problem situations that could be modelled through differential equations. The analysis of the data collected through a written questionnaire enabled us to identify categories of theory-driven and data-driven themes emerging from students' reflections, to highlight the categories that students relate to each digital meta-scaffolding element and to discuss interrelations between different categories.

Keywords: digital resource design, meta-scaffolding, metacognitive activities, problem solving

INTRODUCTION AND THEORETICAL FRAMEWORK

In this paper we present the results of a study developed within the second cycle of a design-based research (DBRC, 2003) focused on the design of digital resources (DR) aimed at providing students with meta-scaffolding to support their reasoning processes when facing problem solving activities.

We refer to Holton and Clarke's (2006) definition of scaffolding as an "act of teaching that (i) supports the immediate construction of knowledge by the learner; and (ii) provides the basis for the future independent learning of the individual" (p.131). Scaffolding is particularly relevant for the design of learning environments aimed at supporting students' problem-solving processes, due to the key role played by metacognition in problem solving (Schoenfeld, 1992) and to the strict interrelation between the provided scaffolding and the corresponding acts of metacognition that could be activated (Holton & Clarke, 2006). Moreover, students' effective use of the scaffolding provided within learning environments requires that they activate themselves at the metacognitive level (Holton & Clarke, 2006). This suggests shifting the focus on a specific type of scaffolding, that is meta-scaffolding, defined as the scaffolding for the scaffolding (Pea, 2004).

Metacognitive aspects play a fundamental role also when the focus is on the design of digital environments aimed at supporting problem solving processes, due to the need for a good balance between procedural and metacognitive scaffolding (Sharma & Hannafin, 2007).

To better characterize the DR-design on which our study is focused, we introduce the notion of digital meta-scaffolding elements (DGMEs) to refer to those elements of scaffolding provided, within digital environments, with the aim of fostering students' metacognitive activities (Cusi et al., 2022). The categorization of metacognitive activities to which we refer is the one introduced by Meijer et al. (2006), who distinguish between orientating, planning, monitoring, evaluation and elaboration (Table 1 summarizes the activities included within each of these categories).

Categories	Activities included within each category
Orientating	Activating prior knowledge, establishing task demands, identifying important information, re-reading questions carefully, establish givens, observing.

Planning	Looking for particular information in text, sub-goaling, using external source to get explanation, backward reasoning, formulating action plan.
Monitoring	Error detection and correction, noticing inconsistency, checking plausibility, claiming progress in understanding, giving meaning to symbols or formulae.
Evaluation	Explaining strategies, finding similarities, interpreting, quitting, self-critiquing, verifying.
Elaboration	Inferring, checking representations, commenting on the difficulty of problems, commenting on personal habits.

Table 1. Meijer et al.'s (2006) categorization of metacognitive activities

When scaffolding is realized within technology-enhanced learning environments, the feedback provided by digital tools plays a central role. This kind of feedback can be considered, according to Nicol and Macfarlane-Dick's (2006) definition, a form of external feedback, which must be interpreted, constructed and internalized by students to have a significant influence on subsequent learning, since the result of this process of internalization is the internal feedback, generated by students' monitoring of their interactions with the tasks and the internal and external outcomes of their work. To distinguish between the different information that feedback could provide, we refer to Hattie and Timperley's (2007) levels of feedback: (i) about the task; (ii) about the processing of the task; (iii) about self-regulation; (iv) about the self as a person.

CONTEXT, BACKGROUND AND TASK DESIGN

The context of this study is a Mathematics course for students enrolled in the "Chemistry and pharmaceutical technologies" degree course at Sapienza University of Rome (Italy). The Mathematics course, which is focused on basic knowledge related to different topics (algebra, analytical geometry, goniometry, probability, statistics, calculus) is scheduled for the first term of the first year. Within the part of the program devoted to calculus, the topic of differential equations is faced, with a focus on linear equations with constant coefficients and on their use in modelling simple problems. Many students enrolled in the Mathematics course face difficulties with this topic, in particular in correctly interpreting the elements that constitute the differential equations that model the problems and in connecting specific properties of the graph of the functions that constitute the problems' solutions to the corresponding characteristics of the represented phenomena. During the first cycle of our design-based research, to support students in overcoming these blocks and to foster their activation of metacognitive activities during the resolution of problems, we designed a specific DR: a GeoGebra applet aimed at supporting students when facing a problem that could be modelled by means of a linear differential equation with constant coefficients. The text of the problem is: "An industry produces mobile phones at a rate of 20% per month. Every month, 150 mobile phones are sold. Suppose that at time $t = 0$ there are 700 mobile phones ready to be sold. Is the production's rate sufficient to meet market needs?".

The main DMSEs included in the first design of the DR were presented in Cusi et al. (2022), together with an analysis of a set of a-posteriori interviews conducted with five pairs of students who tested the DR. Table 2 summarizes the main DMSEs provided through the DR and the metacognitive activities fostered by each of them.

Digital meta-scaffolding elements	Fostered metacognitive activities
A) In case they fail in the initial construction of the differential equation, students are provided with the general form of the equation they have	Orientating, since students are supported in the identification of important information and in establishing given values within the problem's text.

to construct ($y'=py+q$) and guided, by means of specific questions, to read the problem's text, identifying the information that could help them in determine the coefficients p and q .	Planning, since students are guided to look at particular information useful to achieve the goal of constructing the differential equation that models the problem.
B) After the construction of the correct differential equation, students are guided in making the meaning of each term of the equation (y' , py , q) explicit.	Monitoring, since students are supported in giving meaning to the mathematical objects they are working with and in highlighting their understanding of what they are doing.
C) After having drawn the graph of the solution of the differential equation through GeoGebra, students are asked to identify, within a list of properties of the graph, the property to which they should refer in order to answer the problem's question.	Orientating, since students are supported in a careful reading of the question and in the interpretation of the graph in relation to the problem. Evaluation, since the focus is on the interpretation of the result of the processes activated to solve the problem.
D) After having inferred that the production's rate is not sufficient to meet market needs, the students are asked to determine the number of months after which there are no more mobile phones ready to be sold. If they fail in answering this final question, they have the opportunity to choose between two possible strategies and are guided toward the implementation of the chosen strategy.	Planning, since students are guided in formulating their action plan.
E) If students fail in specific steps of the task, they have the opportunity to use theoretical hints to support their resolution or to check the correctness of their work.	Orientating, since students are guided to focus on information that could support their work. Planning, since students are enabled to use external sources to get more explanations to organize the resolution process.
F) At each step of the activity, students are provided with reminders about the main results of the previous steps.	Monitoring, since students are supported in referring to the outcomes of the previous steps of the problem's resolution and in keeping track of their work.
G) Error messages are provided during the whole activity, together with feedback aimed at supporting students in detecting their mistakes.	Monitoring, since students are guided to detect mistakes and understand possible reasons related to partial failures in the resolution process. Evaluation, since students have the opportunity to check their work in progress and to develop a self-critique about the chosen approaches.

Table 2. Digital meta-scaffolding elements and related metacognitive activities

The analysis presented in Cusi et al. (2022) highlighted students' lack of awareness about some of the DMSEs that were part of our DR-design and showed the importance of stimulating their reflections on the scaffolding provided by this DR to make them realize how to take advantage of its use to support both their cognitive and meta-cognitive processes. This result suggested us to re-design the students' activity with the DR with the aim of making them become more aware of the

role of specific DMSEs by stimulating their reflections immediately after their use of the DR. For this reason, the study conducted during the second cycle of our design-based research involved the students enrolled in the Mathematics course in an activity that combined the work with the DR with a phase of individual reflections on the experience of using the DR to solve the problem. The phase of students' individual reflections was engendered by asking them to answer a written questionnaire constituted by 25 questions focused on: (a) the students' general difficulties in solving this kind of modelling problems, (b) the effectiveness of the DR in making them overcome these difficulties and (c) the usefulness of the different DMSEs provided by the DR. In this paper we present the first results of our analysis of the students' answers to the questions focused on (c). Each of these questions refers to a different DMSE and is structured like the one in Figure 1.

(14) After the construction of the graph of the particular solution of the differential equation, it is required to identify the property of the graph which must be referred to in order to answer the problem's question. Do you think this support is useful?

Screenshot from the applet:

What property of the graph that you constructed enables us to identify the answer to the problem's question?

☐ The graph has an horizontal asymptote

☐ The graph intersects the ordinate axis at one point

☐ The graph is always decreasing

What does this property allow us to conclude?

☐ That the production's rate is sufficient to meet market needs

☐ That the production's rate is not sufficient to meet market needs

1

2

3

4

5

Not at all useful

☐

☐

☐

☐

☐

Very useful

(15) Motivate your answer to the previous question.

Figure 1. Example of a question on the usefulness of a specific DMSE

RESEARCH QUESTION AND METHODOLOGY

The aim of the research documented in this paper is to answer the following research question: How do the students enrolled in the Mathematics course interpret the role played by the support provided by the DMSEs included in the DR they used?

To investigate students' perspectives on the DMSEs provided by the DR, we collected the answer to the written questionnaire of two groups of students enrolled in the Mathematics course during the academic years 2021-2022 and 2022-2023, who voluntarily participated in the study. The numbers of students who participated in the study were 83 in the academic year 2021-2022 and 63 in the academic year 2022-2023.

We carried out a qualitative analysis of the students' answers to the questions focused on the DMSEs provided by the DR resource. The analysis, which was both theory and data-driven (Cohen et al., 2007), was developed in two subsequent phases. The aim of the first phase was to identify categories of themes emerging from students' answers, associated with different interpretations of the received DMSEs. In the second phase, we read again through all the data to analyze the students' answers in relation to each of the emerging themes with the aim of highlighting the categories of themes that students mainly connect to each DMSE to characterize the ways in which they interpret the support provided by the different DMSEs and reflect on their use of this support.

ANALYSIS

The following tables summarize the different themes that emerged during the first phase of the data analysis. To provide examples for each theme, excerpts from the students' answers to the questionnaire are inserted in the right columns of the tables.

Table 3 summarizes the theory-driven themes, that is those themes that were a-priori considered by referring to the theoretical elements that guided the DR-design.

Themes	Examples from the questionnaire
(1) Metacognitive activities engendered by the DMSEs of the DR (Meijer et al., 2006)	Students' answers can be divided in two main categories, depending whether or not the students recognize the metacognitive activity that the design of the DMSE was aimed at fostering. The following reflection, for example, is in tune with the aim of the DMSE A-design, that is of supporting the orientating metacognitive activity: "It is an effective guide if you cannot understand how the text should be interpreted." (reflection on DMSE A)
(2) Levels of the feedback provided by the DR (Hattie & Timperley, 2007)	Students' answers can be divided in three main categories, depending on how they interpret the received feedback, at the level of: (a) the task: "Initially I gave the wrong answer and I was not able to understand why. Only thanks to the applet's support I understood my mistake and gave the correct answer." (reflection on DMSE D) (b) the processing of the task: "...since it considerably shortens the time to search for your mistake, it allows a longer and more fruitful reflection on the reasons subtended to the solving process." (reflection on DMSE A) (c) the self-regulation: "Since I understood what are the aspects on which I have more difficulties, thanks to this activity I know what are the aspects that I have to focus on to prepare the examination" (reflection on DMSE A)

Table 3. Theory-driven themes

Table 4 summarizes the data-driven themes, that is those themes that directly emerged from the first reading of the students' answers. Each of the themes in Table 4 is presented in terms of categories corresponding to specific dichotomies between counterposed perspectives on aspects that the students highlighted when they commented about the usefulness of the support provided by the DMSEs.

Themes	Categories within each theme and examples from the questionnaire
(3) Aspect on which the support provided by each DMSE is focused:	(a) <u>Focus on the problem solving process</u> : the role of the DMSE's support is recognised in terms of monitoring the problem solving process and of testing students' understanding: "...taking advantage of the graphical support [Geogebra], we can test the functions previously identified, in order to monitor whether the solving process autonomously considered (we autonomously set) was right. In this way, you can also easily self-assess the correspondence between what you studied and what you really understood." (reflection on DMSE C)
	(b) <u>Focus on the product of problem solving</u> : the role of the DMSE's support is recognised only in terms of making the students complete the task determining the right answer: "If you have some doubts or perplexities, [it] guides you in finding out the right answers and in not making mistakes." (reflection on DMSE A)
(4) Temporal dimension of the effects boosted by the support	(a) <u>Short-term effects</u> : the role of the DMSEs' support is recognized only in relation to the resolution of the specific problem on which the applet is focused: "The reminders help you in not forgetting the results of the previous questions and this can help in facing the next questions." (reflection on DMSE F)

provided by each DMSE:	(b) Long-term effects: the role of the DMSEs' support is recognized in relation to the aim of providing the student with a repertoire of strategies that he/she can activate in the future when solving the same kind of problems: "I found it useful, since the applet placed us in front of two different ways of determining the same solution. This can be useful when facing other problems like this one, since we can choose between one method or another." (reflection on DMSE D)
(5) Kind of understanding (Skemp, 1976) that each DMSE supports:	(a) Instrumental understanding: the role of DMSEs' support is recognized in relation to an idea of mathematical problem solving as a mere application of a set of formulae: "Formulae and theoretical materials are useful to quickly solve the task. Often if you use theoretical hints in facing a task does not mean that you are not able to solve the task itself, but that you do not remember a formula which, in most cases, you need to learn by rote." (reflection on DMSE E)
	(b) Relational understanding: the role of DMSEs' support is recognised in relation to the aim of fostering a deep understanding of the strategies that are activated to face the problem: "...it is useful to deeply understand the meaning of each term of the differential equation. It encourages us to think about it instead of mechanically carrying out the resolution's steps. This leads to a more aware resolution of the problem" (reflection on DMSE B)
(6) Perspective on the impact of the support provided by each DMSE:	(a) Objective perspective: the role of the DMSEs' support is recognised by reflecting on the difficulties that a generic student can face: "It is very useful in this phase, since it forces the student to think about the different aspects that often are only quickly interpreted and not enough considered." (reflection on DMSE B)
	(b) Subjective perspective: the role of the DMSEs' support is recognised by reflecting on the student's own difficulties: "I always have difficulties in setting up the solving process, so this element is very useful for me, since it made me understand that I did not have to take the initial number of phones into account." (reflection on DMSE A)

Table 4. Data-driven themes

The analysis conducted during the second phase enables to highlight how the students reflect on the support provided by the different DMSEs and on their use of this support.

Here, due to space limitations, we limit ourselves to the presentation of some results from the analysis conducted in relation to the DMSE A. The themes that students mainly connect to the DMSE A are theme 1, theme 2, theme 5 and theme 6. The analysis of students' answers to the questions focused on the DMSE A enables us to highlight that most of them grasped that the DMSE A has been designed to mainly foster the orientating and planning metacognitive activities (theme 1). Many of them also refer to the monitoring that the use of this DMSE could activate, focusing on its role in providing a feedback on both the task, by supporting the identification of mistakes, and on the processing of the task, by stimulating reflections aimed at deepening the understanding of the reasons behind specific strategies (theme 2). Their interpretations of the received feedback are often related to the students' personal experience in Mathematics and on their learning difficulties, highlighting a subjective perspective on the impact of the DMSE's support (theme 6). Most of the students' answers are also focused on the importance of being aware of the meanings of the mathematical objects constructed during the resolution process, hence revealing a perspective on

the DMSE's support related to the aim of fostering a relational understanding of mathematics (theme 5).

Besides allowing the identification of the themes that are more recurrent in students' answers on the support provided by specific DMSEs, the analysis developed during the second phase enables us also to highlight interrelations between specific categories of themes.

Regarding the DMSE A, we observed that the idea that the DMSE's support is focused on the product of the problem-solving process (category b, theme 3) is often combined with a focus on the role played by the DMSE's support in providing a feedback on the task (category a, theme 2) and to the short-term effects of the provided scaffolding (category a, theme 4). This is testified, for example, by this reflection: "It allows you to proceed with the right data and, even if it was not my case, it allows you to understand how to correctly write the differential equation and to understand your mistake". On the other hand, the following reflection on DMSE A testifies that often the focus on the problem solving process (category a, theme 3) and on the idea that the DMSE gives a feedback on the processing of the task (category b, theme 2) is combined with a focus on the long-term effects of the DMSE's support (category b, theme 4) and on the idea that the DMSE supports a relational understanding of Mathematics (category b, theme 5): "It is useful since it allowed me to autonomously find the solution, supporting to solve the specific case starting from the general formula. This helped me to improve my capability of problem solving".

FINAL REMARKS

In this paper we investigated university students' perspectives on the support provided by DMSEs included in the design of a DR aimed at scaffolding students' problem-solving processes in the context of problem situations that could be modelled through differential equations. The qualitative analysis of students' answers to a written questionnaire highlighted, in its first phase, theory-driven and data-driven themes emerging from students' reflections on the provided support and categories within each theme. The coding students' answers according to these categories, during the second phase of our analysis, enabled us, on one side, to identify the themes that are more recurrent in students' reflections on the support provided by each DMSE, and, on the other side, to highlight interrelations between specific categories of themes.

This shed light to the student's interpretations of the role played by the different DMSEs and to the possible utilization schemes that students could develop when interacting with specific DMSEs of the DR. In a previous work (Cusi & Telsoni, 2020) we categorized students' instrument-mediated action schemes in their use of a specific element in the design of a DR, associating each scheme to a function of the considered element (replacement function, diagnostic function and elaboration function). In the next steps of this research, we will refer to the results of the second phase of the analysis documented in this paper to characterize possible categories of instrument-mediated action schemes related to the use of specific DMSEs and corresponding functions of specific elements of the DR-design aimed at providing support at the metacognitive level. For this reason, we plan to combine a systematic analysis of students' answers according to the themes and categories presented in this paper, to a quantitative data analysis in order to map the students' answers' distribution with respect to the identified categories.

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THE USE OF FOCUSED AUTHORING DIGITAL TOOLS FOR LEARNING MATHEMATICS IN TASK DESIGN

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This paper focuses on a group of Mathematics teachers that choose a digital medium for teaching probability in the same junior high school. They design interventions with it, and though their discourse, a debate emerges between them. They follow different approaches in the classroom exploitation of these digital medium, they observe the lessons of one another, and they do not share the same view when they reflect on the classroom observations. Using CHAT (Cultural-Historical Activity Theory) we are interpreting this debate and the diversity of teachers' activities, as a chance for change in their professional practices. As a part of a broader one, this study refers to a single aspect of the role of design for teaching and the discourse around it, as a window to teachers' practices and their change.

Keywords: Design of digital media, Teachers' Professional Development, Probability, Covariation, CHAT.

INTRODUCTION

Based on research results, the study of Mathematics' teachers as learning intervention designers, with the exploitation of digital or non-digital media, has significant contribution in understanding of their practices related to teaching, and their views of the curriculum (Remillard, 2018). Moreover, studies on teachers as designers of digital learning resources (collaboratively or on their own), whether they are using them in teaching or not, makes the kind of knowledge they enable as they act as professionals, more transparent to the view of the researchers (Laurillard, 2018). In these two aspects of a teacher as designer, there is common ground; elements of Teachers Professional Development (TPD) may be apparent. Here, we present a part of a broader study, around the practices' developed by a group of Mathematics' school teachers who adapted and redesigned learning digital resources provided by the Ministry of Education to exploit them in their teaching. One of the total study's questions was to track elements of Mathematics' teachers practices that were related to their TPD. In this paper we present a small set from a cluster of results, focusing on two filters we used to analyze the participant teachers' practices: a) their references to the mathematics' curriculum and the school ecology, and b) their reflection on classroom situations, while designing, preparing implementing and redesigning lessons with the exploitation these digital resources.

THEORETICAL FRAMEWORK

In the field of TPD, the study of teacher as designer of learning environments with the use of digital media is not something new. Documentational Approach to Didactics (DAD) had already offered a viewpoint to instructional design and its versions as a passage to 'capture' and analyze teachers' professional development (Gueudet et al., 2012; Gueudet & Trouche, 2009). DAD focused on the schemes of use for certain digital resources, that each teacher enables or constructs; the same medium could be used differently and provide evidence for different professional approaches between teachers or between discrete teaching designs and interventions of the same teacher. Ruthven's approach focused on the teaching practices with the use of digital media, in view of the

school environment that the teaching performance took place (Ruthven, 2009), examining teachers' decisions and actions in this environment, taking in consideration critical aspects like the exploitation mode that can be informative around the issue of technology integration in teaching (Drijvers et al., 2010).

In our previous work we have focused on teacher as co-designer of educational material employing narrative and riddles, in collaboration with students (Diamantidis et al., 2019). In that study, the complexity of design as an activity was apparent; teacher adopted a hybrid role and had to make decisions related not only to learning, but to other factors, like to make a good narrative. So, in the present study, our approach is to capture instances of instructional designs in a holistic view, to grasp as much as possible of these complex aspects of teachers' decisions and actions as a unity; their schemes of use for the available resources, their view of the curriculum implementation, the impact of the school environment and rules on their design decisions or their answers to possible exploitation dilemmas, the interaction between them, etc. So, we made use of third generation cultural-historical activity theory (CHAT) (Engeström, 2001), to have a detailed view of participant teachers design and teaching actions.

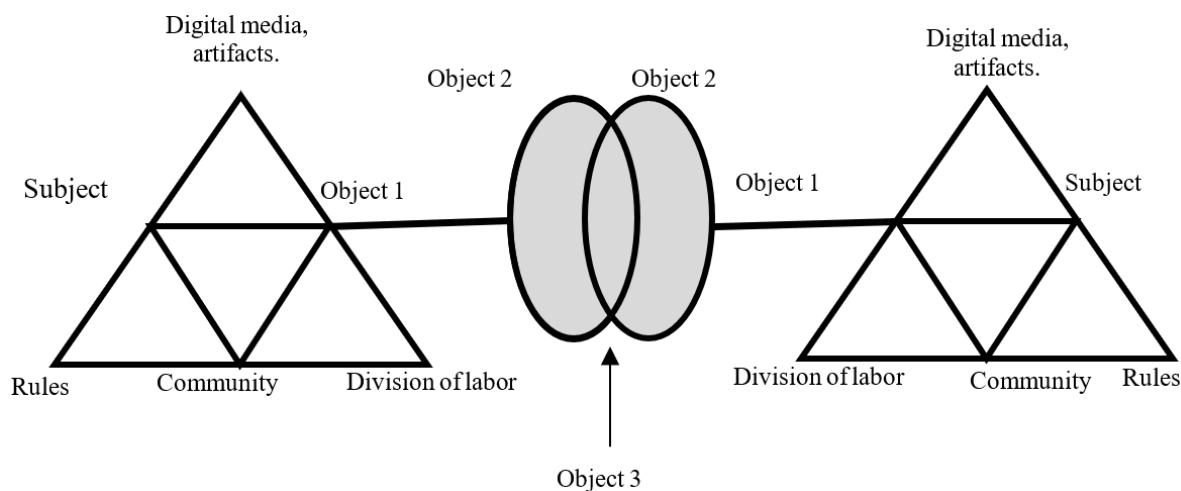


Figure 1: The CHAT 3rd generation activity system.

CHAT in the field of TPD research figure 1, can be used to interpret teachers' activity having in consideration all the elements that constitute it. In our case the activity is designing interventions with digital media for teaching Mathematics (designing/teaching). The subjects are the teachers, while rules and community refer to the school environment, as well as the division of labor (i.e., the teachers has to teach, and the students have to do their assignments, etc.). Object 1 is described by the goal of the activity, while object 2 is the result of the activity. If there are different teachers (subject) engaged in a similar activity (i.e., designing/teaching), then object 2 is not the same for every teacher, since there might be different motives (i.e., different teaching goals), etc. So, object 3 is the shared result, the consensus of objects 2 for each teacher.

From the point of view of CHAT, change in the activity and practices is welcome when tensions in the activity are apparent, that may lead to contradiction. This tensions may emerge in the activity system (the triangle of figure 1), or in the objects 1, 2 and 3 as well. In this study, we are searching for evidence in the activity 'designing/teaching' of mathematics teachers that may lead to change in their practices, according to the approach of CHAT.

METHOD

For this study, three Mathematics teachers of a junior high-school in Athens choose digital learning objects for Mathematics, from a Greek Ministry infrastructure called ‘Photodentro’ <https://photodentro.edu.gr/lor/>, called ‘micro-experiments’. Micro-experiment refers to a digital artefact that simulates the layout of an experiment addressing a challenge and provokes the student to use it and to be involved in inquiry (Kynigos & Grizioti, 2018). Micro-experiments are resources involving a buggy by design artifact, in order to provoke learning (Kynigos, 2007) but containing some closed questions readily matching equivalent questions from traditional curriculum exercises. Only here, there is one or two open-ended, constructionist or exploratory questions at the end (Kynigos, 2020).

The medium that the teachers of study used, was a simulator of a fair coin tossing (figure 2). They designed lessons for teaching probability, which in terms of learning outcomes -according to the curriculum means- that students should use theoretical probability to make models problems, to solve problems and to compare it with the results of the corresponding random experiments.

In the broader study, which this one is part of, teachers were involved to cycles of redesigning micro-experiment, one in each cycle, and using them in teaching, reflecting on it. There were 8 cycles in total. Following design-based research methods (Bakker, 2018) we made modifications after each cycle, while we refined our design principles and the conjecture mapping we started with. The researcher was a teacher of the school apart from the three, and had the role of a participant observer. The corpus of data produced and analysed was voice recordings of teacher’s dialogs during the design, recordings from the classroom during teaching, the researcher’s field notes and the versions of the digital resources that teachers adopted, redesigned, and used in classroom. For the need of this paper, only a part of these data was used.

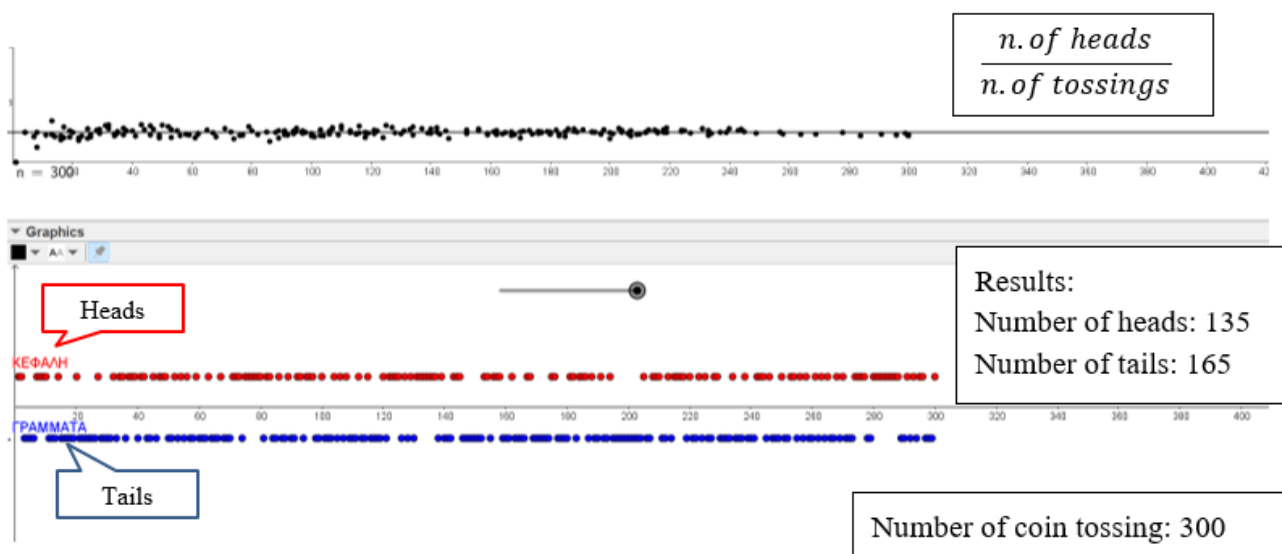


Figure 2: The micro-experiment, the way teachers redesigned it (initially).

RESULTS

During the 7th iteration of the study, the participant teachers decided to use the micro-experiment that is shown on figure 1. The graph on the upper cartesian plane represents the trace of a dynamically manipulated point that its y-coordinate is equal to the frequency of heads $\left(\frac{\text{number of heads}}{\text{number of tosses}}\right)$, while its x-coordinate is equal to the number of coins tossing. The horizontal line $y = 0.5$ that is also drawn, represents the theoretical value of the “head” event’s probability.

A debate on a suggested modification of the micro-experiment

The point’s position can be manipulated through dragging the slider (in the lower graph) which changes the number of coins tossing. The original version of the micro-experiment did not include the upper graph. It was an element that teachers added to the design of the medium.

- 096 T1: We should add a point that shows the frequency of heads, so that students, through dragging the slider, can see the variance of the results around the theoretical value of probability.
- 097 T2: Then, we should represent this value... [...]
- 107 T2: We could use a horizontal line, $y = 0.5$ and then have this trace of the point. Students could realize that there is a natural variability around 0.5.
- 108 T3: Yes, but this is not the spirit of the teaching guidelines we have! They should learn about probability, so I am thinking of asking them to investigate the relation of frequency with probability. I think I prefer two points. One red for the frequency of heads, and one black for the frequency of tails.
- 109 T1: Ok, but if we had that straight line representing the theoretical probability of ‘heads’, students could investigate also what happens when the number of coins tossing increases. Something like the law or large numbers. We could ask them these questions.
- 110 T3: Too ambitious, I think, but ok, let’s try it. However, I would like another version, like the one I said before. I am thinking of asking them to describe the way that these two ratios change, as the number of coins tossing increases. I will be very happy if they tell me something about their difference, and how it changes.

This small debate around the micro-experiment’s usage and learning exploitation led teachers to make another version of it, based on what T3 suggested (line 108). Instead of the upper graph in figure 1, there was another graph (figure 2). However, both versions were available for them to use.

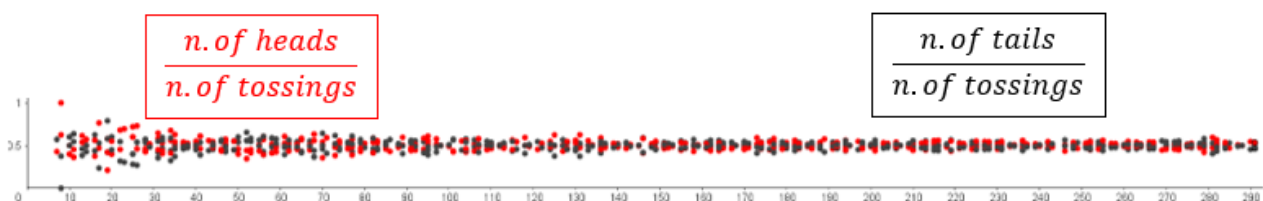


Figure 3: The micro-experiment, and the way teachers redesigned the upper graph, after the suggestion of T3. It has two points, one red for the frequency of heads, and one black for the frequency of tails.

Teachers reflect after their lessons with the micro-experiment

T3 used the version with two points on the upper graph, in her lesson, which T1 and T2 participated as participant observers. Students worked in pairs, in the school PC lab. Each pair had access to a PC with the micro-experiment on it. The main challenge that T3 addressed to them was to

investigate when the two ratios come close to 0.5. After the lesson a discussion took place around the learning outcomes that students achieved, from their point of view. We refer to students using their first names' initial.

- 351 T1: What about M and J, they seemed to be the ones that made a clear conclusion.
- 352 T3: Yes, they said that there was a symmetry between red and black point. And that these points came closer to each other and to 0.5 when the number of coins tossing increased.
- 353 T2: This was a good one. I think that they described the law of large numbers, somehow. And answered fair enough to your question.
- 354 T3: Or they just realised that the difference of two ratios decrease, as the number of coins tossing increases.
- 355 T2: So, from this point of view, they almost answered to your task!
- 356 T3: Almost! At least, they realised that when the number of coins tossing increases, the ratios are mostly near 0.5. But they did not relate this conclusion with the theoretical value of probability.

T1 used the first modification of the micro-experiment (the one in figure 1) for his teaching. The orchestration of the classroom was like the one of T3, while T2 and T3 had the role of participant observers. T1 asked his students to investigate what happens when the number of coins tossing increases. Since the lesson of T1 followed the lesson of T3, the discussion about T1's lesson was somehow comparative.

- 509 T3: I think that you students made it better. For example, K and I said that when the number of coins tossing increases, the point stays close to the horizontal line, which is the theoretical value of "heads" probability.
- 510 T1: Yes, that was a good answer. But also, C and D made a very good conclusion.
- 511 T2: I totally agree! Listen to what they said "if you move the slider to the right, then you know that there is a big number of coins tossing and a small variability of the point around the horizontal line. But when you move it to the left, you have less coins tossing and big variability around this horizontal line". What else do you want them to say?
- 512 T3: Yes, very good observation, but is it what we want them to realise? It seemed to me more related to the graph, than the concept of probability.
- 513 T2: It doesn't matter. The slider helped them conceptualize the connection between the number of coins tossing and the variability of the heads' frequency.
- 514 T1: So, it seems that we could use this micro-experiment in more than teaching probability.

In the extracts above, it seems that the three teachers do not share a common view, neither about the learning goals of probability in 9th grade, nor about the way students addressed the two tasks. In the next paragraph we will try to interpret these findings using the analysis of the designing/teaching activity of T1 and T3.

DISCUSSION

Both teachers' activity systems' elements are common. The digital medium is mostly the micro-experiment, or other digital artifacts that simulate coin tossing. The subject is T1 and T3 in each

case, while all teachers along with the students belong to the community. The curriculum could be construed as a text that describe some rules, along with the duration of the school hours, etc. The division of labor corresponds to the roles of each teacher in the classroom, i.e. during the first lesson T1 and T2 were participant observers, where T1 was the teacher. The object of each activity is described in the table 1.

Subject	T3	T1
Object 1	To design/teach with the micro-experiment	
Digital medium	The micro-experiment, other related simulations	
Community	T1, T2, students	T2, T3, students
Rules	The curriculum, teaching guidelines	
Division of labour	T3 is the teacher, the other teachers are participant observers. Students are addressing a challenge, which should focus on the concept of theoretical probability.	T1 is the teacher, the other teachers are participant observers. Students are addressing a challenge, which focuses on the concept of theoretical probability, but there are more concepts that they are welcome to conceptualize through the investigation.
Motive	Students to be engaged in an investigation around the theoretical value of probability.	Students to be engaged in an investigation around the concept of probability.

Table 1: Designing/teaching activity system of T1 and T2

Also, the last line of table 1 refers to the motive of each activity, which is crucial since the activity can be described by its motive. It seems that T3 designed her teaching intervention having in mind the restriction of the curriculum, according to her (line 108), while T1 seemed to think that it was legitimate to leave an open door for students to investigate the concept of probability in a broader field (lines 096, 109). T1's opinion about students' answers in both lessons (lines 356, 509, 512) showed that she expected from students to focus on the concept of theoretical probability, while T3's view was more tolerant -if not receptive- to answers indicating that students made preliminary meanings about the law of large numbers, or the natural variability of frequency around the value of theoretical probability.

Moreover, in line 510 T1 said that C and D made a "very good conclusion". These two students coordinated two quantities through the movement of the slider; the first quantity was the number of coins tossing, while the second was the variability of heads' frequency around the horizontal line. This coordination could be interpreted as a conceptualization of covariation (Thompson & Carlson, 2017); students used the slider as a representation of a multiplicative object which coordinates the two quantities that vary. Students refer to the slider as an operative image of coins' number and variability covariation. Both quantities having been coupled and each of their values corresponds to the position of the slider (right or left). T1 seems to realize that students' actions and observations may extend the possible use of the micro-experiment (line 514).

Finally, we could say that object 2, for T1 was the designing/teaching for probability and other concepts related to the natural variability of random experiment results, while for T3 was design/teaching for students to be engaged to the investigation of probability as a concept in relation with the frequency of "heads" and "tails" events. So, in the description of object 3 it seems that a contradiction evolves. There is a tension between the description of object 2 of T1 and the object 2 of T2. This tension was apparent in the extracts, above, as well as into their teaching strategies.

Tensions in teachers' actions, like in this study, may end up to contradictions inside the activity system, according to CHAT (Engeström & Sannino, 2010). A contradiction is the starting point of

a novel practice, a new activity. So, in case of professional actions and practices, these tension and contradictions may lead to the change/development in professional level.

CONCLUSIONS

The teachers in this study were engaged in designing/teaching activity with the use of a digital medium. Through this activity a debate became apparent, as an opportunity for the teachers' activity to develop. What was the nature of this debate, and the aspects of the tension that was related to it? One aspect could be the agency of the teachers involved; the tension started to emerge from the diverse design agencies of T1, T2 and T3. According to Monaghan (2016), the different modifications of T1 and T3 for the micro-experiment shows their different views and their diverse agencies of what they wanted to do with it. But these agencies became apparent not independent from the digital medium, the micro-experiment. They probably could not be visible -not the same way- without it. I.e., in the case of T1, his initial design ideas referred to the variability of 'heads' frequency around 0.5. In his lesson students conceptualized the slider as an image of coordinating the number of coins tossing to the variability. T1 seemed to welcome this. A possible explanation could be that T1 thought of variability to be related to the conceptual field of probability (Vergnaud, 2009), but this is not something that was clearly evident. However, the detailed study of the design/teaching activity, with CHAT could be revealing for the nature of these tensions, and correspondingly to the mechanism that may lead to the change of professional practices of teachers that use digital media in teaching Mathematics.

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THE 3D PRINTER IN KINDERGARTEN EDUCATIONAL ACTIVITIES

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Abstract. Nowadays 3D technologies play an important role in the world of education. Students should acquire digital skills that enable them to become producers of digital artifacts themselves. The acquisition of STEAM (Science, Technology, Engineering, Art and Mathematics) skills increasingly leads to integrated learning based on dialogue and critical thinking. Maker education, based on the pedagogy of doing, should fully enter in the daily activity in the classroom. A kindergarten teachers' group through the participation in an INDIRE¹ experimental research project are helping to prepare their children for success in the future by providing them with valuable STEAM skills and fostering their creativity and critical thinking abilities. This paper aims to discuss the value of Maker Education and the main strategies for organizing and planning its implementation to help young children acquiring STEM skills since an early age.

Keywords: Maker Education, 3D Printer, Tinkering, Artefacts.

INTRODUCTION

3D printing technology is revolutionizing the way objects are manufactured, and prototypes are created. It is impacting various sectors, from technology to art, and is significantly transforming the traditional craft and production system. This technology has provided a strong boost to the world of creativity. Incorporating 3D printing into classrooms, from kindergarten to high school, provides the educational system with a new way of teaching digital skills. This enables students to move beyond simply being users of digital technology and become producers themselves. They can design and create digital objects of various types and characteristics on their own, which can help them develop a deeper understanding of the technology and its potential uses. This can be a valuable way for students to learn about design, prototyping, and problem-solving. 3D printing in the classroom can also help students develop skills in creativity, teamwork, and critical thinking. By creating digital objects, students are able to utilize a variety of skills, including design and knowledge of 3D design and modelling software. This process helps students gain technical skills, but it also provides opportunities for them to learn and practice empathy, teamwork, and problem-solving. The process of designing and creating digital objects can foster collaboration and communication among students, encouraging them to work together to find creative solutions to problems. Furthermore, the use of a 3D printer in the classroom can help students develop advanced critical thinking skills. The process of designing and creating objects using a 3D printer requires students to think creatively, analyze problems, and make decisions. They must also evaluate their designs, make adjustments, and troubleshoot any issues that arise. This process encourages students to think critically and develop problem-solving skills. Additionally, the use of a 3D printer can also help students develop spatial reasoning and visual-spatial skills, which can be useful in a variety of fields. Overall, the use of a 3D printer in the classroom can provide students with hands-on learning opportunities that can help them acquire cognitive tools that help them think creatively, solve problems in a more effective and efficient manner, and develop problem-solving skills and ability to think outside of the box. Creativity and lateral thinking, the ability to preview a product are all skills that mature through laboratory activities. Understanding that redesigning an object is a normal part of the process can help foster a growth mindset and reduce the fear of failure. This educational scenario explains why in recent years, in education, the “Maker Movement” (Halverson & Sheridan, 2014) is spreading more and more. The “Maker Movement”, based on the traditional world of “do

¹ the National Institute for Documentation, Innovation and Educational Research – <https://www.indire.it/>

it yourself’ applied to modern technologies, emphasizes hands-on learning, creativity, and problem-solving skills, which aligns with the goals of modern education. The Maker Movement has evolved over time and led to the development of a new type of artisan, the technological artisan. Today, these technological artisans refer to themselves as "makers" and are involved in producing or modifying engineering, electronic, or robotic products, (Guasti, 2017). This shift in the traditional craftsman profession reflects the growing importance of technology and the integration of traditional crafts with modern technologies. This approach to education fosters creativity, critical thinking, and problem-solving skills, which are essential for success in the modern workforce. Additionally, the Maker Movement encourages students to embrace failure as a learning opportunity, promoting a growth mindset and resilience (Capone *et alii*, 2019). By integrating the principles of the Maker Movement into their curricula, schools aim to prepare students for the challenges of the future and equip them with the skills they need to succeed in a rapidly changing technological landscape.

In this work, our research wants to investigate on how *an educational path that involves creating artifacts with 3D printers may increase logical thinking and creativity in kindergartners*.

A Kindergarten teachers group participating in the INDIRE experimental research project “*Building toys with the 3D printer in the kindergarten*”², has been engaged in activities such as designing and printing toys with a 3D printer, building and assembling toys, experimenting and testing toys, collaborating and sharing with their students. The project aimed to help young children develop STEM skills and creativity through hands-on Maker activities, and the teachers participated by incorporating these activities into their curriculum. By participating in this project, the teachers have gained valuable experience and insights into how Maker Education can be effectively implemented in a kindergarten setting. They have also helped their students acquire valuable STEAM skills, including design, engineering, and technology, as well as creativity, critical thinking, and problem-solving abilities. Overall, the participation of the Kindergarten teachers group in the INDIRE research project has helped to advance the field of Maker Education and will likely have a positive impact on the children they teach for years to come.

THE MAKER EDUCATION

The “*Maker Education*” aims to develop practical skills, critical thinking and problem solving through an active and engaging learning experience: to design and create objects through technologies. It is based on the "Think-Make-Improve" cycle (Martinez & Stager, 2013) and promotes collaboration and peer comparison to encourage social learning (Vygotsky, 1962); moreover, this methodology spreads the application of collaborative and inclusive teaching methodologies such as Peer Education (Damon, 1984). This type of teaching is capable of developing a range of competencies, including creativity, collaboration, and technology literacy, that are crucial for success in the modern world. By placing the student at the center of the learning process and providing opportunities for hands-on problem-solving, the Maker Education prepares students for the challenges of the future and helps them develop the skills they need to succeed. The three phases of the "Think-Make-Improve" cycle can be described as follows:

- *Think* - Students engage in brainstorming sessions with teachers and peers to generate ideas and hypothesis for a project they are working on. They also use traditional methods such as drawing and coloring to create visual representations of their ideas.
- *Make* - Students move on to the implementation phase, where they bring their ideas to life. This may involve using software like Tinkercad to model 3D objects or printing out the designs.

² <https://www.indire.it/en/progetto/building-toys-with-3d-printers-3/>

- *Improve* - Students assess the functionality of the finished product and make improvements if necessary. If errors are found, they engage in discussions and further iterations to enhance the original design. This cycle allows for continuous improvement and emphasizes the importance of hands-on learning and critical thinking.

The Maker movement considers invention and creation of objects a fundamental human activity, and technology allows maximize this attitude. Makers are inventors, authors, and researchers who design and self-produce in their workshops, known as makerspaces or fabLabs, all sorts of mechanical equipment, electronic instruments, software, and robotic creations. In schools, the makerspace provides students with a laboratory approach to problem-solving through design and experimentation, combining traditional and innovative aspects of manual work. In the makerspace, students must be stimulated to interpret, to make judgments about the shape, size, movement and relationships between surrounding objects, to predict and manipulate 3D models. In schools, the makerspace provides students with a laboratory approach to problem-solving through design and experimentation, combining traditional and innovative aspects of manual work. The teacher's goal becomes to increase students' design knowledge, operational and instrumental skills in using design programs (such as 3D CAD) and printing procedures, as well as developing a problem-solving approach in their teaching skills.

RESEARCH AND THEORETICAL FRAMEWORK

Hands-on learning experiences with 3D printing technology can help students develop problem-solving skills, critical thinking, and creativity by allowing them to bring their ideas to life. In this work, our research question is “*may an educational path that involves creating artifacts with 3D printers may increase logical thinking and creativity in kindergartners?*”

The theoretical framework where the maker teaching may be included, is the papertian constructionism. According to the theory of experiential learning (Kolb,1984) and the Piaget's theory of genetic epistemology (Kitchener, 1986), individuals construct mental models to understand the world around them, and they learn best through direct experiences and reflection on those experiences. Papert's perspective (Papert, 1986) aligned with these theories emphasizes that children learn by doing and through artifacts. According to Papert, children and adults learn more deeply when they have the opportunity to explore, experiment, and manipulate objects. He proposes that experiences should be at the center of the educational process. It is necessary on providing children with opportunities to engage in creative activities that stimulate their creative thinking. In this model, the act of constructing artifacts serves as a facilitator of learning. As children engage in these activities, they not only acquire knowledge but also structure their understanding of the world parallel to the process of constructing objects. In summary, the theories of constructionism on the role of physical objects in learning, support the idea that a maker-based educational approach, which involves the use of 3D printers in the learning process, can contribute to the development of logical thinking and creativity in kindergartners. This approach is situated in practice and linked to the student's project, allowing them to learn through hands-on experiences and problem-solving. The use of 3D printers enables the creation of practical and real-world learning experiences, promoting a dynamic process of knowledge construction and adaptation.

AN EXPERIMENTAL PROJECT

In this paper we describe the activities of INDIRE³ experimental research project “*Building toys with a 3D printer in kindergarten*”⁴ within the “*Maker@Scuola*”⁵ project.

³ the National Institute for Documentation, Innovation and Educational Research

⁴ <https://www.indire.it/en/progetto/building-toys-with-3d-printers-3/>

⁵ <https://www.indire.it/en/progetto/maker-at-school/>

The Maker@Scuola project is a national ministerial initiative in Italy that focuses on evaluating the effectiveness of "maker space" and Fab Lab models within the Italian school system. The project involved eight comprehensive schools located in different contexts, between large cities and smaller towns. Initially, the project went through an exploratory phase after which the project team gathered valuable insights and data. However, the results of the experimental phase are currently owned by the Ministry. At present, the project team is awaiting authorization from the Ministry to publish the data. Consequently, the information available in this document will be limited to the results derived from the exploratory phase of the project.

The “Maker@Scuola” project examines the impact of the “Maker Movement” on school laboratory teaching. It aims to determine if the use of innovative tools and teaching methods in the classroom can improve traditional frontal teaching and support a more current innovative teaching in which the pupils become the protagonists of the own learning. The “Maker” approach to teaching has been shown to improve students’ logical-mathematical, scientific, and linguistic skills, and emphasizes the development of meta-competences and soft-skills (Dougherty & Conrad, 2016; Martinez & Stager, 2013; Peppler, Halverson, & Kafai, 2016). It encourages a more participatory and problem-solving approach, where mistakes are viewed as opportunities for reflection and growth, rather than failures. The learning process, typically in the laboratory, is based on improvement cycles where errors are utilized as a learning opportunity. Schools that adopt the “Maker” teaching methods view new technologies as a driving force for creating innovative ways of learning and not just as a means to optimize the existing education system.

The “Maker” approach to teaching is based on:

- a “*hacker*” approach to knowledge, according with which pupils acquire knowledge by disassembling things, analyzing their workings, and using the acquired knowledge to create new and more interesting things. This approach emphasizes hands-on learning by modifying software and hardware, with the goal of gaining a deeper understanding of systems and the world around us. This way of learning is in line with the principles of the "Maker Movement" in education, which seeks to create an engaging and participatory learning environment where students take a more active role in their education. (Steven Levy, 1984)
- a “*tinkering*” methodology, based on the think-make-improve cycle that guides students through the entire process and helps them to reflect on their work, making revisions and improvements along the way. Students, infact, first ideate, plan, and brainstorm; then create, program, and prototype; and finally verify and improve their work. Mistakes are seen as opportunities for progress, not failures.
- an open “*sharing*” philosophy, that values collaboration and knowledge sharing, promoting an open and inclusive learning environment where students are encouraged to ask questions and share their ideas with each other. Copying is encouraged as a learning activity because promote dialogue and mutual influence among students.

The experimental research project "Building toys with a 3D printer in kindergarten" was conducted in several phases. The first phase, carried out in the years 2014/2015 and 2015/2016, focused on creating an didactical environment for experimenting with a 3D printer in a kindergarten setting. Since there was no existing literature on this topic, this phase involved establishing the necessary foundations for the subsequent phases. In the following year, 2016/2017, an exploratory trial of the designed didactical instrument was conducted. This phase likely involved implementing the 3D printer and allowing the children to engage with it to gather preliminary insights and observations. The subsequent phases, conducted in the years 2017/2018 and 2018/2019, aimed to really measure the impact of using the 3D printer in kindergarten. Initially, a pilot experiment was conducted to refine the experimental procedures and measurement tools. Subsequently the complete experimentation, where data were collected. The data collected are currently owned by Ministry.

Therefore, the focus of this paper is on analyzing the outcomes the design phase where the learning path was developed and administered to the children.

RESEARCH METHODOLOGY

We describe the educational approach studied by the teachers at the “Mother Teresa of Calcutta” Kindergarten in San Valentino Torio, Salerno, Italy, as part of the INDIRE experimental research project “*Building toys with a 3D printer in kindergarten*”. This project aims to explore the potential of using 3D printers in the learning process for kindergartners and how it may impact their logical thinking and creativity. The methodology employed in this research project is qualitative. Observations were conducted on the pupils who participated in the educational activities. Additionally, the Video Club (Sherin and Han, 2004) technique was utilized to assess the feasibility of the instructional approach and its potential impact on the development of logical thinking and creativity. The video recordings of selected aspects of the teaching practice served the purpose of stimulating discussion and facilitating shared reflection among the teachers. This collaborative analysis aimed to finalize and identify the crucial elements of the conducted experience and to identify areas where new practices or interesting variations were observed.

PARTICIPANTS

The sample consisted of pupils and teachers all from a Kindergarten school located in a rural context of a city in Southern Italy (San Valentino Torio, Salerno, Italy). Participated in the experimental project 5 teachers and 41 children. The average age of the children in the experiment group was 4.30 (SD 0,38). Only one of the participants had neurological problems.

PROCEDURE

The didactic path created by the teachers within this project is an example of “Maker Education” in practice. The “didactic maker” experience in the classroom involves using a 3D printer as a teaching tool to enhance logical thinking in young children. The project, which was implemented in a kindergarten, involved using 3D graphic modeling software (Doodle 3D and Tinkercad) to complete specific tasks based on a story entitled “*A strange theft*” about a family of elves recovering an object stolen by an ogre (Guasti & Niewint-Gori, 2017). The aim of the project was to experiment with the educational use of a 3D printer and see if it could contribute to enhancing these cognitive skills in young children. Overall, the project aimed to integrate the use of 3D printing technology into daily teaching practice to provide hands-on learning opportunities that can help students develop valuable skills that can be applied in various fields. All tasks involved designing a product to be 3D printed. The object design phase was always preceded by preparatory activities functional to learning the concepts subset the realization of the task, while the following one the printing concerned discussions about product evaluation. Teachers, through stimulating questions, encouraged children to reflect on the object correspondence to what was in the story required and on the contrary, to find the errors and the possible solutions to solve them. The error is no longer seen in its negative sense but as an opportunity to stimulate improvement. (Zan, 2007)

- The first task is “Drawing the characters of the story with 3D Doodles”. The teachers introduced the 3D printer during a circle time session and told the part of the story related to the task, which involved printing all the characters with Doodle 3D as the final product.
- The second task: “Building the trunk of a hollow tree, home of a good witch, with the Tinkercad software”, required the printing of a hollow tree as the final product.
- In the third task “Build the layered cake” the children discussed the overlapping of solids.
- The fourth task “Make footprints with the 3D printer” required the construction of a path that would later have to be walked by the characters.

- The fifth task "Building a bridge with Tinkercad" required that at the end of the path there was a river to be crossed by building a bridge.
- The sixth and final task "Making the special game" gave children space to free their imagination by creating one with Tinkercad to be delivered to the mischievous ogre.

The tasks provided the teachers with a valuable learning experience from both a technological and educational perspective. They likely gained hands-on experience with using Tinkercad software and 3D printing technology, as well as insights into how these tools can be used effectively in the classroom to enhance student learning and creativity. The second task, building the hollow tree trunk, involved the children applying the "Think-Make-Improve" cycle. In the "Think" phase, they discussed and agreed on using a cylinder with a hole as the base shape. In the "Make" phase, they showed proficiency in modifying basic shapes, but had difficulty in creating a hole in the trunk. The "Improve" phase involved printing several versions of the trunk and observing and discussing any errors or design flaws, allowing for refinement and improvement of the final product. Throughout the task, the teacher played a *facilitative* role, allowing the children to freely express their ideas and perspectives.

DISCUSSION

During the Video Club Meeting, the teachers utilized their skills in analyzing the interactions between children and new technologies, as well as the thought processes of the children. The purpose was to gain insights into how the implemented teaching practices, specifically those involving the use of a 3D printer, could either confirm or challenge existing methods of teaching intervention aimed at enhancing children's creativity and logical skills. Through the analysis of the videos, certain findings or observations emerged. Particularly, during the activities, the children showed interest and curiosity in printing their own designed objects, but also important reflections on why their printed objects were incorrect. Here are some excerpts from the conversation as evidence of what was said:

Marco:	Wow, I can touch my drawing! (It is the first reaction of the children when for the first time they see their printed object).
Maria:	The tiered cake is broken because I didn't use the Merge function, (Referring to the tiered cake that didn't come out whole but three separate cylinders)
Francesca:	The tree is not a case because the sphere was not transparent. (Referring to the realization of the hollow tree, unsuccessful)
Michele:	The elf's foot is smaller than the ogre's, so the elf has to walk more steps than the ogre to reach the house. (Referring to the comparison between the ogre foot print and the elf foot print for stamp making)

Maria and Francesca's comments are very significant, as they not only highlight the mastery of the technological tool that the children have acquired, but also and mainly the acquisition of a specific mathematical language. At the end of the project, the teachers were able to verify how it had become natural for the children to use terms specific to mathematics teaching.

Michele's comment, instead, shows how the comparison of the footprints is a simple and effective way for young children to learn and understand the concept of measure. By observing the difference in the size of the footprints and the number of steps required to reach the house, children can develop an intuitive understanding of how measure is used to describe and compare physical quantities. This hands-on approach to learning can make the concept of measure more meaningful and accessible to young children. It is an excellent example of how play and exploration can be used to support the development of mathematical thinking.

RESULTS

The results obtained by following an experimental path in functional spaces related to the concepts of making and production workshop through the use of 3D printing push us to continue the research towards the hypothesis of a new curriculum capable of promoting innovative educational scenarios aimed at developing geometric and spatial-visual skills starting from early childhood. Compared to three-dimensional manipulative activities such as Lego and Pongo, which allow modification of the project in progress, working with a 3D printer requires children to pay particular attention during the design phase, which favors the formation of concepts through the recognition of invariants that characterize certain figures. In conclusion, the experimentation showed that new technologies, such as 3D printing and Tinkercad software, can be effectively utilized in the classroom as teaching aids to optimize instructional procedures, as well as to facilitate new modes of learning and conceptualizing. The support from INDIRE researchers and collaboration with teachers from other participating schools helped the teachers come to this realization.

CONCLUSIONS

Maker Education is valuable as it encourages hands-on, experiential learning and helps children develop STEM skills, creativity, critical thinking, and problem-solving abilities. The Kindergarten teachers group of St. Valentino Torio participating in the INDIRE experimental research project “*Building toys with the 3D printer in the kindergarten*” have showed how incorporating a 3D printer in their activities can be effectively implemented Maker Education and help young children develop STEM skills and creativity. Some of the educative activities that the teachers may be using include:

1. Designing and printing toys with a 3D printer: Children can learn about design, engineering, and technology by creating their own toys using a 3D printer.
2. Building and assembling toys: By working with the printed parts, children can learn about construction, problem-solving, and fine motor skills.
3. Experimenting and testing toys: Children can learn about scientific concepts and the process of experimentation by testing and evaluating the toys they have created.
4. Collaborating and sharing: By working together on Maker projects, children can develop teamwork, communication, and social skills.
5. The fact that the children also showed important reflections on why their printed objects were incorrect is also significant. It indicates that they were thinking critically and actively trying to improve their designs. This type of problem-solving and critical thinking is a valuable skill that will benefit them in many aspects of their lives.

By engaging in these activities, young children are acquiring valuable STEAM skills and developing their creativity, critical thinking, and problem-solving abilities. The teachers are also learning how to effectively integrate Maker Education into their curriculum, helping to prepare children for success in the future. Overall, these results suggest that the Kindergarten teachers group of St. Valentino Torio's participation in the INDIRE experimental research project was successful in engaging the children and helping them develop important STEM and problem-solving skills.

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DO DIGITAL TEXTBOOKS OFFER NEW OPPORTUNITIES FOR GEOMETRY EDUCATION? AN ANALYSIS OF TASK FEATURES

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While the tasks in printed mathematics textbooks have been extensively examined worldwide, the requirements of the tasks in digital textbooks are yet to be developed and analyzed. The research presented in this paper utilises a framework developed for analyzing e-tasks and applies it to digital curriculum materials. The aim was to determine what features and potentials are present in the tasks in digital mathematics textbooks. The framework is applied to the content of the geometry chapters of the six digital textbooks for Grade 4 in Croatia. The results show that the geometry tasks in the digital textbooks do not provide a full range of digital potentials, such as dynamics, personalization and cooperation. These findings highlight the potential of digital tasks as an area to be explored and developed.

Keywords: digital textbooks, textbook tasks, task analysis, geometry

INTRODUCTION

In the past two decades, along with the development of digital technology, digital mathematics textbooks have been developed and published in many countries (Fan et al., in press; Pepin et al., 2017). Digital textbooks, as a part of digital curriculum materials, may include new features such as dynamics and interaction, formative assessment and customization, and offer links to multimedia and viral communities (Choppin et al., 2014). These features, not available in printed textbooks, create opportunities for changing mathematics instruction and providing “new educational dynamics” (Pepin et al., 2017, p. 646). These new dynamics also include tasks in digital textbooks.

Textbook tasks create student opportunities for learning mathematics (Sullivan et. al., 2013) and may influence the way students think and “can serve to limit or to broaden their views of the subject matter with which they are engaged” (Henningsen & Stein, 1997). This may be applied to digital tasks as well, particularly because they “can extend and amplify pedagogical features present in non-digital environments” (Leung & Baccaignani-Franck, 2017, p. ix). Potentials such as dynamics are of special interest in geometry education because digital materials may support problem solving, exploring and learning geometry through visual and dynamic environments.

Research on tasks in printed textbooks in recent decades has provided many analyzing frameworks (e.g., Glasnović Gracin, 2018; Zhu & Fan, 2006). However, the new features of digital tasks present new challenges in terms of developing frameworks for the analysis of e-tasks. The already developed frameworks mainly refer to the categories of digital platforms (e.g., Bergval & Dyrvold, 2021; Choppin et al., 2014; Pepin et al., 2017), while the studies which aimed at the analysis of digital *tasks* focused on their particular feature, such as types of feedback in Schute (2008). The aim of this study is to present a multi-dimensional framework for exploring tasks in digital textbooks and in other curriculum materials and to apply it to e-textbooks developed for primary mathematics education.

THEORETICAL FRAMEWORK

Glasnović Gracin and Krišto (2022) provided the framework for the analysis of digital tasks in mathematics textbooks. It is based on the literature review concerning the special features of digital

materials and contains the following categories: Interactive diagrams for exploring mathematics, Personalization in learning, Task form, Feedback, and Cooperation (see Table 1).

Dimension	Details and codes
Interactive diagrams for exploring mathematics	Yes – manipulating objects of dynamical geometry (D1)
Personalization in learning	Yes (P1) Which type of personalization? _____
Task form	Multiple-choice questions (TF1) Fill-in-the-blank responses (TF2) Matching (TF3) True or False (TF4) Put in order (TF5) Other closed forms (TF6) Open-ended questions (TF7)
Feedback	No feedback (F1) Verification (F2) Correct response (IF1) Try again (IF2) Error flagging (IF3) Elaborated (IF4) Hints/cues/prompts (IF5) Other (IF6)
Cooperation	Yes (COO1)

Table 1. Framework for the analysis of digital tasks (Glasnović Gracin & Krišto, 2022)

Interactive diagrams for exploring mathematics refer to tasks which provide dynamics for exploring and understanding mathematical concepts, for example, figure transformations, change of function values, and changing parameters (Usiskin, 2018). Personalization in learning refers to customizing features according to individual student needs and for formative assessment (Pepin et al., 2017; Usiskin, 2018). Task form contains different ways in which a particular digital task is presented, for example, multiple-choice or fill-in-the-blank (Glasnović Gracin & Krišto, 2022; Pepin et al., 2017). Cooperation refers to the possibility for the student to share the workspace with others and provide collaboration (Pepin et al., 2017). The feedback category is organized according to Schute's (2008) categorisation of different feedback types in digital tasks: no feedback means that the student does not know whether or not the answer is correct. Verification feedback means giving a right/wrong response. Correct response means providing the correct answer for the specific task, without additional explanation, and try again refers to the possibility of trying until the correct answer is given. Error flagging highlights incorrect answers in the task, but does not offer the correct answer. Elaborated feedback means giving an explanation why a specific response is correct or incorrect. Hints/cues/prompts are part of elaborated feedback which guide students in the right direction by

giving them, for example, examples or suggestions on what to do next. Still, the correct answer is not explicitly given.

This multi-dimensional conceptualization of task features in digital textbooks raises the question of to what extent the digital textbooks offer the full range of task types. Related to that, the following research question is formed for this study: What task features are present in the geometry lessons of digital mathematics textbooks?

METHOD

This empirical study encompassed analyzing the tasks of all six digital textbooks for Grade 4 in Croatia, which are the most frequently used textbooks for Grade 4 approved by the Ministry of Science and Education. In Croatia, elementary teachers independently choose the textbooks they like each year. According to the Textbook Standard (Ministry of Science and Education [MZO], 2019b), textbooks consist of both the printed and digital versions. Digital versions of textbooks contain at least one of the following features: (a) dynamical representation (sound, animations, videos, etc.), (b) simulation (virtual experiments, interactive videos, etc.), or (c) interaction student–student/teacher and student–content. Student–content interaction refers to interactive quizzes with feedback, didactical games, augmented reality, etc. (MZO, 2019b). In this study, a task is considered as a request for initiating student activity and requiring student answer. The analysis encompassed all the tasks in the digital textbooks provided for acquisition and practice in the geometry chapters in which the student should give their answer in the digital space provided. The six textbooks were coded as A, B, C, D, E and F. Altogether, the analysis involved 551 tasks (164 in digital textbook A, 74 in B, 81 in C, 64 in D, 51 in E, and 117 in F).

The reason for considering only Grade 4 was because, according to the curriculum (MZO, 2019a), this grade contains the most geometry content in Croatian primary education. In Grade 4, students learn about angles, triangle types, the circle, radius, measuring area, units of area, and the square grid.

The analysis instrument is an extended version of the framework of Glasnović Gracin and Krišto (2022) given in Table 1. The extension refers to adding the Measuring response time dimension with its subdimensions focusing on whether the measured time is limited or not (see Table 2). This dimension was added because it represents another feature of digital tasks, not present in previous sources.

Dimension	Details and codes
Measuring response time	Yes – time is measured (TM1) Time is measured, but without a time limit (TM1.1) Time is measured and there is a time limit (TM1.2)

Table 2. The added dimension Measuring response time

Also, the modified framework presented in this paper refers to refining the dimension Interactive diagrams for exploring mathematics (D1) into D1.1 (dynamics is required, e.g. drawing, dragging, but without exploring requirement) and D1.2 (exploring is specifically required).

Each of the tasks was analyzed and assigned codes in each category (see Tables 1 and 2). One of the authors conducted the coding and the other author analyzed 10% of the tasks. The matching was high (99%), and the inconsistencies were discussed. The obtained data were processed using relative frequencies of codes within a particular category.

RESULTS

The examination showed large discrepancies in the number of digital tasks offered in the textbooks. Textbook A has the most tasks (164), while textbook E has more than three times less tasks (51). The results are presented according to the framework dimensions.

Interactive diagrams for exploring mathematics

The results revealed that most of the analyzed digital textbook tasks do not involve interactive diagrams for exploring ideas and relations of geometry (see Table 3). Digital textbook C (DTxb-C) offers dynamics in 23% of geometry tasks for Grade 4, while textbook B has only one such task. The tasks in textbook C that refer to this dimension require drawing geometrical objects in dynamic geometry applications, but mostly without exploring requirements. The other four digital textbooks do not contain any tasks with interaction for exploring mathematical ideas.

	DTxtb-A (n = 164)	DTxtb-B (n = 74)	DTxtb-C (n = 81)	DTxtb-D (n = 64)	DTxtb-E (n = 51)	DTxtb-F (n = 117)
D1	0.00%	1.35%	23.46%	0.00%	0.00%	0.00%
D1.1	0.00%	0.00%	17.28%	0.00%	0.00%	0.00%
D1.2	0.00%	1.35%	6.17%	0.00%	0.00%	0.00%
P1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
COO1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Note. D1 – opportunity for manipulating objects of dynamical geometry, D1.1 – dynamics without exploring, D1.2 – exploring is specifically required, P1 – personalization in learning present, COO1 – cooperation present.

Table 3. Dimensions of Interactive diagrams for exploring mathematics, Personalization and Cooperation in digital textbook tasks

Personalization and Cooperation

The results revealed that none of the textbooks analyzed contains any tasks which support personalization or cooperation in learning (see Table 3).

Task form

Regarding task form, the results (see Table 4) showed that all the examined textbooks contain tasks with multiple-choice questions (TF1), fill-in-the-blank responses (TF2), matching (TF3), and true or false items (TF4). The overall results imply that the multiple-choice tasks are the most frequent in textbooks A, B, E and F, making about 40-45% of all digital tasks in the geometry chapters. Analysis of the textbooks shows that these tasks occur mainly as parts of different quizzes, but also as separate tasks. Fill-in-the-blank tasks make between a quarter and a third of all tasks in each textbook examined, except for textbook D (11%). But textbook D puts strong emphasis on matching requirements, half of its geometry tasks take this task form. About 20% of all geometry items in textbooks E and F require True/False answers, while the other digital textbooks contain this task type to a much smaller extent. The task forms put-in-order and open-ended-questions are minimally or not at all present in the in geometry chapters of the examined textbooks. Interestingly, 17% of tasks in textbook C are other closed forms (see Table 4). These tasks refer to drawing geometrical objects.

	DTxtb-A (n = 164)	DTxtb-B (n = 74)	DTxtb-C (n = 81)	DTxtb-D (n = 64)	DTxtb-E (n = 51)	DTxtb-F (n = 117)
TF1	45.12%	45.95%	14.81%	23.44%	39.21%	44.44%
TF2	31.71%	31.08%	37.05%	10.94%	25.49%	28.21%
TF3	17.68%	12.16%	17.28%	48.43%	15.69%	7.69%
TF4	4.27%	8.11%	4.94%	12.50%	19.61%	19.66%
TF5	0.00%	0.00%	2.47%	4.69%	0.00%	0.00%
TF6	1.22%	0.00%	17.28%	0.00%	0.00%	0.00%
TF7	0.00%	2.70%	6.17%	0.00%	0.00%	0.00%

Note. TF1 – Multiple-choice questions, TF2 – Fill-in-the-blank responses, TF3 – Matching, TF4 – True or False, TF5 – Put in order, TF6 – Other closed forms, TF7 – Open-ended questions.

Table 4. Dimension of Task forms in digital textbook tasks

Feedback

The results showed that all of the examined textbooks contain a high percentage of verification tasks, that is, tasks with feedback on the correctness of the student's answer (see Table 5). In all the textbooks examined, except for C, more than 97% of geometry tasks have feedback. Only textbook C has a significant amount of tasks with no feedback (23%).

	DTxtb-A (n = 164)	DTxtb-B (n = 74)	DTxtb-C (n = 81)	DTxtb-D (n = 64)	DTxtb-E (n = 51)	DTxtb-F (n = 117)
F1	2.44%	2.70%	23.46%	0.00%	0.00%	0.00%
F2	97.56%	97.30%	76.54%	100.00%	100.00%	100.00%
IF1	23.78%	74.32%	9.88%	98.44%	100.00%	90.60%
IF2	54.27%	14.86%	0.00%	0.00%	1.96%	2.56%
IF3	28.66%	85.14%	76.54%	100.00%	100.00%	96.58%
IF4	0.00%	6.76%	0.00%	0.00%	0.00%	0.00%
IF5	0.00%	0.00%	0.00%	0.00%	0.00%	0.85%
IF6	1.22%	0.00%	0.00%	0.00%	0.00%	0.00%

Note. F1 – No feedback, F2 – Verification, IF1 – Correct response, IF2 – Try again, IF3 – Error flagging, IF4 – Elaborated, IF5 – Hints/cues/prompts, IF6 – Other.

Table 5. Dimension of Feedback in digital textbook tasks

Consideration of the subcategories of different types of feedback show that all the digital textbooks contained features IF1 and IF3, although they differed in their proportions. Tasks providing feedback just to correct answers (IF1) are widely offered in textbooks B, D, E and F. From one hand, textbook E offered this feature in all of its geometry tasks, while, from the other hand, it was present in only 10% of textbook C tasks. Error flagging (IF3) only highlights students' incorrect answers and was highly present in all textbooks examined, except for textbook A (29%). All of the geometry digital tasks in textbooks D and E have this feedback feature. Subdimension IF2 (trying

again and again until the student gives the correct answer) is differently distributed over the six textbooks: from no representation at all in textbooks C and D, very small proportions in E and F (less than 3%), 15% in textbook B, to more than half of all tasks (54%) in textbook A. Important and useful features for formative assessment, such as explanations as to why a specific response is correct or not (IF4) and giving hints/cues/prompts (IF5) are not present in the examined textbook tasks. Only textbook B contains 7% of IF4 tasks (see Table 5).

Regarding different textbooks, the results show that textbooks A and B contain the greatest diversity according to feedback features, with four feedback types with proportions greater than 1%. The other textbooks mainly possess features IF1 and IF3 only. Textbook C lacks feedback in about a quarter of its geometry tasks.

Measuring response time

The results showed that the feature of measuring response time was present only in textbooks A, B and D (see Table 6). All of these tasks relate to internet quizzes. In textbooks A and B, the time is measured in about a fifth of all tasks. Tasks with a time limit for responses are provided in quizzes with the aim of encouraging students' procedural fluency and make up 8% of all tasks in textbook A, 11% in B, and 6% in D.

	DTxtb-A (n = 164)	DTxtb-B (n = 74)	DTxtb-C (n = 81)	DTxtb-D (n = 64)	DTxtb-E (n = 51)	DTxtb-F (n = 117)
TM1	21.95%	20.27%	0.00%	9.38%	0.00%	0.00%
TM1.1	14.02%	9.46%	0.00%	3.13%	0.00%	0.00%
TM1.2	7.93%	10.81%	0.00%	6.25%	0.00%	0.00%

Note. TM1 – Time is measured, TM1.1 – Time is measured, but without a time limit, TM1.2 – Time is measured and there is a time limit.

Table 6. Dimension of Measuring response time in digital textbook tasks

DISCUSSION AND CONCLUSION

The results show that the tasks in the geometry chapters of digital textbooks do not provide a full range of features according to the framework applied. Personalization, cooperation and exploring requirements are not used in the textbooks examined. Still, textbook C contained dynamics within tasks with drawing or dragging points in the plane. Regarding the task form, all of the textbooks contained multiple-choice, fill-in-the-blank, matching, and true/false tasks. Most of the analyzed tasks did not have the feature of measuring response time. Regarding the Feedback dimension, all the examined textbooks contained a high percentage of verification tasks, but they differed in the proportions of their subcategories. Most textbooks put emphasis on IF3 (error flagging), and some on IF1 (providing just correct answers).

These results imply that the potential of tasks regarding interactivity and dynamics is not being utilized in digital textbooks, even though these features are highlighted in the national Textbook Standard for e-textbooks (MZO, 2019b). This issue is particularly important in geometry education for visual comprehension of geometrical objects and their relations through dynamics and dragging (e.g., Arzarello et al., 2002) and for children's spatial comprehension (Jones & Tzekaki, 2016). Also, in geometry "dynamic environment can help students to move to higher levels of discourse" (Kaur & Sinclair, 2014, p. 416). Further, the study presented in this paper showed the domination of multiple-choice and fill-in-the-blank tasks. These task types may be helpful in distinguishing and explaining terms, but the other dynamical digital potentials are lacking. Interestingly, the previous

research (Glasnović Gracin & Krišto, 2022), which focused on textbook A through grades 1 to 4, showed that this textbook contained geometry tasks with dynamics only in grades 1 and 2.

The lack of personalization and cooperation features in tasks may be the consequence of the limited capability of the publishers' platforms provided for digital textbooks. These issues make the e-tasks closer to traditional tasks found in printed textbooks, rather than providing new opportunities, which is in line with other studies conducted (e.g., Dryvold, 2022; Glasnović Gracin & Krišto, 2022). These findings also correspond to the results by Pepin et al. (2017), where the digital curriculum resources are found to be "still relatively rudimentary" (p. 652) and lacking focus on the educative nature of digital resources.

This paper deals with the development of the multi-dimensional framework for analyzing the features of digital tasks, as they have not yet been well investigated (Fan et al, in press; Pepin et al., 2017). The further research may encompass exploring which (geometry) content is particularly convenient for the new educational dynamics in e-textbook tasks. Although the focus here was on geometry, the framework can be implemented to other mathematical disciplines, as well as for other school subjects in different countries. The instrument is yet to be further developed by refining the existing dimensions and possibly adding new dimensions. Such a framework may highlight the different types of new educational potentials in digital materials.

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DIGITAL FORMATIVE SELF-ASSESSMENT

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Digital formative assessment tools rarely implement self-assessment, even though it can support students' mathematical learning processes and metacognitive activities. The BASE project seeks to address this gap by developing a digital formative self-assessment tool (BASE tool) in which learners judge the correctness of their task solutions with the help of interactive sample solutions and task-related checklists. The paper first presents the design principles of the BASE tool and illustrates the self-assessment process using a multiplication task. Subsequently, we outline a theoretical framework for capturing the effects of the BASE tool on multiplicative understanding and metacognitive activities.

Keywords: self-assessment, digital feedback, mathematical understanding, metacognition, artefact-mediated activity

INTRODUCTION AND LITERATURE REVIEW

Digital technologies can support formative assessment, for example, through dynamic and interactive visualisations (Yerushalmy & Olsher, 2020), adaptive feedback (Rezat, 2021), and individualised learning paths (Stacey & Wiliam, 2013). However, most digital formative assessment environments do not include self-assessments, even though research has shown that “students benefit most when they develop the capacity to assess their own learning and evaluate the feedback they receive from other external sources” (Hughes, 2010, p. 224). This benefit was explicitly shown in the SAFE project with a self-assessment tool in the field of functional thinking (Ruchniewicz & Barzel, 2019). In line with SAFE, the BASE project addresses this gap by developing and researching a digital self-assessment tool (BASE tool) for basic arithmetic knowledge. The material in BASE and SAFE is based on concepts and tasks from the project KOSIMA which led to a research-based textbook to engage students in active learning (Prediger et al., 2021).

Self-assessment

“Student self-assessment...most generally involves a wide variety of mechanisms and techniques through which students describe...and possibly assign merit or worth to...the qualities of their own learning processes and products” (Panadero et al., 2016, p. 804).

When describing one's learning processes or products, a distinction is made between summative and formative self-assessments. In summative self-assessments, learners evaluate their skills at the end of a lesson, such as giving themselves grades (Tejeiro et al., 2012). Formative self-assessments go beyond this and focus on adapting or changing students' learning processes based on substantive information about their learning (Andrade, 2019). Summative self-assessments do not have the purpose of deepening understanding and improving performance. Therefore, when self-assessment is referred to, it should be taken in its formative form. Various mechanisms and techniques can be used to implement self-assessments. For example, practices can differ based on whether students are given standards to assess themselves (Andrade, 2019). A “checklist-referenced self-assessment” involves providing criteria with which students can evaluate their learning processes or products (Andrade, 2019, p. 3). Checklists could focus on subject-specific or general skills, such as “I can represent multiplications on the number line”, or they can be related to specific characteristics of

tasks, for example, “One of the two factors in the calculation corresponds to the number of my jumps”.

Self-assessments require metacognitive skills, such as observing and monitoring oneself (Zimmerman, 2002). Activating metacognitive skills and taking more ownership can help improve students’ academic performance (Brown & Harris, 2013). However, there is a risk that students may incorrectly assess their learning process or products by, for example, judging a mathematically incorrect task solution to be correct. These misconceptions can lead to inaccurate conclusions. Therefore, students need to receive feedback from external sources, such as teachers, classmates, or digital assessment tools, in addition to their self-assessments (Ruchniewicz, 2022).

Potential of digital technologies for students’ self-assessment

Digital technologies open a variety of options to support self-assessments in mathematics education. For example, Drijvers et al. (2016, p. 12), following Stacey and Wiliam (2013), distinguish two ways of using technology for assessments: (1) assessments *with* technology, which are paper-based and with tools like CAS or geometry software and (2) assessments *through* technology, where technologies are utilised to organise and carry out the entire assessment process (e.g., online diagnosis). If self-assessment processes are conducted through technology, mathematical concepts and relations can be represented interactively and dynamically to support these processes. By incorporating sample solutions into such interactive and dynamic representations, learners can gain a deeper understanding since learning with sample solutions has proven to be a successful way to understand a task’s structure and principles. The presence of explanatory hints, which can be requested on demand when students work with sample solutions in a digital learning environment, has been shown to increase learning (Atkinson & Renkl, 2007).

Automatic analysis of student data allows immediate feedback (Bokhove & Drijvers, 2012) and can support (self-)assessment. Feedback, as defined by Shute (2008) and Narciss (2008), provides students with information on their current learning level and inspires changes in their thinking and behaviour. Narciss (2008, p. 135) distinguishes various types of (digital) feedback, including:

- *Knowledge of result/response*: Students receive feedback only on whether their result was correct or incorrect.
- *Knowledge of the correct results*: Students receive a description or a hint to the correct response, for example, in the form of a sample solution.
- *Knowledge about concepts*: Students receive detailed information, for instance, hints or explanations on subject-matter concepts.

While feedback design is always linked to supporting learning processes concerning the learning objective, its meaning is not solely determined by feedback providers (e.g., students, teachers, and digital tools). The extent to which feedback stimulates changes in students’ thinking and action depends on how they perceive, interpret and use it (Rezat, 2021). Therefore, assessing how students use feedback, particularly when transmitted digitally, and how this affects the learning process is essential (Rezat, 2021). In addition, the automatic analysis of student data can not only be used to provide immediate feedback but also to enable individualised learning paths by automatically allocating suitable follow-up tasks (Stacey & Wiliam, 2013). This can be particularly fruitful within self-assessments because follow-up tasks may be allocated not only based on task performance but also on how well the students self-assessed their solution.

THE BASE TOOL

The BASE tool is designed to help students self-assess their arithmetic understanding and gain a deeper understanding of mathematical concepts.

Design principles

Based on the literature review, six design principles emerge for the design of the BASE tool:

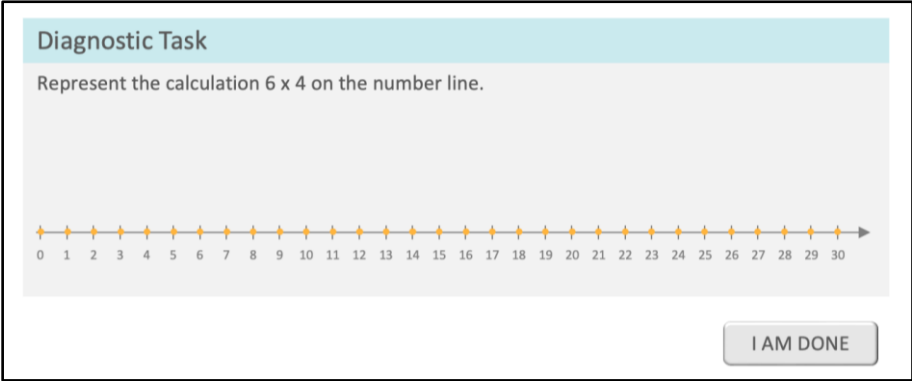
- 1) *Comprehension-oriented tasks* that focus on students' arithmetic understanding
- 2) *Dynamic and interactive sample solutions* for self-assessment
- 3) *Self-assessments* guided by content- and task-related criteria
- 4) *Automatic feedback* provided to students in elaborative and verifying form
- 5) *Adaptive learning paths* formed based on task solution and self-assessment
- 6) *Diagnosis-guided promotion* by eliciting students' understanding

Structure

In the BASE tool, the students first go through a “pre-diagnosis”, comprising three to four diagnostic tasks.

Solving a diagnostic task

Figure 1 shows an example of a diagnostic task, which is part of the pre-diagnosis “Recognising and representing multiplications in pictures” and requires students to represent a given multiplication by jumps on a number line.



The screenshot shows a digital interface for a diagnostic task. At the top, there is a light blue header with the text "Diagnostic Task". Below this, the instruction "Represent the calculation 6 x 4 on the number line." is displayed. The main area contains a horizontal number line starting at 0 and ending at 30, with tick marks every 1 unit. Orange arrows are placed on the number line, starting at 0 and jumping to 6, 12, 18, and 24, representing the multiplication 6 x 4. At the bottom right of the interface is a button labeled "I AM DONE".

Figure 1. Solving a diagnostic task about multiplication

Self-assessment of a diagnostic task

After submitting their solution to the diagnostic task, students are given a self-assessment checklist (Figure 2 - right) which characterises a possible solution to the diagnostic task. A statement of the checklist can be answered “YES” if the students think their solution fulfils the characteristic mentioned in the statement, “NO” if they do not, and “UNSURE” if they are unsure. To support students in their self-assessment, they are provided with feedback in the form of a visualised sample solution to the task (*Knowledge of the correct results*, Figure 2 - below), along with the task-related checklist. The sample solution visually presents a correct way to solve the task, allowing students to play the jumps dynamically. Furthermore, it is possible to display visual and textual cues to highlight and explain certain features of the sample solution (*Knowledge about concepts*). The cues emphasise the concepts addressed in the self-assessment statements.

My solution

Represent the calculation 6×4 on the number line.

Possible sample solution

▶ play
number
length
total length

Learn from your solution

One factor of the calculation corresponds to the number of jumps.

YES NO UNSURE

The other factor corresponds to the length of each jump.

YES NO UNSURE

The result of the calculation corresponds to the total length of all jumps.

YES NO UNSURE

I understood the task and the sample solution.

YES NO UNSURE

I AM DONE

Figure 2. Self-assessment of the task with the help of a sample solution

Reflecting on a diagnostic task

After answering the self-assessment statements, students receive feedback on their task solution (highlighted solution, top left in Figure 3) and self-assessment (highlighted wrong-self assessments, right in Figure 3), indicating if they were correct or incorrect (*Knowledge of result/response*). This feedback can be used to reflect on the diagnostic task.

My solution

Represent the calculation 6×4 on the number line.

Possible sample solution

▶ play
number
length
total length

Learn from your solution

One factor of the calculation corresponds to the number of jumps.

YES YES

The other factor corresponds to the length of each jump.

YES NO

The result of the calculation corresponds to the total length of all jumps.

NO NO

I understood the task and the sample solution.

YES

I AM DONE

Figure 3. Reflecting on the task with the help of feedback on task solution and self-assessment

Once all diagnostic tasks of a pre-diagnosis are done, the BASE tool automatically assigns supportive tasks. The selection of these tasks depends, on the one hand, on the correctness of the solutions to the diagnostic tasks and, on the other hand, on the correctness of the students' self-assessment. After completing the supportive tasks, the students take a "post-diagnosis", which comprises the same diagnostic tasks as the pre-diagnosis but with different numerical values. A complete overview of the structure of the BASE tool is given in Figure 4.

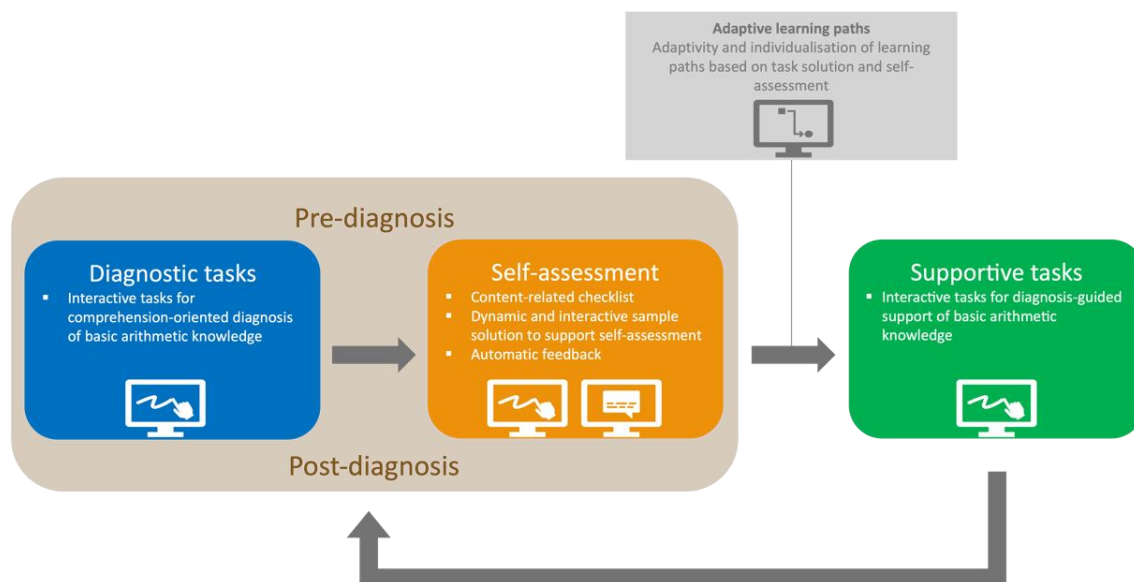


Figure 4. Structure of the BASE tool

RESEARCH QUESTION AND THEORETICAL FRAMEWORK

The research on the BASE tool will focus on the relationship between its use, learners' multiplicative understanding and metacognitive activities in the context of self-assessment processes. The following question will be raised:

How do learners use the BASE tool, and to what extent does its use affect learners' multiplicative understanding and metacognitive activities in the context of self-assessment processes?

We are currently planning a qualitative study to answer the research question. As part of the study, several students will be video recorded, working on diagnostic tasks on multiplication. To analyse the data, we have developed a theoretical framework based on artefact-mediated activity (Rabardel, 2002) and the concept of scheme (Vergnaud, 2009).

Conceptualising the BASE tool as an artefact

Learning processes are often mediated through artefacts. Artefacts are physical or abstract objects resulting from human action and mediate the activity of subjects (e.g. students) directed toward a particular object (e.g. task on multiplication). Following Beguin and Rabardel (2000, pp. 179-180) and Rabardel (2002, p. 63), we distinguish three different forms of mediation in artefact-mediated activity: a) pragmatic mediation from subject to object (PM), b) epistemic mediation from object to subject (EM) and c) reflexive mediation from subject to subject (RM).

Pragmatic mediation describes how the subject uses the artefact to influence the object of the activity. An example of pragmatic mediation with the BASE tool is that students select the corresponding cue in the sample solution before answering a self-assessment statement to receive additional information for their self-assessment process. Epistemic mediation focuses on the artefact's role in developing an understanding of the object of the activity. Here, students' understanding can be reconstructed through the so-called "operational invariants" (Vergnaud, 2009). Operational invariants are composed of concepts-in-action and theorems-in-action, wherein concepts-in-action refer to mathematical objects considered relevant or irrelevant by the subject (e.g., factors, product tick marks on a number line). Theorems-in-action are statements about the mathematical objects that the subject believes to be true but may be mathematically incorrect (Vergnaud, 2009, p. 88). Concerning the task presented above (Figure 1), epistemic mediation can be used to analyse the relationships students establish between the symbolic representation of

multiplication and the corresponding iconic representation on the number line. Reflexive mediation refers to processes in which subjects use the artefact for self-reflection, and metacognitive activities are made possible through interaction with their own cognitive processes. Here, for example, it can be analysed which elements of the BASE tool are included by the students when they evaluate their task solution or self-assessment as correct or incorrect.

An integrated framework to investigate the use and impact of the BASE tool over time

Regarding the previous use of theoretical frameworks based on artefact-mediated activities, two limitations were identified:

1) The frameworks are usually only applied to artefacts or components whose functions and possible uses do not change significantly over time. However, when students complete the diagnostic tasks, self-assess, and receive feedback in the BASE tool, different functions targeting different objects are available at each of the three points in time. For example, students can create jumps on the digital interface when solving the task. However, this function is no longer available once the task is completed, and the students are asked to self-assess their solution. Instead of creating jumps, playing the sample solution or selecting visual cues during the self-assessment is possible.

2) In addition, it can be stated that applying theoretical frameworks based on artefact-mediated activities has predominantly focused on students' mathematical understanding without considering their mathematical understanding in relation to metacognition. However, the use of artefacts - especially when they focus on student self-assessment - can be expected to affect learning at both mathematical and metacognitive levels. Therefore, it is crucial to consider metacognitive activities when exploring using the BASE tool through the lens of artefact-mediated activity. In the following, we explain how we will consider the dynamics of the BASE tool and the possible impact of its use on multiplicative understanding and metacognitive activities.

When students work with the BASE tool, they must first complete a diagnostic task which is carried out via a digital interface. We will investigate how the students solve the task on the digital interface and what multiplicative understanding (concepts- and theorems-in-action) and metacognitive activities they show in this process (Figure 5, first small triangle).

After completing the task, students should self-assess their solution (Figure 2). The self-assessment represents the object of the student's current action. During the self-assessment, students have access to different artefact components: To their own solution, a dynamic and interactive sample solution and the statements of the self-assessment checklist (Figure 5, second small triangle). While the students self-assess, we will analyse which artefact components they use for this and how it affects the multiplicative understanding and metacognitive activities.

In the final step of the diagnostic task, students should reflect on their task solution and self-assessment (Figure 3). To support this, automatic feedback is provided as an artefact (Figure 5, third small triangle). An analysis will determine how students understand this feedback and its impact on their metacognition and understanding of multiplication.

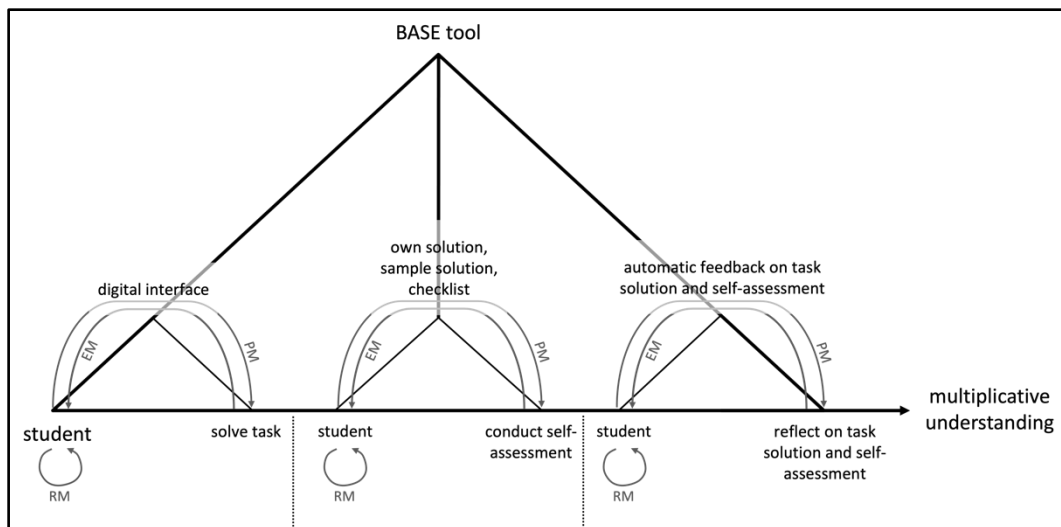


Figure 5. An integrated framework to investigate the use and impact of the BASE tool over time

RESULTS AND DISCUSSION

In a previous pilot study, we gathered data from students who had worked with a preliminary version of the BASE tool outlined in this paper. This pilot study involved a case analysis investigating the impact of the dynamic and interactive sample solution on a student's understanding of multiplication and his metacognitive activities (Graewert et al., 2023). Part of the theoretical framework presented above was used to analyse the student case. However, the analysis results provided evidence for further refinement of the BASE tool. Therefore, as a next step, we need to collect data using the adapted version of the BASE tool presented in this paper and conduct an analysis using the theoretical framework described above.

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ENHANCING STUDENTS' 21ST CENTURY SKILLS THROUGH PLAYING AND MODIFYING EMBODIED DIGITAL CLASSIFICATION GAMES

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This research paper explores the pedagogical potential of playing and modifying a digital embodied classification tool to foster computational and mathematical reasoning skills as high-level 21st-century competencies. In a study involving eight secondary school students, we investigate the outcomes of utilizing a specially designed digital tool for a subjective classification. The outcomes shed some light regarding the role of such a tool in further engaging students with complex multidisciplinary concepts and collaboration development and lay the foundation for further exploration of the connection between digital embodied classifications and the development of mathematical meanings. We elaborate on a case of obtaining classification skills through digital game modding where computational and mathematical ideas coexist.

Keywords: Digital Games, Classification, Collaboration, Designing Technology, Mathematical Concepts.

INTRODUCTION

Many studies have shown the benefits of embodied interaction with technology for deeper learning and meaning generation about complex scientific concepts by students (Papert 1980; Kynigos et. al., 2010). Yet, this promising approach, has mainly been studied in silo contexts of subject knowledge development, such as maths, physics or computer science concepts understanding. In this paper, we argue that enabling embodied interactions in digital educational technologies, apart from concept understanding, can also be a strong tool for supporting children's 21st century skill development, such as classification, in a relatable and meaningful context. Classification is considered a necessary skill for dealing with society and making sense of the world with researchers including it into 21st-century skills subsets (Owen & Barnes, 2019, Vuorikari et. al., 2022). However, there are quite limited concrete examples of educational practice and tools for supporting higher-order classification processes, while most relevant studies concern basic operations with children ranging from infant to primary school age (Milne, 2007, Cao et al., 2017). To address this challenge, we explored middle school students' classification skills through their collaborative embodied interaction within a digital Tetris-like gaming environment, called "SorBET" (<http://etl.ppp.uoa.gr/sorbet>). "SorBET" enables students to play gesture-manipulated classification games and design their own with block-based programming and database affordances.

In this paper, we pose questions about two key factors regarding the specific digital classification tool as a medium for students' higher conceptualization. The first is about classification as a way of engaging with complex concepts in terms of computational and mathematical thinking, and the second addresses the role of embodiment in terms of student collaboration, focus and construction of higher meanings:

- How do students make sense of classification concepts while playing and modifying a classification game in SORBET?
- To what extent can the simultaneous embodied engagement of two students in a digital classification game promote collaboration and meaning-making processes?

This paper aims to showcase the potential of embodied interaction within digital educational technologies in enhancing students' classification skills and overall 21st-century skill development, while also providing concrete examples of educational practices and tools for facilitating higher-order classification processes. Before delving into these specific questions, it is important to have a deeper understanding of the concept of classification and how embodied interactivity can influence students' thinking and meaning-making processes.

THEORETICAL FRAMEWORK

Classification as a Pillar of 21st Century Skills Development

The goal of classification is to break large subjects down into smaller, and more specific components. In our daily lives, we classify things frequently, often without even realising it, which can help us understand the world around us (Owen & Barnes, 2019). This stems from the fact that applying already known patterns and categorising objects allows us to respond quickly to new stimuli (Micklo, 1995; Krnel et al., 2003). According to Piaget, successful classification requires moving from concrete operations to formal operational thinking (Inhelder & Piaget, 1964 as cited in Krnel, Glažar & Watson, 2003) and it involves higher-order mental practices as mathematical reasoning, abstraction, generalisation and design of the classification system (Grizioti & Kynigos, 2023). Classification practices are also closely connected to set theory, as they can be seen as a process of partitioning or classifying data into different sets based on certain criteria. Besides, at its core, classification relies on the principles of logic and inference, which are fundamental components of mathematical reasoning.

Nevertheless, the majority of relevant studies on the development of such skills mainly concern children from infancy to primary school ages (Micklo, 1995; Krnel et al., 2003; Owen & Barnes, 2019). These skills can be difficult for students to adopt, particularly those involved in high-order classification processes like creating classes and patterns, abstracting rules, and implementing relations within the classification system (Armoni, 2013). Classification skills are essential for deeper understanding and are linked to computational thinking, but they are often left out of school education due to their abstract nature (Cao et al., 2017).

It should also be noted that classification and categorization, while both are methods of organizing information, possess semantic differences in structure and meaning. (Jacob, 2004), Classification is rigid, requiring entities to belong to specific categories, whereas categorization is more flexible, based on recognizing similarities among entities (Jacob, 2004). Combining these two methods can assist individuals develop predictive and knowledge production abilities (Owen and Barnes, 2019). In this context, the digital tool used in this research, incorporates both the structure of classification and categorization (Grizioti & Kynigos, in press), allowing an object to be assigned to just one, several or all of the available categories without restriction.

The Role of Embodied Interactions in Learning

In order to enhance students' 21st century skills, using immersive digital technologies that utilize physical interaction to help them comprehend complicated ideas and generate scientific meanings, can be an effective educational strategy (Weisberg & Newcombe, 2017). As these digital tools have an impact on the education sector, it's important to consider how students engage with them. When playing a digital educational game, most of the time students have a passive role by simply using the mouse and keyboard. Such actions may have a negative effect on them, spending a great amount of time staring at the computer screen. According to Shapi'i & Ghulam (2016), a good learning experience ought to involve all the senses and activate the extremities by making them react to stimuli. This is followed by the Natural User Interface (NUI), giving a new way of learner-computer interaction. NUI refers to sensory inputs such as touch, speech and gestures (Shapi'i & Ghulam, 2016), freeing users from

peripheral computer devices, enabling an intuitive and virtual experience with digital content (Johnson-Glenberg et al., 2014). This interaction has its basis in the theory of embodied cognition.

When we refer to embodied cognition, we are discussing experiences and actions that involve either the whole body or the use of just the hands manipulating objects (Fyhn, 2007; Johnson-Glenberg et al., 2014). The theory of embodied cognition, or embodied learning, is studied by cognitive and social psychology as well as by neuroscience and linguistics, and argues that human knowledge is not isolated as a perceptual action, but is closely linked to the activities of the body which is the “mediator” between the mind and the environment (Barsalou, 2010; Shapiro 2011; Weisberg & Newcombe, 2017). Piaget (1952) was one of the earliest proponents of sensorimotor activities as a means of constructing knowledge, defending the view that bodily actions are not separate from the mind. Memories of movement have proven to help students for future action (Tran et al., 2017), constituting “alternative channels of learning” that offer students easier understanding of educational material, as they connect the oral discourse with their learning environment (Hwang & Roth, 2011). In this way, students gain the ability to collect, compare, disagree with their peers' ideas, and adapt the information they receive through their personal interests. Most embodied digital environments support visualization of body movement as a function of learning goals to be achieved, influencing the learning process (Tran et al., 2017).

THE DIGITAL CLASSIFICATION TOOL “SORBET”

The digital tool used for this research is called “SorBET”, as for “Sorting Based on Educational Technology”. It is a software of the Educational Technology Laboratory (ETL) of National and Kapodistrian University of Athens, purposing to operate as a “game generator”, supporting teachers with no prior programming knowledge, create their own categorization games (Grizioti & Kynigos, 2023). It was inspired by the interactive game “Koskino” (Kynigos et. al, 2010), where the players push and place objects falling at a constant rate in the appropriate categories using collaborative body movements. The prototype version was developed as part of a Master thesis (Giama, 2020) and it was further developed to include block-based programming and embodied interactions (Nikolaou, 2022). In “SorBET”, users have the ability to play, modify, and design any type of classification games. As players, they need to make quick decisions, by classifying as many of the falling objects as possible into the correct categories, while achieving high levels in their score. As designers, they can design game elements such as density, speed, rules, object and category definitions, with block-based programming and database affordances. The field of objects and categories is up to the student to define, with no restriction to the topic. The classification model is not a one-to-one relationship but instead permits an object to be assigned to multiple categories, if it satisfies the criteria of the framework, resulting in a one-to-many relationship. Consequently, a category can overlap with or exclude another. This structure is intended to spark debate among players about the intersections or mutual exclusions of available categories based on object properties.

To promote embodied collaborative learning, we extended the initial “SorBET” tool, by incorporating gesture recognition as a means of categorizing objects. This enhancement is intended to improve the communication and interaction among players during the learning process, as well as to facilitate the development of strategies for classification and the interpretation of concepts. This new version allows players to handle and classify the falling objects using their palms, by recognizing hand gestures via users’ web camera without any further equipment needed (Figure 1a). The current system's built-in interaction simulates the actual movement of a human palm, closing and opening to grasp and release an object, perceiving the closed fist/open palm gesture as a more intuitive and physical interaction type of system handling.

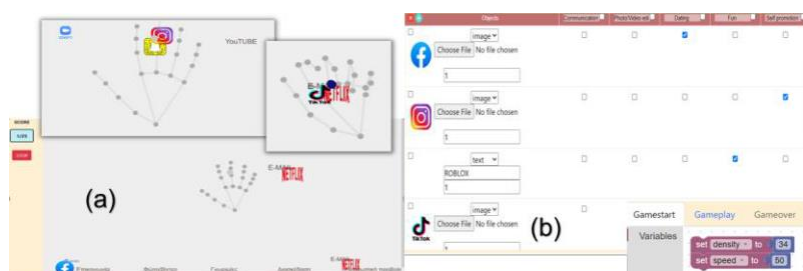


Figure 1: (a)Hand gesture tracking while playing the “App Game” in SorBET (b)Edit Mode

PILOT STUDY

To develop our understanding on children’s classification and formation of meaning process, while playing and designing embodied classification games, we conducted a pilot study with secondary school students. The present study is the pilot part of a larger ongoing design-based research, studying the development of student 21st century skills in different domains through emerging technologies. The results will inform the redesign of the activities and the “SorBET” .

Context and Game

The study involved eight secondary school students (five boys and three girls) ages 13 to 17 working in groups of two. The duration was two hours for each group. Students experienced the kinesthetic version of the tool, by first playing and then modifying a subjective SorBET game called “App Game”. The game involved classifying popular mobile applications, such as YouTube, Instagram, and Facebook, according to their primary way of use, to categories such as “communication”, “self-promotion”, “entertainment” and “dating” (Figure 1a,b). The applications were deliberately chosen so that the way they are used is ambiguous, with the aim of provoking debate among players about which category suits best, based on their personal experience. In each phase, one of the students would initially be the operator and the other one would observe and advise. Afterwards, the roles would be reversed, so that all participants could experience the activity equally. In the modification phase (Figure 1b), students had the opportunity to redesign the game according to their own criteria, by adding new applications or removing some of them, changing the object – class assignments and selecting the amount of times each application will display during the game. Most importantly, they had access to the original assumptions of the classifications, challenging themselves to think critically about the changes they would make, based on their own interpretation and applying logical reasoning.

To have a comprehensive view of student activity throughout the activities we collected multiple types of data. The most valuable of them, involved recorded group conversations, videos and photos to record students’ hands and body movements, the games created by students along with their log files, and an individually semi-structured interview at the end of the study. We analysed the collected data in Atlas.ti software following the thematic analysis approach. First we triangulated the transcribed dialogues with the interviews, the photos and videos. Then two researchers analysed the data looking for critical episodes of student interaction with each other or with the tool that were relevant to the research questions. Since this approach is innovative and there was no known framework or coding scheme for these specific questions, the researchers followed a bottom-up abductive coding method for coding the incidents. After both researchers had coded the dialogues, they compared their codings and concluded to a final coding list from which they generalised the main themes of the results.

Initial Results

The following table shows the emergent codes and the documented relevant incidents as derived from the qualitative data analysis (Table 1).

Themes	Codes
Classification	“Comparison”, “Discrimination”, “Generalization”, “Intersection”, “Inclusion”, “Exclusion”
Usability & User Interface	“Changes - Additions”, “Difficulty”, “Aesthetics”, “Usability”
Students Engagement	“Creative Freedom”, “Dedication”, “Disappointment”, “Personal Gratification”,
Social Context	“Competition”, “Mutual Support”, “Social Interaction”

Table 1: The codes resulting from the data analysis in Atlas.ti, per theme

The students played “AppGame” in pairs, without knowing that classification assignments were clearly subjective. It should be noted that in the initial design of the game, only the “one-to-one” category assignment was used in each application, in order to increase the difficulty, so that only one choice would be considered correct during the playing process. This resulted in students’ low scores and cognitive dissonance about what the game considered as “correct”, but rich dialogues as students collaborated by classifying the applications with their palms. One example is the following utterance from a student while playing the App game that was coded as “Intersection”:

Student 3.1: “Instagram as communication and entertainment? But it's also a medium for photos and videos”.

In the design phase students appeared to have a better understanding of the underlying abstract concepts of the study. While editing the game, they discovered that certain sets of objects may correspond to more than one category, which completely altered their perspective on the classification process. Students also realized that one category does not exclude another or that some categories are inherent in others. In many cases students suggested changing the existing categories with other more specific or more inclusive ones. This is evident in the following incident from Group 4 that was also coded as “Exclusion/Inclusion”.

Student 4.1: “If it had “online lessons” it would fit, let's put it in...”

Student 4.2: “Should we add a new “music” category?...”

Student 4.1: “Should we change the title in the category “Photo/Video”, make it only “Video””

Giving students access to the game structure, sparked debate about the game's legitimacy. In most cases they modified the game based on their perspectives and view on how to use the applications.

As far as it concerns the game handling, the addition of gesture recognition for moving the objects to be classified had a genuine impact during the game process (Figure 2). Students engaged in an interactive classification experience using their palms which provided a more playful and interesting environment for them. This experience offered fertile ground for their interaction, collaboration, but also the formation of meanings about arbitrary concepts, such as the notions of classes and

objects. One of the most interesting observations of the researcher who was present in the study, was the cooperation and mutual support that occurred between teammates. Although only one hand could be detected by the camera, the player who had the role of observer was actively involved by giving advice, encouraging and helping his teammate to sort using correct gestures:

Student 1.1: "This is how I did it, I think it works better, try it" (shows the movement of his hand to the teammate who was struggling).

Student 4.1: "Well done, you got it straight away, let's go!.....Close your hand and go like this to get in the center (shows how the camera should see the palm to detect the spot)".

While some characteristic replies in the interview regarding their experience of working together are the following (coded as "Social Interaction" and "Mutual Support"):

Student 3.2: "This game enhances cooperation because let's say that I didn't know a piece of information, but my teammate knew and help me and vice versa".

Student 2.1: "It would have been a lot of fun if we could both play together at the same time and I realized that as soon as I got involved".



Figure 2: Secondary students interact with the digital classification tool using hand gestures

DISCUSSION & CONCLUSIONS

The findings reveal that the tool provided students with a flexible and interactive environment that stimulated innovative thinking and enabled them to create their own categories and assignments. Through lively debates on the validity of classifications (Grizioti & Kynigos, 2023) and active manipulation of entities and categories, students gained a deeper connection with classification concepts such as comparison (Cao et al., 207), inclusion and exclusion, fostering their learning experience. By joining in arguments, explanations, and discussions, students actively engaged with the underlying concepts of the classification process, a fact that seemed to help them uncover hidden meanings and enhance their comprehension (Weisberg & Newcombe, 2017). The embodied interactions within the tool, both between students and between students and the system, confirms previous related studies (Johnson-Glenberg et al., 2014; Tran et al., 2017) as they seemed to play an important role in facilitating expression, meaning-making, and collaboration in a more concrete and tangible way (Fyhn, 2007; Barsalou, 2010; Shapiro 2011; Hwang & Roth, 2011; Weisberg & Newcombe, 2017). The presented pilot study will inform the iterative process of the design based research regarding the re-design of the activities and the digital tool in order to enhance students' game and learning experience. Some suggestions for improvement that came out of this research include, for example, changing the way objects are manipulated such as dragging instead of grabbing with the palm, simultaneous recognition of two or more palms, the ability to personalize the user interface (UI) and having more options available in blockly commands.

The exploration of classification as a versatile and higher-order skill holds significant value and warrants further investigation for advancing computational and mathematical reasoning. However, it is essential to ascertain whether the challenge of allowing students to explore, express, and combine computational and mathematical ideas through digital embodied classification games sufficiently fosters comprehensive skill development. While this pilot study did not specifically focus on individual mathematical skills, such skills are inherently embedded within the classification process itself. Therefore, extensive research and empirical evidence are needed to precisely define the key characteristics of digital classification and embodied cognition as an integrated approach for cultivating 21st-century computational and mathematical thinking skills. The insights gained from these studies will further contribute to the design and development of effective educational tools and strategies that promote higher-order thinking and skill development across various domains.

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THE MOVING ARROWS ENVIRONMENT: A DIGITAL ARTIFACT FOR MEDIATING THE MEANINGS OF VARIABLE AND UNKNOWN

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This study, part of a greater research project, describes a digital interactive artifact designed for mediating the mathematical meanings of variable, constant, functional dependency, structure and value of a linear literal expression, and relationship between two expressions depending on the same variable. We present the a-priori analysis of the semiotic potential of this artifact and the a-posteriori analysis of its actual unfolding in the context of a pre-experimentation with low achieving students (aged 15-16). In particular, we focus on the signs produced by these students during the accomplishment of exploratory tasks involving the artifact.

Keywords: variable, linear literal expression, semiotic potential, low achievement in mathematics

INTRODUCTION

Wide research has documented students' difficulties in the domain of school algebra, that include giving meaning to algebraic symbols, unknowns, variables, equations and inequalities (e.g., Arcavi et al., 2017; Kieran, 1992). In particular, algebra can be extremely challenging for low achieving students (Xin et al., 2022). Yet, studies investigating the implementation of appropriately designed digital artifacts in the teaching-learning of algebra suggest that they can be helpful for students with a history of low achievement in mathematics (e.g., Baccaglini-Frank, 2021; Mariotti & Cerulli, 2001). We engage in this line of research, since further studies are still needed, to shed some light on what aspects of such artifacts seem to help students the most, in order to guide the design of appropriate instructional interventions. This study is part of a greater funded research project (DynaMat) that, through a design-based methodology, is conducting case studies of second year high school students with a history of low achievement in mathematics. These students, volunteering from different Italian high schools, participate to an intervention conducted by researchers during which they engage in a set of newly designed digital activities in the context of algebra. In this study we describe a specific digital interactive artifact designed for mediating the mathematical meanings of variable, constant, functional dependency, structure and value of a linear literal expression, relationship between two expressions depending on the same variable; and we explore its semiotic and didactic potential.

THEORETICAL FRAMEWORK

We adopt the lens provided by the Theory of Semiotic Mediation (TSM) (Bartolini Bussi & Mariotti, 2008) in order to explore the affordances of a specific digital artifact. The TSM finds its roots in the socio-cultural frame for learning developed by Vygotsky, according to which signs play a crucial role for learning. According to Vygotsky (1978) signs (symbols, texts, formulae, graphics, tables, and most fundamentally, language) are *psychological tools*, that mediate the development of cognitive processes such as thinking, concepts formation, voluntary attention, and so on, in ways appropriate to our cultures. Within such a general frame, the TSM conceives teaching-learning as a semiotic process which originates from the activity with an artifact and develops through a continuous interplay of signs. Three main categories of signs can be distinguished in this process: *artifact signs*, which refer to the context of the use of the artifact, *mathematical signs*, which refer to mathematical context, and *pivot signs*, which act as bridges between the artifact signs and the

mathematical signs. The potential of an artifact to trigger such a complex process rests on its so-called *semiotic potential*: the use of an artifact for accomplishing a task can foster the development of personal signs which, from the expert's point of view, can be put in relation to and evolve towards specific mathematical signs. It is a double semiotic link that can be established by an expert, as a mathematics teacher, between the artifact and the task, and between the artifact and a piece of mathematical knowledge, as culturally and historically established; this double semiotic link is the semiotic potential of the artifact with respect to an activity and a specific mathematical knowledge. As emphasized by Bartolini Bussi and Mariotti (2008), the semiotic potential of an artifact is not naturally activated and exploited but the teacher must take advantage of it to foster the transition of signs produced by the students towards the mathematical signs which are objectives of the didactical intervention.

The research objective of this paper is to study the semiotic potential of the “Moving Arrows Environment” (MAE), a digital artifact that we designed for mediating specific mathematical knowledge in the context of school algebra. We would contribute to the stream of research about the implementation of digital resources, which other studies has shown to be promising for algebra and calculus teaching-learning processes (e.g., Falcade et al., 2007). More specifically, we analyze the semiotic potential of the artifact MAE and its actual unfolding in the context of a pre-experimentation conducted with pairs of students in a laboratory setting. In order to do so, we focus on the artifacts signs produced by students during the accomplishment of exploratory tasks involving the artifact.

THE ARTIFACT DESIGNED FOR THIS STUDY

We designed a GeoGebra applet that we call MAE (Figure 1). The applet presents a red arrow, labeled a , on a horizontal oriented number line; the origin of the arrow is the point 0, the endpoint is marked with a red tick mark and is directly draggable bound to the oriented number line. Then, there are two black arrows labelled a and $-a$ whose endpoints move indirectly, one opposite to the other, as the endpoint of the red arrow is dragged, maintaining the same length of the red arrow, and two other black arrows representing 1 and -1 that are fixed.

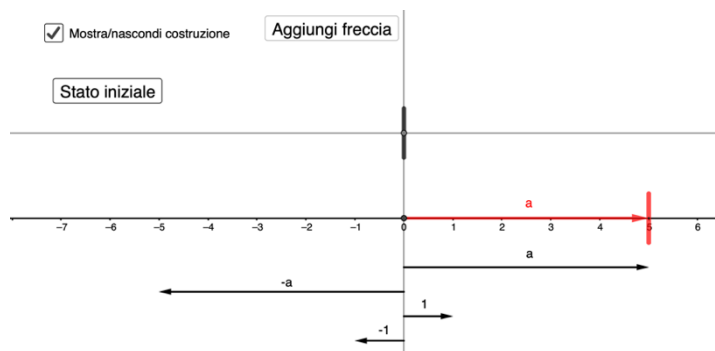


Figure 1. Screenshot of the moving arrows environment

New arrows can be constructed on a horizontal non-numbered line through the “Add arrow” button (in Italian “Aggiungi freccia”). This button allows the user to create different consecutive copies of the black arrows (that is arrows with the same length and same orientation); the origin of the first copied arrow is at the intersection of the horizontal line with a vertical line passing through the origin of the oriented number line, the origin of each other arrow is the endpoint of the previous one (Figure 2). The endpoint of the last copied arrow is marked with a black tick mark. The endpoint of the copied arrows cannot be directly dragged but moves accordingly with the dragging of the red arrow. The user can choose to visualize either the entire construction, that is, all the arrows copied on the horizontal non-numbered line, or the black tick mark alone, that is, only the endpoint of the last arrow, by clicking on “Show/hide construction” (in Italian “Mostra/nascondi costruzione”). The

“Reset” button (in Italian “Stato iniziale”) restores the initial configuration by deleting any construction made. Figure 2 shows the result of the construction of two consecutive copies of the black arrow a and a copy of the black arrow 1 . In Figure 2a the red tick mark is on number 3, in Figure 2b it is on -2 . In this latter case, the last copied arrow (a copy of 1) overlaps a copy of the arrow a ; however, the black tick mark still denotes the endpoint of the last arrow.

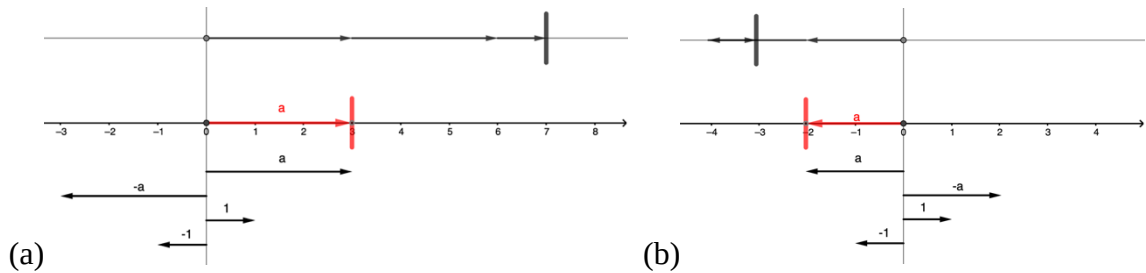


Figure 2. Two screenshots of the moving arrows environment in which two consecutive copies of a and a copy of 1 have been constructed, and the red tick mark is on the number 3 (a) and -2 (b)

Moreover, we created another version of the same applet, with two horizontal lines parallel to the line containing the red arrow, so that two expressions dependent on the same variable a can be represented simultaneously. In this version, there are two “Add arrow” buttons that allow the user to choose on which line to create a copy of the selected arrow. The endpoints of the last copied arrow on the two lines are marked respectively with a black and a blue tick mark. As in the previous version, the constructed sequence of arrows moves accordingly how the endpoint of the red arrow is dragged.

Analysis of the semiotic potential of the artifact

The MAE artifact is designed to mediate the mathematical signs of variable, functional dependence between variables, value and structure of a linear literal expression, and reciprocal relations between two expressions depending on the same variable. In this section we analyze the semiotic potential of the artifact with respect to particular tasks, seeking to highlight the semiotic link that can be established firstly between the artifact and the mathematical knowledge, and secondly between the artifact and a task. Specifically, in Table 1 we report the representations provided within the MAE, the possible actions on them or feedback of them, and in the third column we relate these to the mathematical knowledge they might be exploited to mediate.

In GeoGebra, it is possible to distinguish two types of motion: direct, when the user acts directly on a base object, as the red tick mark in our applet; and indirect if the observed movement is obtained as a consequence of dragging another object (Falcade et al., 2007). We exploited this functionality for designing the MAE, where the dependence relation can be experienced, thanks to the use of dragging, in terms of these two different types of motion. Moreover, a characterizing feature of our artifact is that it provides a twofold representation of algebraic expressions. In functional terms, the tick marks represent two variables, one dependent and the other independent, and their movement represents the functional relationship between them. Moreover, the construction of the arrows obtained with the “Add arrow” button provides a representation of the structure of the expression. For instance $2a+1$ can be obtained adding two copies of a and a copy of 1 . The order in which the user copies the black arrows determines the structure of the expression, but it does not affect its functional aspect.

Representations in the MAE	Related possible actions / feedback	Related mathematical signs

Red arrow, red tick mark	Can be directly dragged on the oriented number line	Independent variable
	Can be dragged to specific numerical values or positions on the number line	Assignment of a value to a variable
Black arrows a and $-a$, and their copies	Change length as an effect of the dragging of the red tick mark	Dependent variable
Black arrows 1 and -1 , and their copies	Fixed constant length	Constant numerical value
Sequence of arrows on the non-numbered line and its corresponding black tick marks	Can be constructed through the “Add arrow” button	Construction of a linear literal expression/Definition of a linear function in one variable
	Change length as an effect of the dragging of the endpoint of the red arrow	Dependent variable
		Evaluation of an expression or function for specific instantiation of the independent variable
	Show/hide construction	Consider the explicit analytical definition of a function / consider function as a set of ordered pairs
Red and black arrows, red and black tick marks	Simultaneous change of the length of the arrows / displacement of the tick marks as an effect of the dragging of the endpoint of the red arrow	Functional relationship between variables
Two sequences of arrows on two horizontal lines and their corresponding blue and black tick marks	Can be both constructed through the “Add arrow” button on the two lines	Construction of two linear literal expressions/Definition of two linear function in one variable
	Simultaneous displacement of the blue and black tick marks, as an effect of the dragging of the red tick mark	Dependency of both the two expressions/functions on the same variable
	Reciprocal position of the blue and black tick marks, at a given position of the red tick mark	Equal-/greater than-/less than-, relationship between the values of the two expressions/functions for a given value of the variable
		Values of the unknown that verify the equality/inequality of the two expressions

Table 1. Overview of the semiotic link between the artifact and the mathematical knowledge

METHODOLOGY

We conducted a pre-experimentation in October and November 2022 that took place in extra-school hours, in a learning center outside of school. There were involved 12 students (aged 15-16) from different schools, volunteering, with a history of low achievement in mathematics. They worked in pairs, under the guidance of a researcher, for five sessions of 2 hours each. They had at disposal two touch-screen tablets: one to interact with artifacts and one for writing their own notes. Data collected consist of video-recordings of the sessions, tablets' screen-recordings, and students' writings.

The activities we designed within the MAE consist of two tasks, simultaneously given to students, written on a single page on the screen of a tablet. In the first task one construction (or two, in the case of the applet in the second version) is required, with the “Add arrow” button, representing a certain expression in the variable a , given in words. For example: *We want to construct the arrow representing the double of a , minus 1. To do this, use the “Add arrow” button.* In the second task, a description of the observed movement is required, after dragging the red tick mark: *How does this new arrow move as a varies?* Moreover, during the activities the researcher guided students’ explorations through some questions, orally posed, asking for predictions, comments or synthesis. Preliminarily, the first time that students were introduced to the MAE, the researcher asked them to explore it by dragging and to describe what happens, before doing any construction.

In this paper, we focus on the preliminary request (episode 1) and on two activities (episodes 2 and 3). Both these activities consist of the two mentioned tasks but we focus on the second one, which we expected may support a rich semiotic production by students. In particular, the task is designed to foster the production of signs, related to the activity with the artifact, that have potentials consistent with the embedded mathematical knowledge and the mathematical signs that an expert can use to express that knowledge. In our a-posteriori analysis, to gain insights onto the actual unfolding of the hypothesized semiotic potential of the artifact, we investigate the possible emergence of signs:

- Referring to the distinction between direct dragging of the red arrow and indirect movement of the constructed sequence of arrows
- Referring to specific positions of the tick marks or to the length of the arrows
- Indicating the impossibility to stretch the 1 and -1 arrows
- Expressing that the relationship considered in the construction of the sequence of arrows is invariant under dragging of “ a ”
- Relating the movements of the arrows constructed on the two lines or of the relative tick marks

DATA ANALYSIS

In the following sections we present three short episodes in which three pairs of students (names are pseudonyms) interact with the MAE. We chose these episodes since in a few turns they allow to have an overview of students’ rich semiotic production.

Episode 1 – Andrea and Hugo

This episode comes from the students’ preliminary exploration of the artifact. The students are requested to drag the red tick mark, without having yet used the “Add arrow” button.

Researcher: What does it happen to the arrows one and minus one?

Hugo: They remain there

Researcher: Why?

Hugo: Because these lines, let’s say, tell us that up to here its value is one or minus one

Researcher: Okay, try to tell me this again

Hugo: These remain still because, let’s say, they tell us that this arrow makes, is... the length of this arrow is one or minus one

[...]

Researcher: Why are these ones called a and minus a ?

Hugo: Because these one, let's say, they have no measure, in fact a and minus a change from the position of where the red line goes, indeed for example if I put it on two a and minus a go always onto the same position

Researcher: Okay, so you said that a and minus a are called like that because they have no measure, you said? Okay what do you mean by that?

Hugo: That they have no value. Because their value depends on where the red line goes

The request gives rise to an intense production of signs referring to the activity with the artifact. Among these we point out the artifact signs “these remain still” and “they have no measure, in fact a and minus a changes from the position of where the red line goes” with which Hugo describes the difference between the arrows 1 and -1 , and those a and $-a$. In these turns, the different dependence relationships on the red tick mark are made explicit in terms of no movement (“these remain still”) or dependent movement (“their value depends on where the red line goes”). These signs could be exploited to mediate the difference between constant numerical value and variable.

Episode 2 – Grazia and Lucia

This episode follows the students' construction corresponding to the “double of a , plus one”. The red tick mark is now on the value 2 and the researcher asks what they expect will happen if it is moved.

Lucia: Unless also the arrow that we constructed moves, this arrow will no longer be the double of a plus one, that is, because we constructed it considering a as two, but if we move this onto four this one will no longer be the double of a plus one

Researcher: Okay do you agree Grazia?

Grazia: It makes sense what she said, however if I move, for example, I point a , the red arrow, for example on three, in turn, a also moves on three and then, in turn, also

Lucia: You didn't listen to me well... I said unless it also moves, that is, unless the arrow above will also update, so, this a , that eventually is this one, will change in size along with the guide

Grazia: But this is what she asked!

Lucia: No... I mean, unless it keeps moving according to a

Researcher: And do we expect it will move according to a , or not? For how we constructed it

Lucia: One didn't move even before, so it should stay still and, instead, a in principle should move according to how the guide moves, and so yes, it would remain the arrow corresponding to the double of a plus one

This episode is full of artifact signs used by Grazia and Lucia to express their prediction about the behavior of the sequence of arrows representing the double of a plus 1 as a moves. In the opening part, the students describe in detail two possible behaviors of their construction under the dragging of a : either the constructed arrows will not move and therefore “this will no longer be the double of a plus one”, or “the arrow above will also update so [...] this will change in size along with the guide”. The students' description of these two possibilities involves many artifact signs. For example, they refer to the red tick mark as the “guide”. This sign, with the mediation of the teacher, has the potential to evolve towards the mathematical signs of independent variable and functional relationship. In the concluding part, the students observe that since each arrow a in the construction should move under dragging of the red tick mark, the construction made “would remain the arrow corresponding to the double of a plus one”. Therefore, they seem to recognize the functional relation

between a and their construction and also the invariance under the dragging of a (it keeps being the double of a plus 1).

Episode 3 – Cora and Letizia

This episode follows the two students' constructions corresponding to the “double of a , minus one” and “ a plus three” with the “Add arrow” button on two parallel lines (Figure 3).

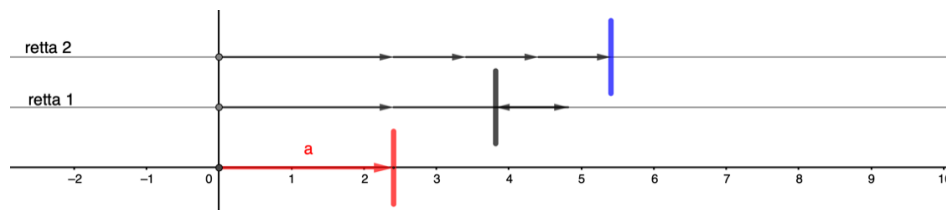


Figure 3. Screenshot of the “moving arrows” representing $2a-1$ (line 1, below) and $a+3$ (line 2, above)

Initially, as asked by the task, the students describe the observed movements by dragging a .

- Cora: The first part of the arrow that indicates a , which is present in all three, remains the same, so it goes together with the red arrow. Then also three always remains three and doesn't get... I mean, while this arrow gets longer, the three remains, that is, the part of the arrow that indicates three... these ones here [pointing to the three 1 arrows] remain the same [...] The black one, practically, both the first part of arrow that is up to here and the second part of arrow that is up to here get longer with a . Then, instead, the one stays still, that is, it is always the segment and goes back

There are many artifacts signs similar to those highlighted in the episode 1 referring to the dependency on a . The signs produced by Cora seem to be related to the structure of the expressions, since she not only describes how the blue and black tick marks move in dependence on the red one, but also how the respective arrows involved in the construction vary (“while this arrow let's say gets longer (...) these ones here remain the same”). Concerning the black tick mark, Cora's last words can be read as signs about the behavior of the a and -1 arrows following the same direction and opposite orientation.

Then the researcher suggests hiding the construction with the “Hide construction” button and, focusing on the tick marks, describing their reciprocal position when dragging the red one.

- Letizia: It seems that here at four they are one above the other, while as they move or it increases, I mean that one, they are always like consecutive

Cora and Letizia focus on positive a -values and Letizia finds a value for which the tick marks are “one above the other”. An expert can recognize that it is possible to establish a link between this artifact sign and what can be expressed through the mathematical sign “the equality is verified for the value 4 of the unknown”. Finally, Letizia, referring to a -values greater than 4, describes the reciprocal position of the two tick marks as being “always like consecutive”. An expert can recognize a possible link with the inequality that is verified in that interval $a > 4$. In a classroom context, the teacher can exploit this potential link to promote the evolution of these signs towards target mathematical ones.

CONCLUSION

In this paper we presented the a-priori analysis of the semiotic potential of the MAE (Table 1) and the a-posteriori analysis of its actual unfolding in the context of a pre-experimentation with low achieving students. In particular, driven by our research objective we focused on the artifacts signs produced by these students during the accomplishment of an exploratory task involving the artifact.

The episodes analyzed show how an expert could recognize these signs and, through purposefully designed actions (Bartolini Bussi & Mariotti, 2008), promote their evolution towards the mathematical signs of variable, functional dependency, linear literal expression, and reciprocal relations between two expressions depending on the same variable. This is particularly relevant considering that the students participating in the DynaMat project are students with a history of low achievement and difficulties in mathematics (e.g., Xin et al., 2022).

Although our results come from a preliminary study, they provide grounds for hope with respect to the didactical implementation of the MAE for the teaching and learning of school algebra. Indeed, the activity with the artifact fostered a rich production of signs by the students, that could become effective teaching resources for the mediation of mathematical knowledge. This study opens the door for further research investigating the possible evolution from artifact signs to mathematical signs, which in tune with TMS, will bring us to deeply focus on the role and actions of the teacher.

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FOSTERING JOYFUL PRACTICE WITH DIGITAL EDUCATIONAL GAMES: THE FUNCTION DUNGEON GAME

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In this paper we present an educational digital game, Function Dungeon, that we developed for learning about functions in a playful way. Results from a pilot conducted with fourteen 13-year-old students in a school in the Netherlands show that the game can foster 'joyful practice'. That is, students experience fun and sense of control while practicing mathematics. Findings from this study contribute to the development of knowledge about digital educational games and its potential to transform traditional practices in education.

Keywords: game-based learning, digital games, mathematics, linear functions

INTRODUCTION

One central concept in mathematics is the concept of function. It enters the curriculum from the beginning of secondary school, and it is needed to understand many topics (e.g. derivatives, equations, modeling). It is also a complex concept for students and not easy to understand for some. A wide range of research on the learning of functions provides directions to design tasks that are suitable to support the understanding of functions (Günster-Weigand, 2020; Janvier, 1987; Oehrtman et al, 2008; Schoenfeld et al., 1993). However, it remains a problem to engage students in thinking about and solving these tasks. Digital games have the potential to engage learners in several dimensions of academic domains such as cognitive, emotional, behavioral, motivational, and social dimensions (Boyle et al., 2016; de Frietas, 2018). A well-designed game can exercise higher-order skills, reasoning, problem solving and collaboration in ways that traditional pedagogy often does not (Gee, 2005; Van Eck, 2006). Digital game-based learning (DGBL) connects better with the way students learn in informal settings in daily life. Instead of learning through explicit linear instruction (e.g., by reading a manual first), many children are used to solving problems through trial and error, recursively collecting evidence which they test through experimentation (Prensky, 2012). In particular, digital games have the potential to enthusiasm students towards playing, while encouraging the development of specific mathematical knowledge and skills (Jensen and Hanghøj, 2020).

In our research we examined the potential of a prototype version of the Function Dungeon game to motivate students 12 to 16 in learning functions in an enjoyable way. The game was developed within Erasmus+ project GAMMA (GAME-based learning in MAThematics) as a collaborative initiative in which researchers, educators and teachers teamed up with one game developer to develop games for upper secondary education. In this paper we report the results of a pilot in which we investigated how the first two levels of the game enacted students' learning experience. The insights from this study are relevant to understand in a broader sense the usefulness of digital educational games for learning mathematics.

THE FUNCTION DUNGEON GAME

In Function Dungeon the player is an explorer trying to find a way through the dungeon. In order to accomplish this game goal, the player has to traverse through a labyrinth of different rooms. Many rooms are originally locked so it is up to the players to find a way to open them. The players can achieve this by interacting with different objects in the rooms. This will allow them to find

function-related problems hidden throughout the rooms. Solving these problems will in turn open the way to other rooms. The main educational goal of the game is to provide students with an opportunity to practice with and learn about linear functions in an appealing and engaging gaming environment. The game is designed to help students strengthen their understanding of the (fundamental) characteristics of linear functions in different representations: textual, numerical, graphical, and analytical.

The game (see Figure 1 for some screenshots) can be played online using the following link: <https://play.unity.com/mg/other/function-dungeon>.

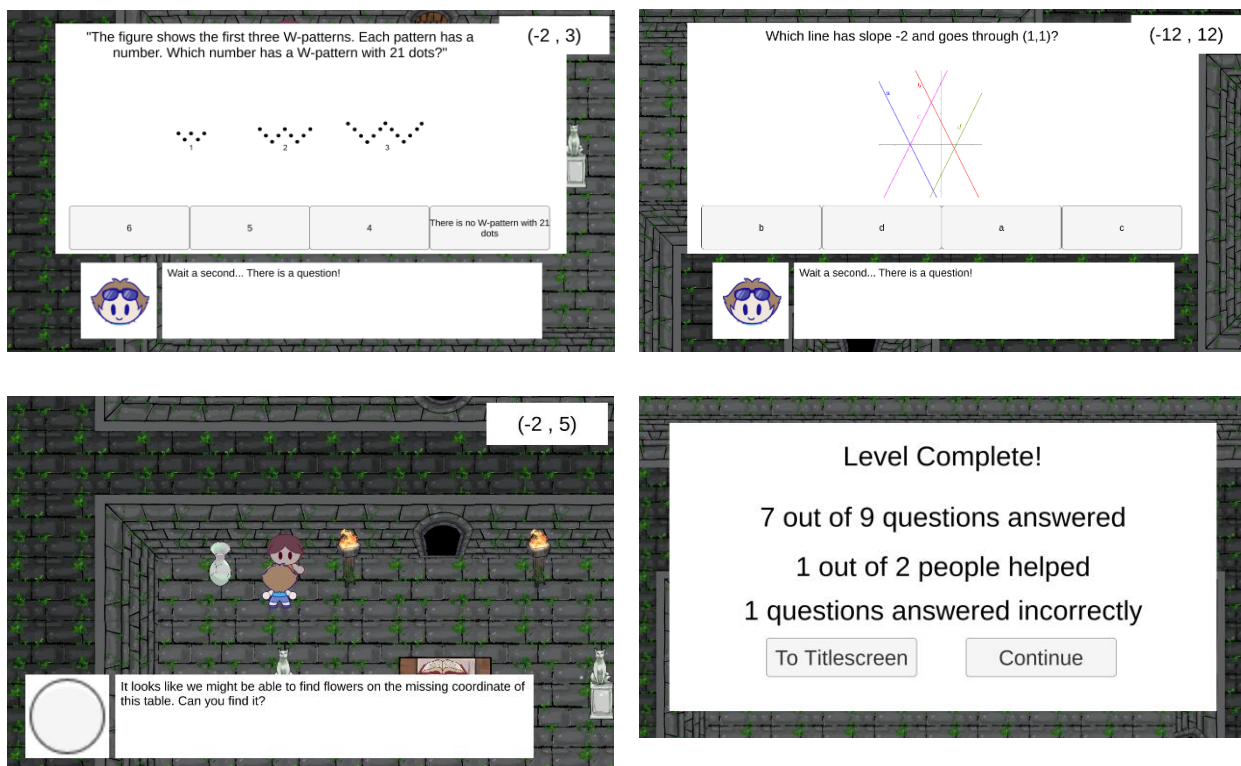


Figure 1. Screenshots of the Function Dungeon game

CONCEPTUAL FRAMEWORK

Literature review on DGBL and learning functions

Several studies report the effectiveness of digital games in mathematics education (e.g. Byun & Joung, 2018; de Freitas et al., 2018; Kangas et al., 2016). In a meta-analysis Byun & Joung (2018) points that games can improve student's motivation, positive attitudes and students' development of specific mathematical skills. Digital games can foster students' interest; motivate them to develop clear goals, and encourage persistence through goal-pursuing activities. In addition, when learning with digital games the student can experience control of his or her learning and consequently remain motivated to solve tasks as part of the game context. Digital games can also provide opportunities for students to make critical decisions in order to solve problems, which can lead to the emergence of problem-solving strategies. Digital games have also the potential to support learning in terms of process skills such as imagination and creativity. However, Byun & Joung (2018) also points that most of the games used in DGBL studies were related to procedural fluency only and few DGBL research had investigated other aspects of mathematical proficiency such as conceptual understanding, strategic competence, adaptive reasoning and productive disposition.

Research on the learning of functions distinguishes between different views on the object function: an action or assignment view, covariational view and object oriented view (Günster-Weigand,

2020). All three views are valuable and depending on the task one view is more adequate than other. An action view involves understanding of the functional notation as an input/output process which helps to organize and to carry out a calculation process. A covariational view is based on causal and dependency relationships between input-output pairs while co-variational understanding. Covariation reasoning is needed e.g. to coordinate an image of two varying quantities and how they change in relation to each other (Oehrtman et al., 2008). An object-oriented view involves awareness of local and global properties and ability to analyze functions by identifying the presence or absence of growth properties. The development of conceptual understanding of function is thus reflected in the capacity of the student to integrate different views of function and coordinate adequately the different representations of the same concept. Examples of learning activities creating opportunities for learning linear function, in particular, involve creating opportunities for students to apply concepts of slope, y-intercept, identify and connect different representations of the same function and reflect on the properties of linear functions (Günster-Weigand, 2020; Janvier, 1987; Schoenfeld et al., 1993).

Framework for the design of an educational game

To design the game, we used a design model developed by Aleven et al. (2010). The method distinguishes three main components in the design process: *educational objectives*, *game-design framework* and *principles of instructional design*. We framed the development of the game Function Dungeon using this method. In the following paragraphs we describe each component and explain how we applied it in the design of Function Dungeon. Although the process is explained in a sequential way and in well-defined steps, in reality the process was iterative, cyclic and often messy with the designers and teachers jumping between different components. . Table 1 provides an overview of the design elements of the game in relation with the educational objectives.

Specification of educational objectives

Specifying the educational objectives helps the designers ensure that the game they create actually meets an intended and coherent set of educational goals. It involves (a) providing a written specification of the prior knowledge and skills; (b) providing examples of tasks by which a student/player will improve the given knowledge and skills and (c) reflecting about the potential transfer (What knowledge and skills might they learn that go beyond what they encountered in the game?). The main goal of the Function Dungeon is to provide students with an opportunity to practice with and learn about linear functions in an appealing and engaging gaming environment. The prior knowledge needed to play Function Dungeon involves a basic understanding of linear functions, and the notion of a function as a mathematical object with different but equivalent representations (see conceptual framework). In the first level of the game, the players strengthen their understanding of the fundamental characteristics of linear functions: constant rate of change, y-intercept, sign, and magnitude of the slope. An example of a multiple-choice question that requires reasoning with the concept of constant rate of change is depicted in Figure 1 (top-left). In the second level, players enhance their ability to connect representations of linear functions: verbal, table, graph, and formula, An example is shown in Figure 1 (top-right). Reflecting on the potential transfer of the game we expect that the reasoning skills practiced in the game (e.g. analysing the function object; relating different representations) are also applicable to other types of mathematical functions.

Applying a game-design specific framework (the MDA-framework)

In the second step, the MDA framework (Hunicke et al., 2004) was used to support the game design. The Mechanics (M) of a game refer to the basic components out of which the game is made, which are the materials, rules, explicit goals, basic moves, and control options available to the players. The Dynamics (D) of the game refer to the behaviors that result when applying the game's mechanics with player input during game play. The Aesthetics (A) of a game refer to the subjective experience

of the player such as emotions and pleasure and it comprises the items: sensation, fantasy, narrative, challenge, fellowship, discovery, expression, and submission. When a game is played, the game’s mechanics give rise to its dynamics, which in turn evoke a certain aesthetic. In the design of the game, the three levels will not be considered just in this order. In fact, the designer has only control over the mechanics while the player’s experience of a game is at the aesthetics level. The MDA framework allows a designer to articulate aesthetic goals (e.g., challenge), consider possible dynamics that may give rise to this aesthetic (e.g., time pressure), and try out different mechanics that may give rise to this dynamic (e.g., use of a timer, or of parallel play by multiple players with bonus points for whoever finishes their move first) (Aleven et al., 2010).

In the Function Dungeon the *aesthetic experience* is centered around discovery and challenge. The *dynamics* of the game is slow-paced; there is no time pressure in the game. The players can take as much time as they need to explore the rooms, solve the tasks, and think about the feedback. Many rooms are originally locked so it is up to the players to find a way to open them. The players can achieve this by interacting with different objects in a room. This will allow them to find function-related problems hidden throughout the rooms. Solving these problems will in turn open the way to other rooms. While exploring the players can make friends with Non-Playable Characters (NPC: a character in a computer game that is not controlled by someone playing the game) - see Figure 1 (bottom-left). Besides moving the character and interacting with objects, the core game *mechanics* in Function Dungeon include answering multiple-choice questions. By answering correctly, a door is opened. When answering incorrectly, a player is thrown in the vault where a book with explanations can be consulted. The player can also find hidden objects in the dungeon. When a hidden object is returned to a NPC, a power-up is collected: the player can phone the NPC to ask for help answering a multiple-choice question (two wrong answers are identified by the NPC). At the end of the game, statistics are shown to the player: number of questions answered, number of NPCs helped, and number of questions answered incorrectly (see Figure 1, bottom-right).

Applying principles for instructional design and mathematical learning

The Function Dungeon game has been developed along with constructivist perspectives, which emphasize that meanings and understandings grow out of interaction with the learning environment and in the presence of adequate feedback. The player is triggered by the tasks in the game to recall and apply concepts of slope, y-intercept, identify and connect different representations The game environment provides immediate feedback on task (e.g. the doors go open if the answer is correct); reflective feedback (explanation when consulting the book) and social-support (the player can ask help from a NPC). In addition, we applied the suggestions for learning activities from the literature review on the learning of functions.

Knowledge and skills	Gameplay
- recognize, remember, and connect function-related concepts - relate different representations of a function - problem solving & recognizing patterns - perseverance, enjoyment and motivation - effort to improve wrong answers	- explore and complete tasks at slow pace - obstacles that force the player to employ their skills to reach mastery level - virtual characters to motivate immersion in the game - narrative motivates to explore and complete the mission - appropriate feedback and scaffolding - control and choice

Table 1. Design elements of the game in relation with educational objectives

METHOD

The game was tested with students and teachers several times during its development. In this paper we use data collected in a pilot (Nov. 2022) conducted in a classroom with fourteen students (13-14 years old, 8th grade) in general education in The Netherlands (Figure 2). The students had previous knowledge on linear functions, which is a topic usually introduced in the 7th grade). The first author assisted with the lesson. The teacher explained briefly the goal of the game, that the game was still being developed and that they could consult the game menu “how to play” in case they needed. The students could choose the level themselves and most of the students (12 students) played both levels; sometimes more than once. The teacher walked around in the classroom, assisting students when they had questions or got stuck in the game. The lesson took about 50 min. At the end of the lesson the students were asked to fill in an online survey to evaluate the pilot.

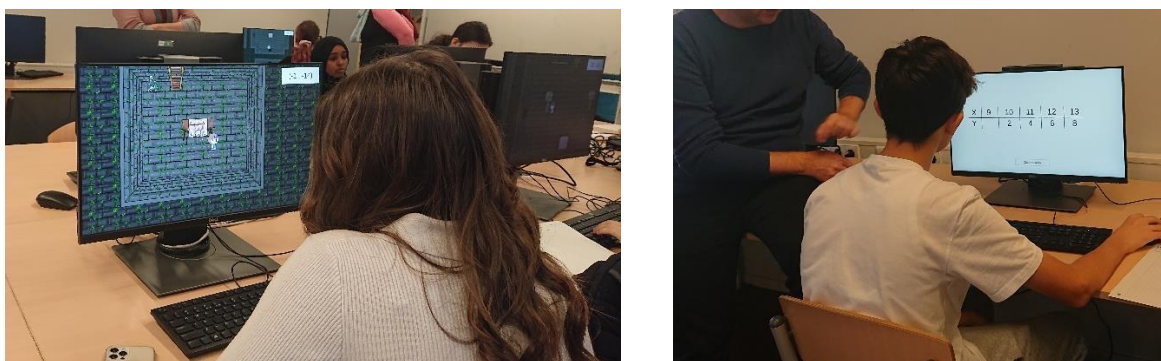


Figure 2. Students play the Function Dungeon game in the classroom

The online questionnaire included open- and closed-ended questions. By using open-ended questions and qualitative analysis procedures, our intent is to develop an in-depth understanding of the way that the Function Dungeon game can foster mathematical knowledge and skills (Tabel 1) from the perspective of the students. In the first questions (Q1-Q3), participants were asked to provide information on aspects related with their learning experience with the game (*What level did you play? What can a player learn with this game? Explain; What did you think of the game? Explain*). Fourth and fifth questions (Q4-Q5) focused on aspects of the game environment (*Was it clear how to play the game? Does the player, in your opinion, get enough feedback to improve his or her answer?*). The last two questions (Q6-Q7) were meant to collect suggestions to improve the game and any other information that students would like to share (*Do you have any suggestions for improving the game? Which one? Do you want to add something else?*). The data collected was analyzed subjected to thematic coding analysis (Gibbs, 2018). According to this method text passages that express equal or similar meaning are identified and combined into categories.

RESULTS

The four categories that emerged from our data were: mathematics learning, fun, agency and other skills (Table 2). The categories mathematics learning, fun and other skills concern *students' mathematical learning* (Q1-Q3) while the category agency concern *students' experience of the game environment* (Q5-4-Q5). In the following of this section we present both results.

Table 2. Learning experience enacted by the game environment according to the students

Categories	Aspects referred and their frequency (N=14 students)
Mathematical learning	Formulas, graphs, functions (5 students) Mathematics (5 students); Mathematics in a better or different way (3 students);

	Learning (1 student)
Other skills	Looking closely to own work (1 student)
Fun	It is fun (6 students) It is fun and you learn from it (4 students) It is fun but unnecessary to learn (1 student) No opinion about (3 students)
It was clear how to play the game (agency)	Clear (6 students) Clear because of in-game explanation (3 students) Clear but needed help at first (3 students) No opinion (2 students); Unclear (1 student)
Enough feedback to improve the answers themselves (agency)	Enough feedback in-game (8 students) Enough feedback but it can be better (6 students) No opinion (1 student)/Not enough feedback (3 students)

Regarding *students' mathematical learning*, all students considered that players learn mathematics from the game (second row). Some students stated that with the game they learn better or in a different way than in the regular lesson. For example one student considered that he “solved sums faster” with the game. Some students referred to have learned about different function representations: “practicing with formulas and graphs”, which was one of the learning objectives of the game (see Table 1). Most students found the game fun (Table 2, third row). One of the students stated: “It makes mathematics fun through fun questions and a beautiful design. You learn something from it and that is very useful”. Some students were also critical about the usefulness of the game as learning tool: “It's a lot of hassle to make a few questions because you have to look for them. I'd rather just have the questions directly. I do like that we then work online instead of in the book.” Another student was critical about the tasks in the game: “on the one hand nice on the other hand boring because you get the same things”. One student mentioned to have learned other kind of skills. Namely “looking more closely at your answers”.

With relation to the *game environment*, most students found clear how to play the game (Table 2, fourth row). They valued the 'how to play' option, explanations in the game and the book in the room. Few students needed help from the teacher or classmates: “At first I didn't know what to do, but through help I was able to do it”. Some students faced some difficulty at first but they solved it themselves: “Yes, when I pressed how to play I got it a bit but with those coordinates [it] was [quite] difficult”. Students found the feedback (Table 2, fifth row) enough to improve their wrong answers but they pointed that the explanation could be better: “If a question is wrong, you will not get an explanation why it is wrong”.

DISCUSSION AND CONCLUSION

Digital game-based learning environments have potential to foster broader and specific mathematical skills. However, most of the games used in DGBL studies are related to procedural fluency only and few DGBL research has investigated other aspects of mathematical proficiency such as conceptual understanding, strategic competence, adaptive reasoning and productive disposition (Byun & Joung, 2018). The Function Dungeon game has been developed with the objective to foster a broader range of mathematical skills with regard to the concept of function. In this study we examined the kind of (mathematical) learning experience enacted by the game according to the students.

The first main result is that students feel that they learn mathematics in a fun, better or in a different way with the game. The game environment enact students experience of agency (a sense of control on their actions) when solving the mathematical tasks in the game. The players have fun in the Function Dungeon. The story and the graphics are nice and appealing. Students find it fun because

it encourages them to exercise knowledge on linear functions. The player needs to master basic concepts of linear functions to engage with the game and have fun. In addition, function dungeon is a game in which the player has the possibility to make some decisions and therefore experience, in a certain way, control over their own learning. The players can decide which level they play; they can go through the game at their own pace; the game can be played at home; the players can decide if they get help from the NPC or not. The players have access to feedback in case they fail to answer a question correctly, while consulting the feedback being not mandatory. The character falls into a room with a book containing more information on what was done wrong, but it is possible to get out of the room without looking in the book. Although it is possible to finish the level without solving all the tasks, the player must solve most of them to open the doors and move forward.

The second main result is that the game environment fostered several kinds of mathematical fluency. Most students succeed in playing both levels of the game and more than once, which means that they succeed in remember function-related concepts, relate different representations and recognizing patterns. These are activities that foster conceptual understanding (Günster-Weigand, 2020; Janvier, 1987; Schoenfeld et al., 1993). Students also showed perseverance and made effort to improve wrong answers, which are aspects of productive fluency and strategic competence (Byun & Joung, 2018).

The game version that we used in this study is still in development. The coordinates task- see Figure 1 (bottom-left) is so far the only game mechanics that directly involve the player in the mathematics. The main mechanics in the game (solving math questions) do not differ much from the traditional practice (answering questions from the book or teacher). This explains also the criticism of few students that found the game fun but not really necessary to learn. Also, the feedback provided to students in case they fail to answer the question correctly should be more elaborated. In the next design- iteration we will focus on the implementation of in-game mathematical-mechanics and improving the feedback. Future research should also investigate the game as integral element of the learning process and not only as a stand-alone activity. The teacher is essential for the success of learning through games and play (Kangas et al., 2016) and teaching experiments are an effective way to develop examples and practical knowledge that can inspire and motivate teachers to use DGBL.

Finally, digital games are examples of digital media for mathematics education that can be transformative (Bray & Tangney, 2017; Kebritchi et al., 2010). Fostering transformational practices in the classroom involves empowering students with knowledge, attitude and skills that prepare them to be successful in their lives and to contribute in a productive way to our society. This implies many more aspects than we can reach with a single digital game. However, digital games such as Function Dungeon, that create a learning environment in which students enjoy learning and practicing mathematics contributes to empowering students as learners. This is an important step towards transformational mathematical practices.

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USING DIGITAL CURRICULUM RESOURCES IN CHALLENGE-BASED EDUCATION: A CASE STUDY WITH APPLIED MATHEMATICS AND PHYSICS STUDENTS

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In this paper, we report on a study carried out in the context of the use of technology to support the work of applied mathematics and applied physics students in a CBE course, in relation to understanding a real-world problem and how to transform it into mathematical objects and relations. Using a case study approach, we investigated how a team of five students and tutors used a digital curriculum resource (Dashboard) to identify and define a problem in a CBE course. Results point to the crucial importance of: (1) the design and use of the Dashboard to foster effective and efficient feedback from the three lecturers of the course; and (2) student knowledge of the Dashboard's affordances that helped to structure students' activity and to use it for their own purposes.

Keywords: Digital Curriculum Resource, Challenge-Based Education, feedback, technological agency

INTRODUCTION

Based on the new needs and concerns in society and industry, new approaches in university (mathematics) education have been developed to better prepare students for the important challenges of this century (e.g. global warming, sustainability, transport efficiency) (Pepin et al., 2021). In these student-centered approaches, learning takes place through open-ended collaborative inquiry, often combined with the design of a prototype solution to a problem (van Uum & Pepin, 2022). One of these approaches is Challenge-Based Education (CBE), a term used in this paper to refer to both teaching and learning processes. Typically, in CBE students are given a broad challenge, from which they identify and define the problem that will be the focus of their project. CBE seeks to help students acquire and deepen disciplinary knowledge and professional competencies and skills (e.g., problem resolution, ethical awareness) while interacting and collaborating with multi-stakeholders (Gallagher & Savage, 2020). In this pursuit of the CBE approach, we highlight two issues that are important to address, specifically related to the fundamental subjects of mathematics and physics: (1) teaching and learning practices through CBE is not obvious for these subjects (Dahl, 2018), even though in practice the CBE approach often involves significant physical-mathematical thinking and work (e.g., learning how to abstract a real-world problem in mathematical terms through the construction and application of models inspired by physics, and the use of theoretical concepts and techniques); (2) It is often difficult for students to identify and define a problem from a given broad challenge that will be solved mathematically (e.g., through modelling). These highlights the pedagogical challenge in line with the new educational approach: how to make interactions between teachers and students more efficient and productive by providing effective feedback, guidance and support to help students in their self-defined projects to make sense of the mathematical activity involved, that is, in the process of modelling and mathematizing the reality. To address these issues mathematics and physics need to be approached in a way that is in line with CBE, in the context of the acquisition and development

of knowledge, competencies, and skills (professional and mathematical) and the use of different resources which mediate the students' and tutors' activity. Niss and Højgaard (2019) point out that mathematical competence "is someone's insightful readiness to act appropriately in response to all kinds of mathematical challenges pertaining to given situations." (p. 12).

Thus, in relation to how digital technology becomes an integral tool in the ways problem-solving and mathematical thinking is supported in CBE university courses, in this study we are interested in analyzing the use (by students and tutors) of a digital curriculum resource (DCR), called "the Dashboard", that was designed to effectively support students in a CBE course involving mathematics and physics. We present a case study of a multidisciplinary CBE course at a Dutch university of technology in which students gained hands-on experience while encountering content from mathematics, physics, psychology and ethics.

We seek to answer the following research question (RQ):

In which ways are students supported in a CBE course by a digital curriculum resource to identify and define a modelling problem?

THEORETICAL FRAMEWORK

Challenge-based education (CBE)

CBE is one of the student-centred approaches that have been developed under the umbrella of inquiry-based education (Martin et al., 2007). Malmqvist et al. (2015) state that problems in the context of CBE, being social challenges, involve a greater complexity than those structured, for example, in problem-based learning. Regarding the learning experience, they point out that besides being typically multidisciplinary it "takes place through the identification, analysis and design of a solution to a sociotechnical problem." (P. 87). Hence, in many CBE courses, student groups are given a broad challenge, in which they identify a particular problem they want to address. Initial support is directed at helping student groups to clearly define this problem. Different CBE learning environments can be described using of a set of criteria that each learning environment fulfils to a greater or lesser extent. Van den Beemt et al. (2022) distinguish three dimensions of CBE, particularly in engineering education: (1) Vision/Challenge, (2) Teaching and Learning, and (3) Support, each with different "indicators".

Instrumental Approach and use of resources

In relation to the analysis of disciplinary knowledge (e.g., mathematical knowledge), we follow a cognitive approach oriented to the analysis of knowledge in practice in different situations. As Kynigos (2022) points out in relation to the way in which learning should be analyzed: "We should not be thinking of whether students learn how to factorize or learn how to solve a quadratic equation, but rather of situations resolvable by dense sets of concepts around a central one." (p. 15). We then draw on the instrumental approach (IA) (Rabardel & Boumaud, 2003), which considers both the relevance of technology in the context of the development of cognitive processes and the fact that essentially every human activity is mediated by the use of artifacts. IA characterizes the interaction with an artifact, called Instrumental Genesis, as the conjunction of two processes: *instrumentation* and *instrumentalization*. For the purposes of this study, we understand that an artifact can consist of a set of resources. We take the notion of resources as "anything (digital, cognitive or material) likely to resource the students' mathematical practice such as a textbook or discussions with other students." (Gueudet & Pepin, 2018, p. 58). In particular, we consider a *digital curriculum resource* (DCR) as the "organized systems of digital resources in electronic formats that articulate a scope and sequence of curricular content" (Pepin et al., 2017a, p. 647). Thus, interaction with a resource corresponds to: (1) the instrumentation process, where the affordances of resources influence

student practice and knowledge; and (2) the instrumentalization process, where students adapt the resources to their own needs.

Feedback

In the study, we consider feedback as proposed by Hattie and Timperley (2007). They conceptualize feedback as “information provided by an agent (e.g., teacher, peer, book, parent, self, experience) regarding aspects of one’s performance or understanding.” (p. 81) and point out that effective feedback must address three major questions: *Where am I going?*, *How am I going?*, and *Where to next?* These questions are related to learning goals, the learner’s progress and the learner’s next step and work together at four levels: the level of tasks (how well tasks are understood/performed); the level of process (the main processes needed to understand/perform the tasks); the level of self-regulation (the way students monitor, direct, and regulate actions towards the goal); and the level of self (the person of the learner; the least effective type of feedback). Therefore, effective feedback needs to address questions about the learning goals, the learner’s progress and the learner’s next step.

METHODOLOGY

In this study, we used a qualitative case study approach. The study was conducted at a university of technology in the Netherlands, in a second-year bachelor course “Sociophysics 1”.

Sociophysics 1 and the DCR “the Dashboard”

Sociophysics 1 is part of a Learning Line of three courses which as a whole follows the CBE approach and incorporates the Scrum framework. The latter was used as a project management framework that helps students to develop and organize collaborative work in an efficient and agile way. The main student learning goals of this course were to gain experience in observing, describing, characterizing and measuring a social system, and, particularly, to identify and determine the problem addressed by the students themselves. They also had to consider the psychological aspects necessary to describe the behavior of the social system, and had to start thinking about the ethical implications of conducting a challenge with human subjects. The students grouped in teams had to formulate an Essential Question (EQ) they were going to address mathematically with their team. They were also asked to formulate Guiding Questions (GQs), Guiding Activities (GAs) and Guiding Resources (GRs) to guide them through the project. Other objectives of the course were to familiarize students with the CBE approach and the Scrum framework. The course lasted for eight weeks. The teams made an intermediate presentation and at the end they presented a poster summarizing their results and submitted a written report. With the aim of helping students to learn how to mathematise the real-world problem, two academics involved in the course (also co-authors of this paper) designed a DCR: the Dashboard. With this DCR, the students were able to communicate with their different tutors regarding the process of identifying and defining the problem. Using the Dashboard the lecturers (and sometimes the TAs) would give written feedback, after which the students could post their revisions.

Data collection and data analysis strategies

The participants in the study were two teams (out of 15) of 5 students, and tutors (the 3 lecturers of the course: physics, psychology and ethics; and one TA per team). The data collection methods are summarized in Table 1 (below).

Data sources	Data collection methods
Students: 2 teams of 5 students each	Observation, interviews, SRRSs (drawings), students’ products (presentations, poster, and report)

Tutors: 3 lectures and 2 TAs (one per team)	Observation, interviews
Dashboard	Students' posts, tutors' feedback

Table 1. Data collection methods

The interviews were conducted at the end of the course. During the interview, each student was asked to make a drawing (Schematic Representation of Resource System (SRRS), Pepin et al., 2017b). These drawings provided schematic representations of how students used and integrated different resources throughout the course. All the data were coded into three categories of analysis related to student interaction with the Dashboard: feedback, instrumentation, and instrumentalization. To carry out the content analysis based on the RQ, first, we conceptualized *students' learning experiences* as the way students structure and develop their activity using and integrating different resources (including DCRs); second, we analyzed this activity through the conjunction of two processes: a process where different resources contribute to structuring the students' activity, and a process where students adapt the resources to their own needs and objectives.

For this study, we have focused on the students' and lecturers' posts in the Dashboard and the student drawings in connection with the students' interview data.

RESULTS

We report results from one team of five students (S1-5) who worked on the challenge: *Design methods to optimally (not necessarily homogeneously) distribute passengers on a train platform to improve the boarding time.*

Feedback

Following the CBE approach students were not given a pre-defined question for their challenge. Instead, the team formulated an Essential Question: *How to improve the efficiency of the train boarding process?*, and five Guiding Questions, eight Guiding Activities, and five Guiding Resources. Through the Dashboard, the students received rapid feedback -from each lecturer- on each formulation in quick feedback loops that helped them develop their Guiding Questions, Guiding Activities, and Guiding Resources more quickly, and thus identify the challenge they aimed to solve (Figure 1).

Figure 1 shows how the interaction between the students and the three lecturers of the course developed through the Dashboard. This case shows that after the students' posts, the team received quick feedback on their GQ from each lecturer (one of them even responded in less than an hour). It also shows how the lecturers addressed the three major questions identified by Hattie and Timperley (2007): *Where am I going?* is addressed by Lecturer 3, who asks whether "measure" is in line with the objectives of the challenge; *How am I going?* is addressed by Lecturer 2 who gives information on the progress of the GQ and makes students notice that they need to be more precise ("again a bit vague") and with his question "How to quantify and evaluate (when is it fast enough and when slow) boarding time?" gives information on how they might proceed; and Lecturer 1 addresses *Where to next?* providing information looking for students having greater possibilities of learning from the challenge by also considering "which type of passengers" and contextualizing this observation more broadly.

Version 5			
How to measure and assess the flow of passengers on the train platform in terms of boarding time (while detecting when the train arrives) ?			
Written by Student 4 on Sept. 20, 2022, 6:10 p.m.			
Text	Rating	Teacher	Date
again a bit vague. How to quantify and evaluate (when is it fast enough and when slow) boarding time?	★★★★★	Lecturer 2	Sept. 20, 2022, 7:03 p.m.
Boarding time is probably simple to measure. Maybe you want to measure and understand the flows of human crowds as this is key to the duration of the boarding process. I can imagine that not only the (local) densities (i.e. how unevenly passengers are distributed on the platform) may play a role, but also which type of passengers. Where do 1st class passengers wait? Maybe on the platform there are also passengers for the train on the other track, or waiting for the following train.... etc etc	★★★★★	Lecturer 1	Sept. 21, 2022, 8:25 a.m.
OK, is "measure" the right word. Measurements are done by the camera already?	★★★★★	Lecturer 3	Sept. 22, 2022, 8:20 a.m.

-Students' post-
A GQ (out of 5) formulated
by a team of students
(not the final version)

-tutors' feedback-
Feedback provided by
each of the lecturers

Figure 1. Display of the Dashboard interaction for the case of the fifth version of a Guiding Question

Regarding the number of interactions on the Dashboard, these were related to the expected development of the students' activity throughout the course. Students were expected to formulate their Guiding Questions during the first part of the course (first 2-3 weeks) to have enough time to formulate their Guiding Activities and Guiding Resources related to each Guiding Question, as well as to work with the data they were provided with and with which they developed models (not analyzed in this article). Figure 2 shows the number of interactions on the Dashboard along the course.

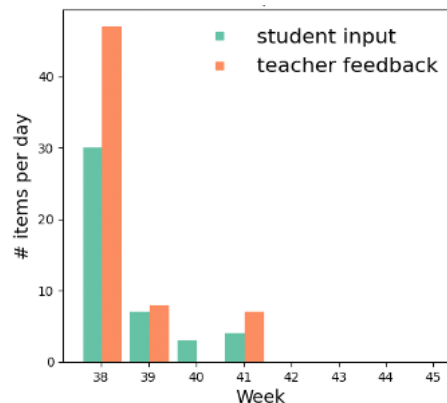


Figure 2. Number of interactions on the Dashboard during the course

We can observe that the number of interactions was highest in the first four weeks, with a peak at the beginning. These changes in the number of interactions per week are consistent with the way students were identifying their problem by formulating first their Guiding Questions and then their Guiding Activities and Guiding Resources that together defined their problem.

Instrumentation process

Regarding the instrumentation process, as expressed by the students, it was important for them (1) to first recognize the characteristics of the Dashboard in the context of their activity and the course, e.g., S1: *“Once I got used to it, I found some features that I didn’t know where they were, then I found it useful”* and S5: *“this is a very feedback centered course, in that sense I think the dashboard is definitely essential”*; (2) to recognize how it helped them with the development of the challenge, e.g., S2: *“I think is really nice to keep everything organized and have a nice interaction with the teachers easily through the comments that they can give from the guiding questions”*.

Instrumentalization process

Regarding the instrumentalization process, the student perceived that the Dashboard helped them particularly in the formulation of their Guiding Questions, which they had to formulate and reformulate based on the rapid feedback received from the tutors via the Dashboard; e.g., S1: “At first there was chaos (see Figure 3-left) then the teaching assistant helped us very much, especially with redefining the guiding questions. Then, the dashboard feedback came, which got us into some right track.”

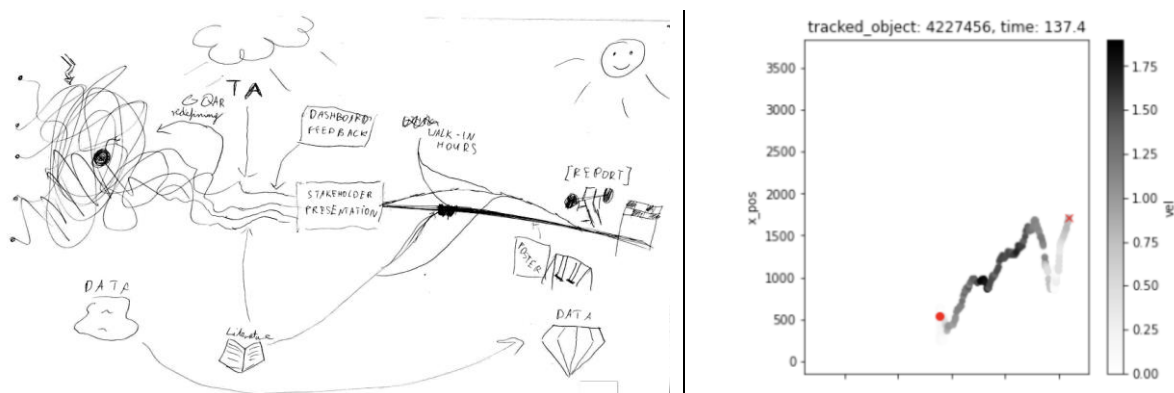


Figure 3. Example of a Schematic Representation of Resource System (SRRS) from S1 (left); and Students-made graph of the trajectory of a person (randomly chosen) on the train platform (right).

What is expressed by S1 becomes more relevant in the light of what Lecturer 1 said (Figure 1). Through this interaction using the Dashboard, we observe how the lecturer made it clear that the students should measure and understand “the flows of human crowds as this is key to the duration of the boarding process” even though the aim of their challenge is “to improve the boarding time”. This insight is taken up by the students when working with the data (see Figure 3-right). In this way, it can be seen that the data became more meaningful after the problem had been identified and defined. This is also represented in the SRRS of S1 (Figure 3-left). In the SRRS of S1 we can see that it was in the first part of the course that work was mainly done on the formulation and reformulation of Guiding Questions, which is consistent with Figure 2 and the expected progression of the course.

CONCLUSION AND DISCUSSION

This paper investigated in which ways the students were supported in a CBE course when using a digital curriculum resource (the Dashboard) to identify and define a modelling problem. For answering our RQ, the interaction with the Dashboard and its use were analysed through the feedback received by the students and two processes: instrumentation, and instrumentalization in the context of identifying and defining a modelling problem.

The students considered the Dashboard as an important DCR (e.g., S5: “the dashboard is definitely essential”) and it helped them identify and define their challenge through the quick and effective feedback they received from the tutors to formulate their Guiding Questions (Figures 1 and 3). We observed that most of the interactions on the Dashboard took place in the first 3-4 weeks of the course (Figure 2). This is consistent with what is expressed by S1 (Figure 3-left), where it is observed that it is in the first part of the course that students have more difficulties in identifying and defining their problem. We identify that the feedback received by the students was effective in that it addressed the three major questions stated by Hattie and Timperley (2007): Where am I going?, How am I going?, and Where to next?

Finally, regarding students’ interaction with the Dashboard, we place our discussion in the context of IA and mediation. Rabardel and Bourmaud (2003), drawing on Vygotski's work developed in the

1930s, point out that one cannot ignore that our actions are shaped by cultural tools. These cultural tools (e.g., the Dashboard), are oriented towards the goal of the activity, towards others and towards ourselves. And they help to shape our cognitive structures which, among other things, allow us to know and identify our objective (*epistemic mediations to object*). Thus, focusing on students' appropriation of DCRs, the results of our study show that the processes of instrumentation and instrumentalization do not take place in isolation between the resource and the student(s) but through the mediation with different agents (e.g., interaction with tutors and resources) who regulate these processes with the aim that the students learn to mathematize the reality (Fig. 4). That is, the agents mediate between students and their objectives and in turn help to shape the way they reflect on their challenge both by themselves and as a team.

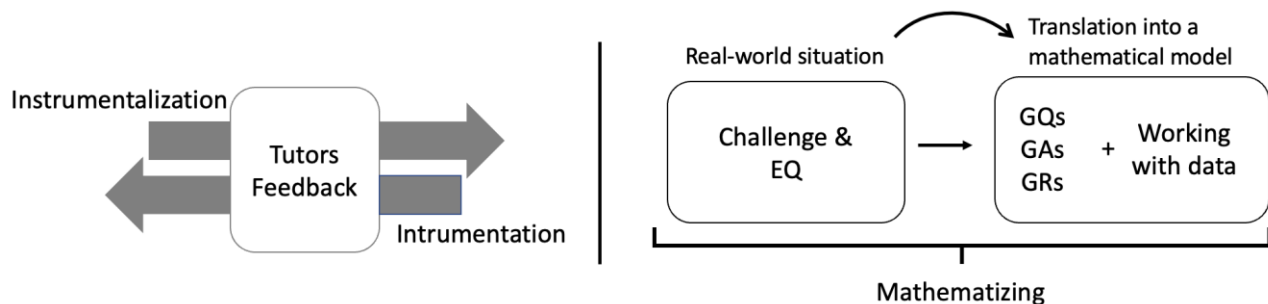


Figure 4. Student appropriation of resources

The results have implications for practice and are useful for course designers: e.g., the use and design of digital resources for providing effective feedback for CBL courses. Future research would deepen into how a DCR interacts with other resources (e.g., Teams and face-to-face sessions) as part of a broader resource system focused on providing effective feedback, and would also consider the students' comments after interacting with the DCR (e.g., likes or dislikes of some features of the Dashboard) for possible future developments.

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DESIGNING A DIGITAL LEARNING ENVIRONMENT FOR COMPUTATIONAL THINKING: THE FOUR PILLARS OF <Colette/>

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Computational Thinking is an essential skillset for participating in and shaping our increasingly digital world, and it needs to be explicitly taught in schools. However, teaching material for Computational Thinking is sparse, especially for contexts outside of Computer Science, like Mathematics and sciences. In this article, we present our approach to designing the digital learning environment <colette/>, which facilitates teaching Computational Thinking in schools. We discuss the design rationale behind the four core pillars (app, web portal, didactic content, and teacher training), and illustrate how they interact with each other. Communicating our vision of <colette/> will be useful for conceptualizing, implementing, and comparing similar digital learning environments.

Keywords: Computational Thinking, Learning Environment, Mobile Learning

INTRODUCTION

Computational Thinking (CT) is a term coined by Jeanette Wing in 2006, conceptually defined as:

Computational thinking is a fundamental skill for everyone, not just for computer scientists. [...] Computational thinking involves solving problems, designing systems, and understanding human behavior, by drawing on the concepts fundamental to computer science. Computational thinking includes a range of mental tools that reflect the breadth of the field of computer science. (Wing, 2006, p.33)

Since then, several researchers have created more explicit, sometimes incompatible, definitions of CT (Weintrop et al., 2016, Fraillon et al., 2020, and see Bocconi et al. 2016 and 2022 for a more detailed overview). According to Bocconi et al. (2016), who synthesized multiple definitions to find a common ground, CT comprises six core skills: Algorithmic Thinking, Debugging, Abstraction, Decomposition, Generalization and Automation.

Abstraction is the skill applied to construct a fitting, mental model for a real-life object by discarding unnecessary details. Sometimes it might be helpful to *decompose* the given problem into smaller problems, solving the smaller ones and combining these solutions to a solution of the original problem. The skill of writing down the way to a solution by using clearly defined steps such that human beings as well as machines can understand and follow them is called *Algorithmic Thinking*. Creating algorithms also involves mentally working through them step by step to see where a problem might occur as well as how the algorithm can be designed better or more efficiently, which is called *Debugging*. Exploiting the fact that machines can perform repetitive tasks very well is the concept of *Automation*. After solving a problem, one may ask if the solution can be *generalized* to a larger set of similar problems.

Note that CT is not necessarily related to Computer Science (CS); at its core CT simply is a skillset for solving problems (Wing, 2006), and, hence, can be taught and applied in other scenarios and subjects too, e.g., in Mathematical contexts (van Borkulo et al., 2020, Weintrop et al., 2016). Although there is a huge movement to include CT in classrooms, it is still unclear how to proceed with this. Bocconi et al. (2016) suggest that the educator has three options: introducing CT in a new subject, in an already existing subject or as a non-subject related skill but as a cross-curricula theme. In practice teachers often have to educate themselves on how to actually introduce CT in their classes. Coding learning environments such as Scratch, code.org, codeHS, pocket code, or MIT app

inventor allow teachers to focus on CT in the CS context, but do not offer teaching material for CT for other school subjects. This means, as teachers do not necessarily have coding expertise, some of them are left behind. In fact, teachers ask for easy-to-use material as one of the biggest hurdles to teaching CT (van Borkulo et al., 2020, Bocconi et al., 2022).

This paper introduces <colette/>, a digital learning environment designed to facilitate the teaching of CT in a variety of school subjects and contexts. The platform comprises four core pillars: a web portal for task creation, a smartphone app for task completion, educational materials for teachers, and didactic content. By describing these four pillars in detail, we lay out our design rationales and outline how the different components of <colette/> interact to create a seamless and engaging learning experience. We hope sharing the concept of <colette/> can serve as a model to inspire the development of digital learning environments, and contribute to the widespread adoption of innovative and effective educational tools.

THE DIGITAL LEARNING ENVIRONMENT <COLETTE/>

<colette/> is a digital learning environment for CT, conceived by a consortium of six organizations from five countries (Goethe University Frankfurt (GER), Johannes Kepler University Linz (AUT), Constantine the Philosopher University in Nitra (SVK), Claude Bernard University Lyon 1 (FRA), Utrecht University (NED), autentek GmbH (GER)), partially funded as a three-year Erasmus+ project (2020-2023). <colette/> is at the development stage, currently being created and refined by working on the material and further adapting the web portal and the app.

The design of <colette/> consists of four not entirely disjoint pillars (figure 1). The first one is the smartphone app which students use to work on the tasks assigned by the teacher and which is downloadable for free for Android and iOS devices. The second one is the web portal where teachers act as an author and design the learning paths for their students, create new tasks and share best practice. The third one is the didactic content, from which teachers can use the given, adaptable *Task Families* to design their own tasks and the fourth one is teacher education. We designed a teacher training which can be attended in person or can be watched online. Additionally, a handbook will be made available in the near future, guiding teachers to create meaningful learning paths. In Bocconi's latest report (2022) teacher education was mentioned as the biggest challenge educators are facing in the lower secondary schools for introducing CT into their classes, warranting the need for CT-specific teacher education.

THE 1ST PILLAR: THE APP

After having seen the rough project architecture, we start with the first pillar by looking at <colette/> in classrooms in action. We developed the app to be compatible with both tablets and smartphones, and it can be downloaded for free on both iOS and Android platforms. By enabling students to use their own devices ("Bring-your-own-device" approach), <colette/> eliminates the need for schools to invest in expensive technology. Not requiring the creation of any account allows for a simple and streamlined onboarding process.

The learning environment is meant to host also various non-coding exercises to provide a comprehensive CT learning experience also outside of CS education. However, when coding is involved, a block-based language is used, meaning that coding novices have a low threshold to getting started (Weintrop et al., 2019).

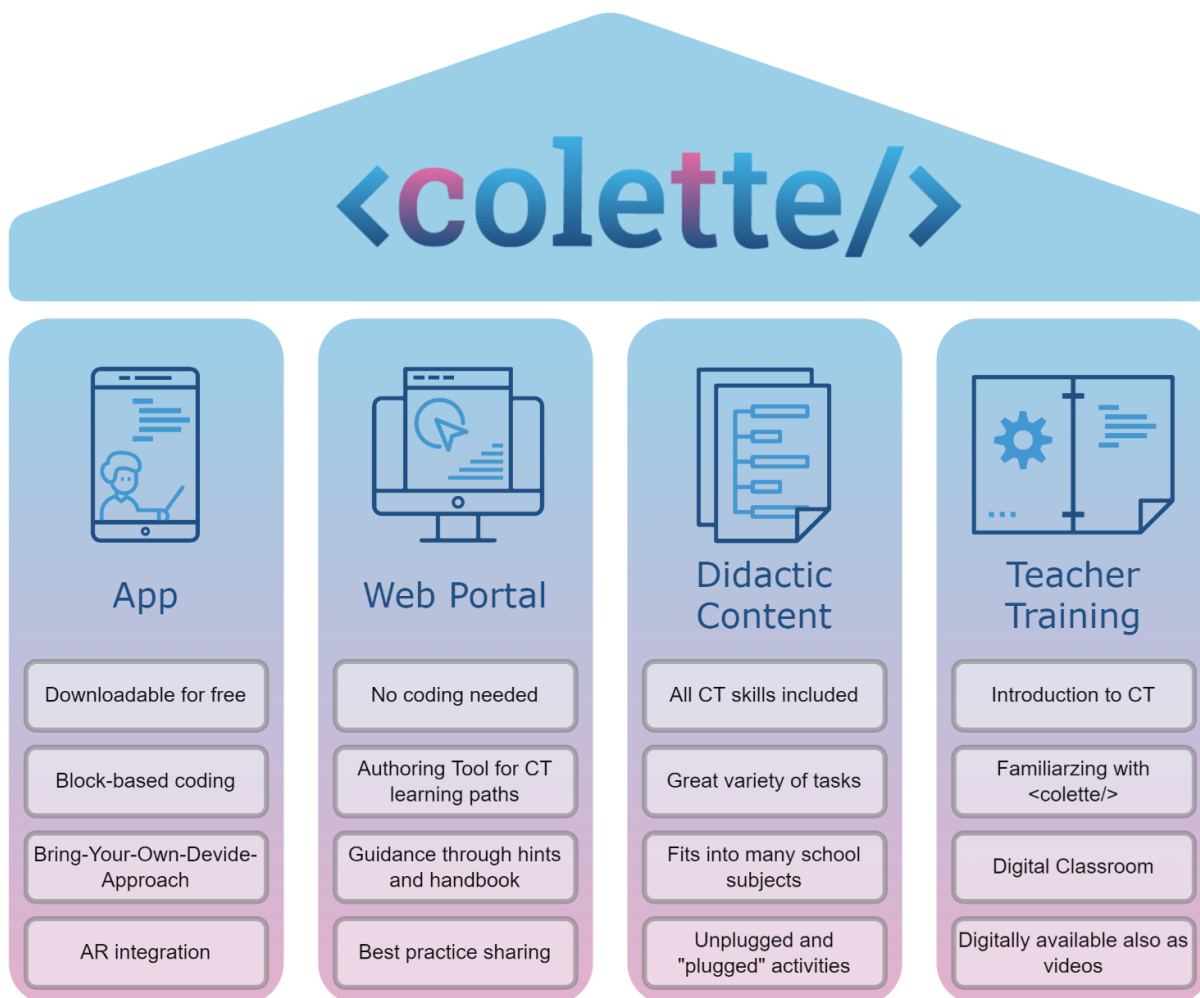


Figure 1: The four pillars of the learning environment <colette/>: App, web portal, didactic content, and teacher training.

The app offers an AR-view enabling the students to verify the function of their code before submitting it. For example, students can see a 3D structure created by their code and see as a ghost-view the desired structure, so that students can see which part of their code isn't working yet, helping them validating their solution. In another *Task Family* a drone executes their code step-by-step and capture images along the way. In both *Task Families* is the CT skill *Debugging* fostered through the AR view.

THE 2ND PILLAR: THE WEB PORTAL

We have seen that the app offers the students' perspective on <colette/>. The web portal acts as an authoring tool for the educators letting them adapt the available tasks to their liking. It is designed to being easy to use, no coding is needed to create meaningful CT tasks. The didactic content is being designed by the consortium, so the educators only have to adapt them for their classrooms. The web portal guides the educator to create those tasks and paths by giving hints along the way facilitating the onboarding for teachers and the creation of learning paths.

Teachers are able to register in the web portal and tasks they created can be made accessible for other educators. This way, best-practices can be shared easily and will make the work with <colette/> even more efficient.

The web portal gives also access to the handbook which is currently being designed and refined. The handbook has two components. The first component is explanatory text and images which can be found in the web portal itself. The second component are videos which will be created and available in all project languages including English and made accessible on our project website. With this guidance and hints in the portal itself we are creating a low threshold learning environment for CT not only for students but also for educators themselves.

THE 3RD PILLAR: THE DIDACTIC CONTENT

The web portal gives access to so called *Task Families* being a general set of tasks (Stäter et al., 2023) all having a certain common core. For example, the common core of the *Task Family Building Cubes* is that all derived tasks ask the student to place cubes on a 3D grid building a shape. Within this *Task Family* different *Assignment Types* are available (figure 2), for coding-based tasks there are implementation, analysis, find the error, etc. For each of the *Assignment Types* different *Scenarios* (meaning different structures) can be selected and edited by using the available set of settings (e.g. size, position, height).

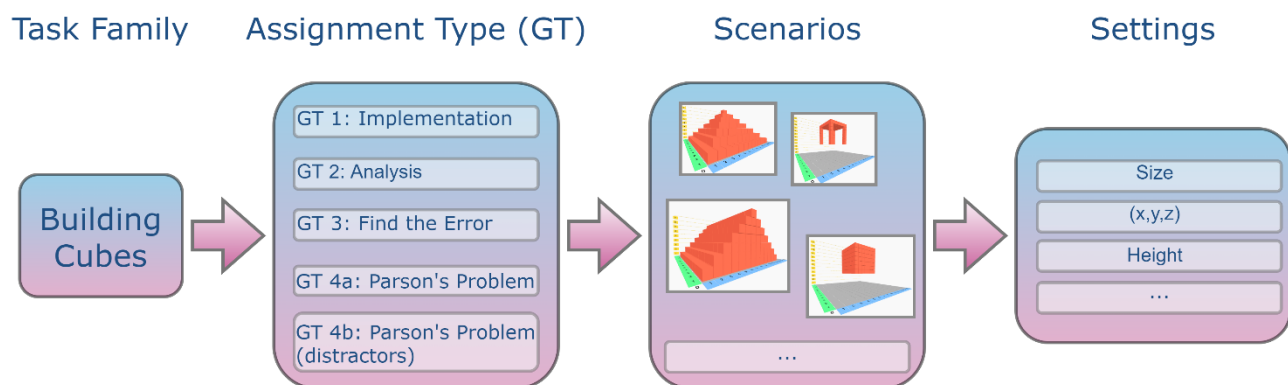


Figure 2: A Task Family includes different Assignment Types (for coding-based Task Families these are the Generic Task ideas) and Scenarios which can be edited using different Settings. In this way from one Task Family several different tasks can be derived.

When designing tasks specifically fostering CT skills not only in the context of coding and being usable in the digital learning environment, certain design choices had to be made. In what follows we'll lay out some design considerations for tasks that can be automatically corrected, created, and easily varied. First, we will explain the design concepts for creating CT fostering tasks, then considerations for creating coding tasks that will be automatically validated.

Design Concept of Task Families and Their Connection to CT-skills

For designing the *Task Families*, we used the backward approach: We had the learning goal in mind and created a task that was fitting to offer access to this learning goal (Lyon et al., 2020). Hence, we designed a *Task Family* having the specific CT skill in mind that students should develop working on the given task derived from the specific *Task Family*.

For addressing the skill *Algorithmic Thinking*, e.g., we created among others the Task Family *Drone* where in one of the *Assignment Types* the students code the flying route of a drone which takes pictures on the way of certain spots of a building or structure. Because this *Task Family* with the

Assignment Type of Implementation focuses on the design of the algorithm using block-based language, we assume that this *Task Family* will highly address the skill *Algorithmic Thinking* (Milicic et al., 2020).

Another example for addressing the skill *Algorithmic Thinking* but without coding is based on process diagrams or mathematical algorithms (e.g., classical constructions, Newton's or Heron's method). For this, one can take any real-life process (using a coffee machine, making dinner, ...) and create a process diagram out of it, explaining this real-life process to any other human being or machine. *Algorithmic Thinking* is fostered as the students must think about each small step a computer or another human being would have to execute and find one correct order of it. An algorithm, at its core, is nothing else as a set of precise instructions in a given order where one produces from given input the desired output. This *Task Family* is available as an unplugged version now and can be used without having any coding experience in nearly every school subject, even language lessons (Rottenhofer et al., 2021).

When making use of the skill *Abstraction* one takes a real-life situation and takes out any unnecessary details to look at a more abstract version of the real world. For addressing this skill, the *Task Family Graphs* was created. In one of its scenarios the student looks at a real-life birds-view picture of a city and has to abstract it in such a way that in the end one has a proper graph giving only the cities as nodes and their distances between them along the vertices (Medová et al., 2022). In this process the student creates from the given picture a more abstract version of it in form of a map and then abstracting it more getting to the graph. Analyzing the graph can be a follow-up task, e.g., finding the Eulerian path.

Connecting Task Families and Generic Tasks

Generic Task is a concept introduced by Milicic et al. (2020) in which one can create different standard CS-tasks simply by using three inputs (figure 3): Algorithm, description, and an optional image. Combining these inputs one can create five different output tasks: Implementation, analysis, find the error, Parson's problem with and without distractors. In <colette/> we use this concept as *Assignment Types* to enrich *Task Families* that are coding-based.

Like mentioned above, <colette/> offers a variety of tasks by using the different *Task Families*. Making use of the concept of *Generic Tasks* offers a bigger set of *Assignment Types*. The web portal only needs three inputs: Algorithm, description, image. Using these three inputs the portal can create five different *Assignment Types* simply by either leaving one of the inputs out (e.g. the portal doesn't show the algorithm to students for creating an implementation task) or changing one (e.g. Find the Error, the portal changes something in the code, like a value, and creates, hence, an error which the student has to correct). Those different *Assignment Types* are being included for all of the coding-based *Task Families*.

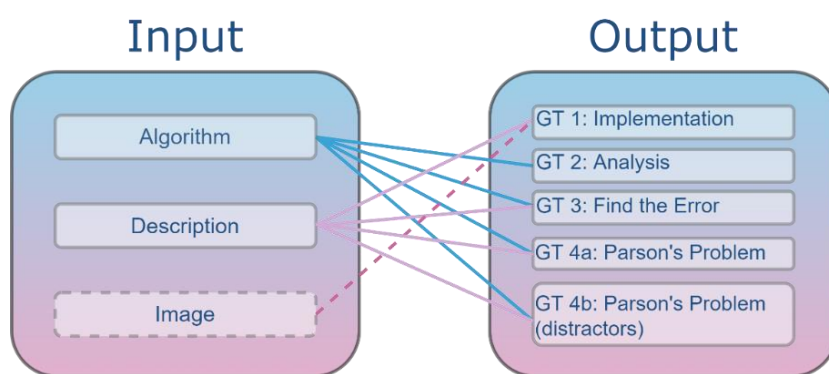


Figure 3: The given input for the task is combined to derive different output tasks. For example, taking the input "Algorithm" and "Description" you can design a "Find the Error" task.

THE 4TH PILLAR: THE TEACHER TRAINING

In the first three pillars, we described the structure of the technology behind <colette/> and the didactic material being used in lessons. <colette/> does not only offer this technology and didactic material but also educates teachers on how to use this technology in a meaningful way to integrate CT in their classrooms. As mentioned previously, the lack of adequately trained teachers was found to be the biggest challenge for lower secondary level teachers to include CT in their classrooms (Bocconi et al., 2022). We hope that the teacher trainings we offer in combination with our handbook and openly accessible videos created by the project will help to overcome this challenge in the future.

Teacher trainings are designed as in-person day-long workshops offered regularly by the project consortium. The training comprises four modules, best taken in sequence and within close proximity (e.g., weeks or months). The schedule of the teacher training is designed such that the teacher can familiarize themselves firstly with the technology from the students' perspective (figure 4). After having seen the *Task Families* and the AR-view in action they will start to learn about the web portal and how to design meaningful learning paths. In the following sessions the focus will be on embedding <colette/> in the classroom, be it the creation of learning paths or the usage of the digital classroom. The digital classroom will help teachers including <colette/> in their offline classrooms. In this digital classroom students can send teachers their solutions, ask questions and the teacher can see the progress students are making and on which task the individual student is working on. The educators can even see the code the students submitted.

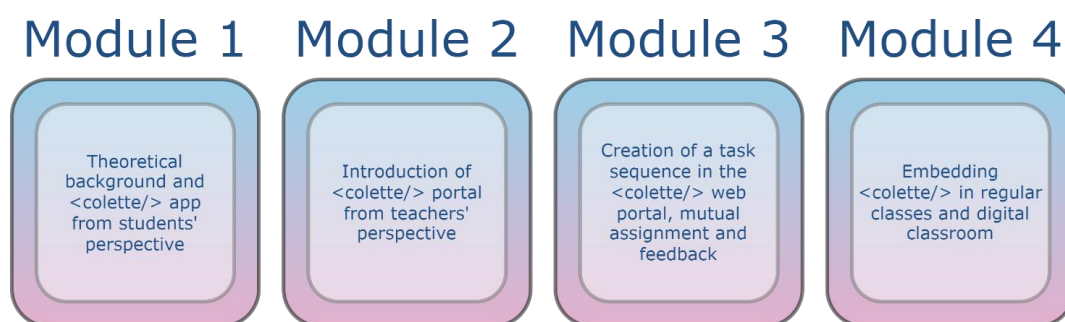


Figure 4: The four modules of the teacher training. The educator starts with the students' perspective, gets to know the authoring tool (the web portal) and starts creating meaningful learning

paths. Lastly, educators will learn about the possibilities on how to embed <colette/> in their regular classes.

HOW THE PILLARS WORK TOGETHER

After having seen what the pillars are meant for and which area of the learning environment they are defining, we are now looking at their interactions and how they are the base for <colette/>.

The teacher can attend one of our teacher trainings to know better what CT is and how they can integrate this concept in their lessons using <colette/>. The educator has, hence, learned how to use the web portal and create meaningful CT learning paths.

The first and second pillar, so the app and the web portal, are the ones actively being used in and for the lessons. The educator uses the web portal to create learning paths which will then be worked at by the students using the app.

The third pillar, the didactic content, is mainly created by the project consortium and is then adapted by the educators to fit their curriculums and students best. Easy-to-use material is one of the teachers' requests for a better inclusion of CT in their classrooms (van Borkulo et al., 2020, Bocconi et al., 2022). For this matter, the tasks are presented as blueprints which makes them easy to use. The educators can adapt this material to their liking and curriculum having learned how in the teacher training.

OUTLOOK

Having described the four pillars, their interaction and concepts behind them, we hope that the communication of the design of this learning environment will be useful for conceptualizing similar digital learning environments, being it for CT tasks or for fostering other skills. In our point of view, the teacher training is not only a good dissemination activity but it is an important and valuable resource for feedback. Teachers and their students are the target group so we need to have their needs in mind implementing and designing the learning environment.

In the remaining project lifetime, we plan on adding more *Task Families* (like interpolation of functions or linear patterns) in the portal and as unplugged *Task Families*. This way we are opening <colette/> to a broader spectrum of school subjects.

More information about the project and its progress can be found on the website colette-project.eu.

ACKNOWLEDGMENTS

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OBSERVING THE SPACE THROUGH THE PLANE: ANALOGIES PROMOTED WITH DYNAMIC GEOMETRY ENVIRONMENTS

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Research in mathematics education on the use and benefits of dynamic geometry environments is notable in 2D geometry, but its development is not similar in 3D geometry. Although some proposals have been formulated to explore 3D geometry, in which dynamic geometry environments are used and the analogy between objects and relationships between the domains of 2D and 3D geometry are involved, there are aspects about the use of these environments in 3D geometry that need to be studied, given the differences in the representation and manipulation of geometric objects. In this document we show the analogical reasoning displayed by a mathematically gifted student when solving construction-and-proof problems in a 3D dynamic geometry environment, how this was promoted by some limitations imposed by the software, and problems the student faced in this process.

Keywords: Analogical reasoning, three-dimensional geometry, dynamic geometry environment, construction-and-proof problems

INTRODUCTION

Nowadays there is abundant specialized literature that shows the benefits and drawbacks of integrating dynamic geometry environments (hereafter, DGE) in the teaching and learning of geometry (Sinclair et al., 2016). Although research has made great steps in 2D geometry, the advances research has made in 3D geometry is not similar (Gutiérrez & Jaime, 2015). Intuitively, it could be considered that the research results involving 2D-DGE could be extended to configurations in which 3D-DGE is used. However, practice shows that some characteristics of the latter, such as the way of representation and manipulation of objects on the screen, make the relationship between an individual and the 3D-DGE be quite different from what happens when using 2D-DGE. It configures a terrain with characteristics that deserves to be studied.

3D geometry is not frequently studied in schools, due to the difficulty of making plane representations of 3D geometric objects capturing all their relevant properties or allowing make a correct reading of the properties that characterize the embodied object (Parzysz, 1988). As 3D-DGE offer a possibility to interact with 3D geometry objects (Sinclair et al., 2016), some proposals for the study of 3D geometry have involved this digital resource, as well as analogies between objects and relationships of 2D and 3D geometry (Ferrarello et al., 2020). This kind of approaches to teaching 3D geometry may allow students discover some properties preserved in both 2D and 3D geometries and understand the reasons for other properties to be valid in 2D geometry but not in 3D geometry, or vice versa.

As part of a research project in which we analyze the processes of reasoning and the progress in learning to do deductive proofs by several mathematically gifted students when solving construction-and-proof problems [1] in a combined 2D and 3D-DGE, the research objective of this document is to analyze the analogical reasoning exhibited by a student when solving the problems, to show how this reasoning is restricted by the lack of some tools in the 3D-DGE that he had used in the 2D-DGE. The analysis also alerts on the difficulties faced by the student when trying to extend some properties from 2D geometry to 3D geometry and the role played by some 3D-DGE functionalities in this process.

THEORETICAL BACKGROUND

What is analogy

According to (Schlimm, 2008, p. 178) an *analogy* is “a relation of similarity between two *domains*, where a domain is a fixed representations of certain aspects of a phenomenon, situation, process, model, problem, conceptual structure, etc.”, through objects and specific relationships. By establishing a set of relationships in one domain and taking them to another domain, in which these relationships are valid, analogy allows the formulation of hypotheses and the simplification of complex mental operations in the second domain, when these are performed in the first domain, that is already known (Fishbein, 2002).

Making analogies is also a product of human activity, with several benefits (Richland & Simms, 2015). This process demands recognizing corresponding conceptual structures between different domains and their similarities or differences, thus advancing from a comparison based on superficial aspects or characteristics to one based on relationships between those domains. It means that this process has implications for the learning of mathematics (English, 1997), since new objects and relationships in an unknown domain can be discovered by extending ideas from a known domain.

In our study, we employ this idea of analogy to introduce students to spatial geometry, using a combined 2D and 3D-DGE. Analogy becomes a tool that allows students explore and discover relationships in the unknown (for them) domains of spatial geometry, starting from known domains of plane geometry. Likewise, the analogy offers the possibility of solving problems in a domain of spatial geometry, when they are formulated and solved in the corresponding domain of plane geometry, so that the results obtained in plane geometry can be interpreted in light of spatial geometry.

Analogical reasoning: establishing and using of analogies

Analogical reasoning is the process through which information from different domains is represented as relationships systems for study and comparison according to certain objectives (Richland & Simms, 2015). Students’ mathematical problem-solving activity has been analyzed to recognize the nature of the analogical reasoning exhibited by them. Although there are different processes with a specific role in this scenario, major emphasis has been placed by researchers on five of these processes: representation, retrieval, mapping, adaptation, and learning (Novick, 1988). These processes are described below according to our research interest.

At the beginning of a problem-solving process, in our case situated in 2D and/or 3D geometry, the problem is *represented* to identify the domain knowledge involved and the ways the elements provided by the statement of the problem may be related. With this representation, the solver can *retrieve* representations of knowledge or problematic situations that are relevant to solve the problem, which belong to 2D geometry. Having identified a domain in 2D geometry that provides information to determine the solution to a problem located in a related domain of 3D geometry, leads to the need of *mapping* the 2D and 3D domains, i.e., to establish links between objects and relationships in the 2D domain that will be useful with objects and relationships of the 3D domain. This does not imply that the relationships coming from the 2D domain are transparent to solve the current problem, since they are in a specific configuration, which may not be near the 3D configuration that is being addressed. This leads to the need to *adapt* the retrieved information (objects, relationships, procedures), so that it can be used in the 3D domain that is now being explored. Finally, when a problem in a 3D domain is solved, the strategy used, the information retrieved, and the relationships established between the 2D and 3D domains lead to new *learning* in which these elements intervene. Across this learning, there is an acquisition of new knowledge, as well as a possible modification of the previous conceptual representations of the solver.

All the analogical reasoning processes that have been described lead us to suppose that analogical reasoning is present in situations in which the representation and analysis of two domains leads to the establishment of analogies or to use an already existing analogy between two domains to solve problems or broaden existing relationships between those domains. The route that these processes outline is not easy to follow; English (1997) mentions the complexity that novice students face when carrying out this activity. For English, students require a cognitive maturity that allows them to focus on structural aspects of the compared domains and not only on contextual and superficial elements. This cognitive maturity is a characteristic differentiating mathematically gifted students from the average students in their grade or age.

METHODOLOGICAL ASPECTS

We present a case study, drawn from a broader research project in which we analyze the learning of proof, in the context of 3D geometry with the mediation of GeoGebra, by four Spanish mathematically gifted students. The students were 11 to 14 years old and studied in grades 1 to 4 of secondary school. We recognize them as mathematically gifted because, besides the ordinary schooling, the students had participated in programs of attention to general giftedness (AVAST) and mathematical giftedness (ESTALMAT).

We designed and implemented a sequence of 18 construction-and-proof problems in 60-minute sessions. Some problems requested the construction, first in 2D and then in 3D, of a geometric object satisfying some properties associated with equidistance (e.g., an equilateral triangle given its side). Other problems requested the construction of a 2D object and an analogous 3D one (e.g., the center of a circle in 2D and a sphere in 3D). In this way, these problems established relationships in 2D that were then analyzed in a 3D configuration, showing common and uncommon aspects; this was a relevant aspect when developing the problems. For each problem, the students first had to solve it and then to discuss the solution and justify its correctness with the teacher (the first author of the paper), who led the conversation. It provided students with useful instrumental and conceptual elements to solve subsequent problems. The sessions were audio and video recorded after informed approval by students and parents. As students were in different school grades and had different previous knowledge, the teaching sessions were organized as individual clinical interviews.

We present episodes of the solutions of three non-consecutive problems by one of the students, named Hector (pseudonym). We have chosen these problems because they illustrate the elaboration and use of analogies, not all expected a priori, between domains of plane and spatial geometry, as well as the influence of GeoGebra in this process. To analyze the analogical reasoning exhibited by Hector in solving the selected problems, we consider the five processes of analogy presented previously.

THE SUBSTITUTION OF THE CIRCLE BY THE SPHERE

Prior to the implementation of the sequence, Hector's knowledge of 3D geometry was scarce, it came from his school experience and was limited to recognizing the sphere and simple solids. Hector's experience with GeoGebra was limited to the use of some 2D tools; GeoGebra 3D was unknown by him. The problems that precede the first problem we presented gave Hector a first contact with GeoGebra 3D, its *Parallel Line* and *Perpendicular Line* tools, and point and perspective dragging [2].

Problem 04: An analogy between circles and spheres

The objective of this problem was to introduce the circle and sphere as loci and suggest geometrical relationships between them. The first part of the problem, to be solved in GeoGebra 2D, asked to create a point at the same distance from three given non-linear points. Solving this part of the

problem led Hector to involve the definition of circle as a locus, known by him from his school experience, as support for the properties that the requested point should satisfy.

The second part of the problem, to be solved in GeoGebra 3D, presented a red point A and black points B, C, D, and E (Figure 1a). Hector had to drag points B, C, and E so that the distance between them and A was equal to the distance between A and D. After building the segments determined by A and each black point, as well as measuring their lengths (Figure 1b), Hector looked over the tools associated with circles, although he did not find useful any of them. When the teacher asked him about the aim of this search, he expressed *the need for a circle with two points, the center, and the point through which it passes ...*, which could not be built because GeoGebra 3D does not have a *Circle (Center - Point)* tool. The conversation with the professor led Hector to understand that this tool was not available because it is possible to create several circles in space with the same center and containing the other point: *I already know why it is. It is because the circle can be this way* [outlining a circle with the pointer]. *The circumference can [also] be like this* [changing the perspective and sketching a second circle] ... *it can be in many ways, rotating*.

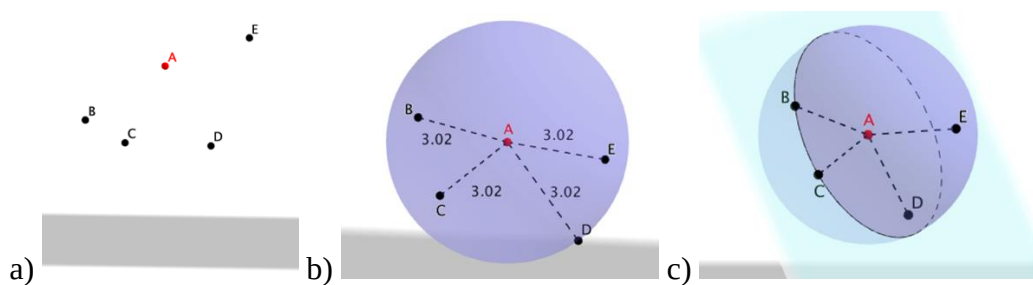


Figure 1. Substituting circles by spheres

Hector changed his strategy, he selected the *Sphere (Center - Point)* tool and constructed the sphere with center at A that contains point D. Points B, C and E were then linked to the sphere with the *Attach/Detach Point* tool (Figure 1b). Hector justified his actions by saying that *in the end it's the same. I just have to make [the points] fit exactly there [on the sphere]*. He explained his construction assuring that *A is undoubtedly the center, because I have used it as such. So, just as it happened with the circle, there is a property that is that A is at the same distance from any point that is on the circle, that is, on the surface of the sphere*.

Until now, the circle and the sphere had not been discussed, so the teacher asked Hector to express what each of these objects meant to him. Hector said that the sphere is a solid in which *the center is equidistant from all the points*, while the circle is *a geometric shape... the center is equidistant from all the points*. In the end, in what would seem to be a way of seeing the sphere as a general object that encompasses the cases of the circles that he previously pointed out, Hector mentioned that *in the circle that is in the plane shared by B and C [and A], A is in the center* [Figure 1c], so [points B and C] *have the same distance from A. On the circle that is in the plane of B and E, A is in the center, so they are equidistant from A. [On the circle that is] In the plane that E and C [and A] or D and C [and A] can share [plane determined by these points], well, the same thing [A is in the center] ...*

Problem 07: Using an analogy to construct a triangle in 3D

The objective of this problem was to construct an equilateral triangle given one of its sides in GeoGebra 2D (segment GH, Figure 2a) and later in 3D (segment AB, Figure 2b). The following episode shows the use of the analogy between circle and sphere that was elaborated in problem 04. In the 2D part of the problem, Hector built circles with center in G and H and radius GH. Then, the point I, intersection of the two circles, allowed him to build the requested triangle (Figure 2a). In the 3D part, although Hector looked at the available circle tools, he selected the *Sphere (Center-*

Point) tool and built spheres with center in A and B and radius AB (Figure 2b). Hector created a point D at the intersection of the spheres and the triangle ABD.

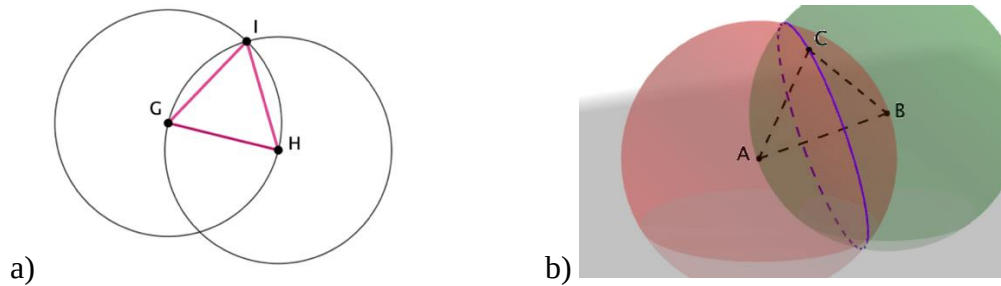


Figure 2. Construction of an equilateral triangle in 2D and 3D

Hector justified the validity of his construction by using the analogy circle-sphere to prove the congruence between the sides of the triangle: *I have done the same as in the 2D version, only that instead of circles I have used spheres... I have used spheres of radius AB, so these two segments here [pointing to AD and DB] are radius of circles with radius AB. So, they are all the same.*

Problem 16: A problem using the analogy

The last episode we present is the solution to problem 16, which asked to construct the center of a given sphere (Figure 3a). Hector made two unsuccessful attempts to use analogy to solve the problem, based on procedures that Hector had used to solve other problems. The first attempt corresponded to a construction that allowed him to obtain the center of a circle in the plane with the support of the tangent lines from two external points and the bisectors of the angles determined by these tangents (Figure 3b). The second unsuccessful attempt was based on the property that any angle inscribed in a semicircle is a right angle (Figure 3c).

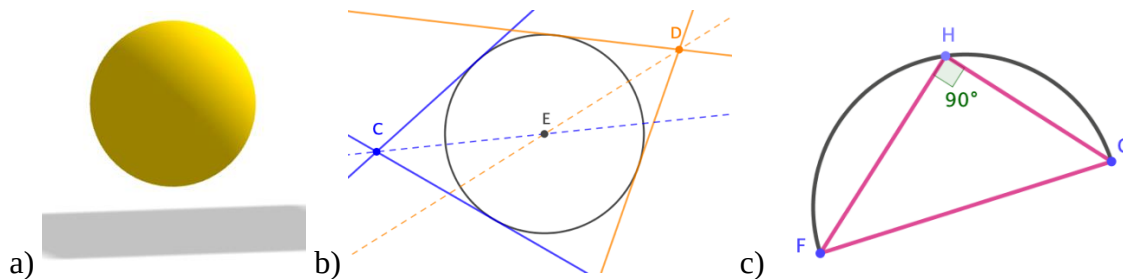


Figure 3. Two successful strategies in 2D

In his first attempt to obtain the center of the sphere, Hector constructed a point external to it, selected the *Tangent* tool, and clicked on the point and the sphere several times, without any result. His intention to recreate the construction he had made in 2D was not successful and, as he recognized, the reason for his procedure not working was that now there was a sphere: *I choose a point and then a circle... ah! It cannot be done with a sphere.*

In his second attempt, Hector constructed points E, I, and D on the sphere and point H as the midpoint of E and D; then he determined the measure of angle EID (Figure 4a). Without changing the viewing perspective of the construction, Hector dragged point I until the measure of angle α was close to 90° (Figure 4a) while stating that *If I get point I [so that] angle I to measure 90° , H will be the center of the circle.* However, the teacher asked him to change the viewing perspective of the construction, with which he noted that this strategy was not useful either (Figure 4b).

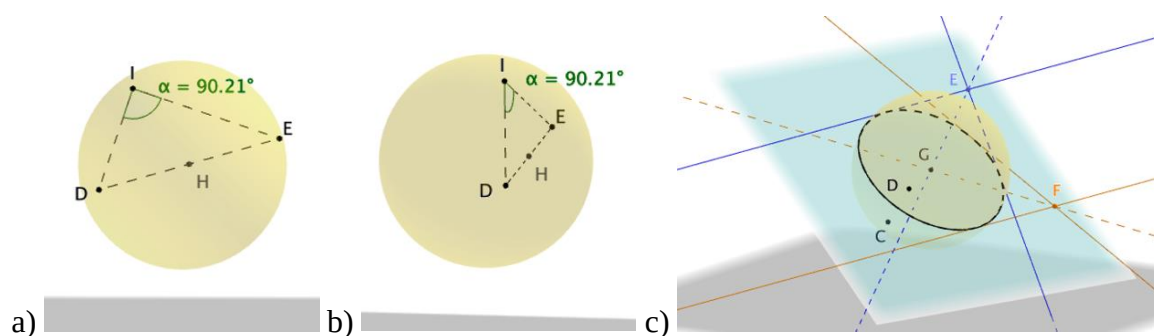


Figure 4. Modifying unsuccessful strategies in 3D

Hector expressed his aim to have a circle contained in the sphere: *the largest, that is, the Equator* [since this circle would have the same center as the sphere] ... *If I find the center [of the circle], I will find the center of the sphere*. To get this circle, Hector built the bisector plane of two points C and D in the sphere and the circle intersection of the sphere and the plane. He justified the correctness of the construction: *I knew that the center was going to be ... in the bisector plane of those points that I have made, because as the center [of the sphere] is equidistant [of the points in the sphere], it must be in that bisector plane*. Finally, Hector recreated on the bisector plane the construction he had made in 2D to determine the center of the circle and the sphere (Figure 4c). Hector justified the validity of his construction by stating: *I have treated it as if it were in 2D. I made a procedure that I also used in 2D ... When making the tangents, the bisector line of [the angle between] the tangents always pass through the center, that creates a diameter. And if I repeat it twice, it creates another diameter... the intersection of those two diameters gives the center of that circle, which is the largest and is like the Equator, has also been applied to the sphere*.

ANALYSIS

The episodes that we have presented show the elaboration and use of analogical reasoning, involving circles and spheres, as a tool to solve construction-and-proof problems in 3D geometry. This reasoning had its origin and development in the fact that some GeoGebra 2D tools are not available in its 3D configuration, as well as in the student's intention of taking advantage of his knowledge of plane geometry relationships to use them in space geometry, to simplify the solutions of the problems.

The episode from problem 04 shows the establishment of the analogy between the circle and the sphere. Although Hector knew well what he had to do and he contemplated a strategy to obtain the desired result (representation), not having the *Circle (Center-Point)* tool in 3D prompted him to provide an explanation for this fact and to introduce the sphere (recovery), arguing that the result obtained with circles or spheres would be identical, which was based on the definitions that he knew of these geometrical objects (mapping). Hector transformed the problem into an equivalent one when he considered the sphere and its definition to obtain the desired result (adaptation). The interpretation of the sphere that Hector used (the sphere contains every circle with center A and containing each given point), provided evidence of the generality attributed to the 3D construction, distinguishing it from the 2D construction, which would be a particular case that occurs in the plane (learning).

When solving problem 07, Hector made use of the analogy to build the equilateral triangle in 3D. The experience of building this triangle in 2D (recovery) gave Hector elements about the properties that should be satisfied (representation) and how these could be obtained in 3D with the help of spheres and the procedure used in the plane (mapping). Due to the nature of the problem, the need to adapt the procedures involved was not perceived, as this was almost immediate. In the end, the validity of the construction carried out in 2D with circles and the analogy between these and the

spheres were used by Hector to support the validity of the result he obtained in 3D. Thus, Hector used the analogy for the establishment of results in a domain (3D) when making the comparison with what he knew in another domain (2D) (learning).

However, the last episode (problem 16) shows the difficulty that can be faced when using analogies and the inappropriate results that can be obtained. To construct the center of the sphere, Hector used two 2D strategies to determine the center of a circle (recovery). However, the way in which these strategies were incorporated into 3D did not yield the expected result, because adequate links between the relationships in 2D and the characteristics of the 3D objects available (mapping) were not established; this difficulty was evidenced when Hector used the *Tangent* tool, or when 3D objects were manipulated from a single viewing perspective on the screen, as seen when Hector tried to construct a right triangle.

DISCUSSION

Based on the comments above, we have showed that trying to solve construction-and-proof problems in 3D-DGE can evoke building strategies known in the 2D-DGE. However, not having the same construction tools in the 2D-DGE and 3D-DGE or having these tools different performance in both contexts, can become an opportunity to understand mathematically why this happens and to promote processes of analogical reasoning that help modify and amplify the construction strategies known in 2D to make them functional in 3D. Hector's solutions showed that analogical reasoning may not require a deep analysis of the domains involved. On the other hand, it is possible that mapping and adaptation processes are deep and necessary, and not take them into account or not carried out properly would provide different results from those expected.

The latter idea is related to the treatment of the representations of 3D objects provided by the 3D-DGE on the screen. The actions carried out by Hector when trying to construct the center of the sphere cannot be generalized, but we can conclude that extending known procedures from 2D to 3D is not straightforward and can lead to a simplification of the properties of the 3D objects involved by relying on a single screen representation of them, which may originate incorrect constructions. This may be due to an inadequate process of mapping or adaptation of the domains involved. However, some 3D-DGE features that allow the manipulation of objects on the screen can help to reduce this problem, an aspect that highlights features of the 3D-DGE such as perspective dragging.

The results presented here come from the analysis of the case of a mathematically gifted student and are not intended to be generalized. However, they offer evidence of the nature of analogical reasoning when solving construction-and-proof problems in a 2D and 3D-DGE and how such kind of reasoning is favored or hindered by limitations imposed by the software, such as the unavailability of some construction tools, and the continuous support of the teacher who controls this process. The results also offer elements in favor of the need to study more deeply the influence and risks of using a 3D-DGE in the exploration of 3D objects, given the differences in the representation and manipulation of geometric objects in 2D and 3D.

Acknowledgment

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NOTES

1. In a DGE, construction-and-proof problems ask i) to create on the DGE a geometric figure having some properties required by the problem, that must be preserved under dragging, and ii) to prove that the procedure used to create the figure is mathematically correct, by explaining and validating the way of construction (Mariotti, 2019).

2. *Perspective dragging* is a term used by Leung and Or (2009) to refer to the possibility to change the viewing perspective of 3D objects on screen when a three-dimensional dynamic geometry environment (Cabri 3D, GeoGebra 3D) is used.

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HOW COMBINING SELF-ASSESSMENT AND AUTOMATIC ASSESSMENT MIGHT HELP TO SUPPORT STUDENT ENGAGEMENT

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While self-assessment and automatic assessment both carry much potential to enable rich formative assessment, there is a lack of research investigating the combination of both aspects to support student learning. In this paper, we present a digital self-assessment module in the context of Example-eliciting-Tasks using the STEP platform in which learners self-assess their work and subsequently receive automatic feedback about conflicts between their self-assessment and the automatic assessment. Furthermore, we present results from a study that shows that this combination of self- and automatic assessment can stimulate cognitive and metacognitive activities. Finally, we outline the first steps to a more extensive mixed-method study investigating the self-assessment module with a more comprehensive set of learners and comparing other forms of assessment.

Keywords: self-assessment, automatic feedback, digital learning environment

INTRODUCTION& THEORETICAL BACKGROUND

Previous studies have shown that digital formative assessment can support student learning, for example, through an automatic assessment of learners' solutions (Olsher et al., 2016). Self-assessment is important in developing learners' metacognitive skills and promoting ownership of their learning (Andrade, 2019). However, little is known about the combination of automatic and self-assessment (Olsher & Thurm, 2021). The project "Interplay between Self-assessment and Automatic Digital Assessment" (ISAA) (ibid.) aims to address this research gap by investigating the combination of automatic assessment and self-assessment in the context of Example-eliciting-Tasks (EETs) using the digital formative assessment platform STEP (Olsher et al., 2016)

Formative self-assessment

Self-assessment can be conceptualized as a process in which learners reflect on the quality of their work, assess how well it aligns with stated goals or criteria, and make revisions accordingly (Andrade, 2010). Hence, formative self-assessment emphasizes the role of the learners as independent actors. Self-assessment is considered one of the five key formative assessment strategies (William & Thompson, 2008) and can be implemented through various techniques and mechanisms (Panadero et al., 2016; Andrade, 2019). Self-assessment can enhance learners' metacognitive and self-regulatory processes (Ruchniewicz, 2022; Panadero et al., 2017). However, it is essential to note that self-assessment may be difficult for learners due to the high cognitive load they have to deal with (Andrade, 2019). Furthermore, there is the risk that learners may draw incorrect conclusions about their own learning process (Ruchniewicz, 2022). However, despite the potential of self-assessment, it is still rarely implemented in classrooms (Kippers et al., 2018).

Potentials of technology for formative self-assessment

Technology has various potentials to support formative assessment and self-assessment processes (Olsher et al., 2016; Ruchniewicz, 2022). In addition to interactive sample solutions or dynamic visualizations (Ruchniewicz & Barzel, 2019), interactive tasks and adaptive real-time feedback (Yerushalmy & Olsher, 2020; Olsher et al., 2016) are particularly relevant in the context of this study. One specific example of the potential of digital technology for formative (self-) assessment

is the use of digital EETs (Nagari-Haddif & Yerushalmy, 2019), which are evaluated in real-time using “attribute isolation elaborated feedback” (AIEF; Harel et al., 2022). For example, in the EET depicted in Figure 1 (Harel et al., 2022), learners are asked to formulate a conjecture about which types of product functions can be obtained when multiplying two non-constant linear functions and to construct three examples that support their conjectures using a multiple linked representation (MLR) interactive diagram. Subsequently, learners receive AIEF that provides feedback on ten predefined characteristics (e.g., the product function has no zero point; the two linear functions have a positive slope) that are fulfilled in their examples. Harel et al. (2022) show that iteratively working with the EET task and the AIEF can support learners in improving their conjectures and extending the variety of the constructed examples.

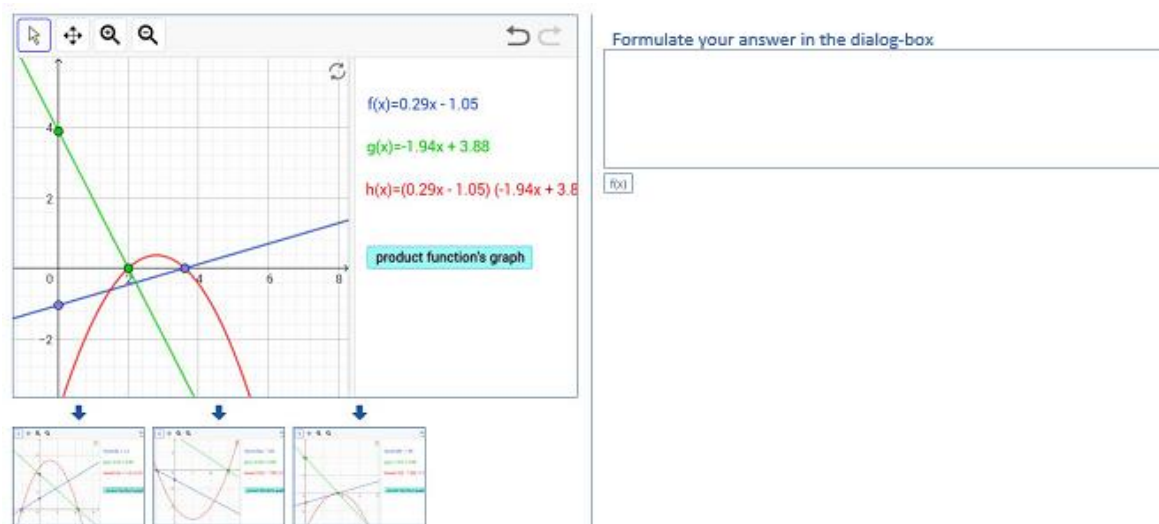


Figure 1. Task

MODULE COMBINING SELF-ASSESSMENT AND AUTOMATIC ASSESSMENT

The present study builds on the work of Olsher & Thurm (2021), who proposed that a combination of self-assessment and automatic assessment may allow to exploit the benefits of both forms of assessment. Self could help to support meta cognitive skills while the automatic assessment can aid in the potential risk of students wrongly self-assessing themselves. In particular, contrasting self- and automatic assessment may help to spark cognitive conflicts.

Based on this work, we conceptualized a self-assessment module (integrated into the STEP platform) that combines self- and automatic assessment. After completing the task in Figure 1, learners first evaluate which predefined characteristics are present in their constructed examples (Figure 2). If they are unsure whether a characteristic is present in their example, they can choose the question mark. Subsequently, learners receive a report consisting of three parts: a) an overview of their self-assessment (no longer modifiable, Figure 3), b) an overview of the automatic assessment (Figure 4), and c) an overview highlighting conflicts between self- and automatic assessment (Figure 5).

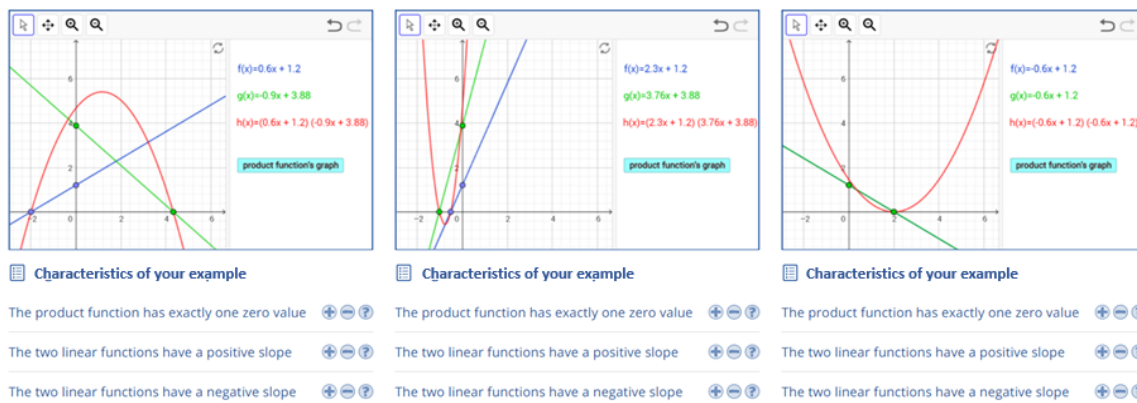


Figure 2. Self-assessment (in the picture, only the first three out of ten characteristics are displayed)

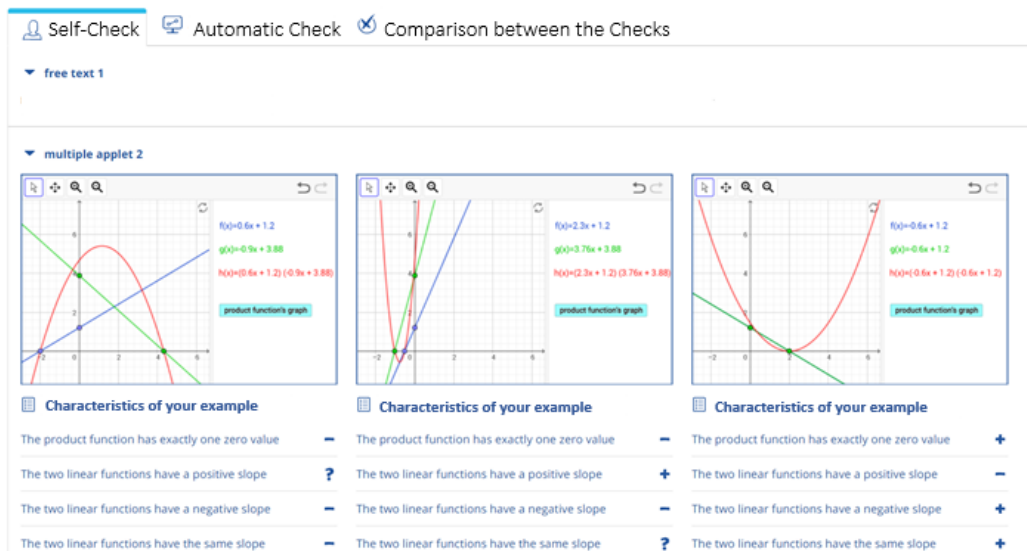


Figure 3. Overview of the self-assessment (in the picture, only the first four out of ten characteristics are displayed)

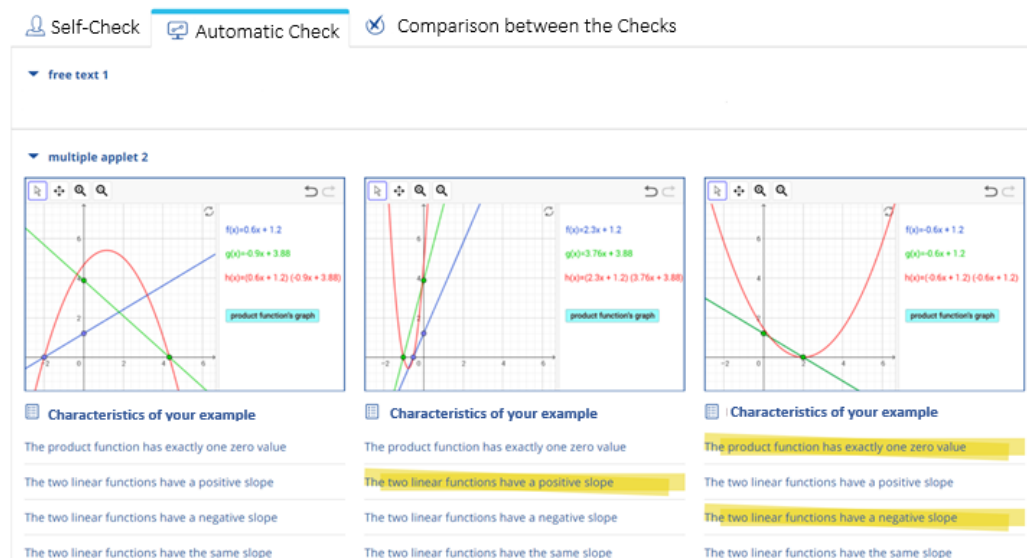


Figure 4. Automatic assessment (in the picture, only the first four out of ten characteristics are displayed and available characteristics are highlighted in yellow)

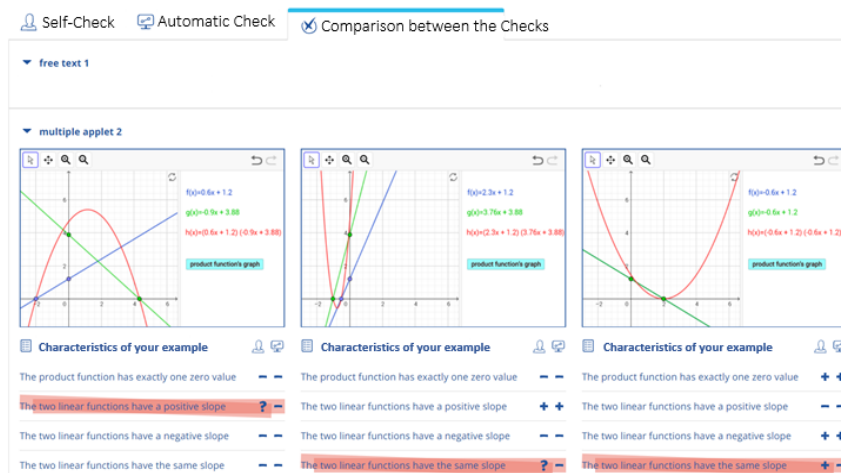


Figure 5. The combined summary (in the picture, only the first four out of ten characteristics are displayed)

PILOT-STUDY

In a first qualitative pilot video study with 16 learners (Tusche et al., 2022), we investigated to what extent conflicts between self-assessment and automatic assessment sparked metacognitive strategies and cognitive activities. For this, we developed a category system capturing students' metacognitive activities. The results showed that most learners were motivated by the feedback given in the report (Figure 2-4) to investigate the conflicts between self-assessments and automatic assessments. For example, two students stated that the linear functions in their generated example had the same slope. However, the functions only looked like having the same slope but were actually not parallel. Due to the shown conflict in the report, the learners started to analyse their mistake and to find possible explanations on both the graphical and symbolic level. Overall, the learners had fewer conflicts (i.e., a more accurate self-assessment) when running through the same EET and the self-assessment module a second time. Hence, the pilot-study suggests that the combination of self-assessment and automatic assessment can encourage learners to be more engaged and active in their formative self-assessment process and that the self-assessment module helps to stimulate cognitive and metacognitive activities.

MAIN-STUDY

In the following step, we aim to investigate the self-assessment module with another EET and with a more extensive set of learners. Furthermore, we aim to compare how learning processes differ between a) learners who work on the EET followed by the *self-assessment module* (Figure 2-5), b) learners who work on the EET followed by receiving only the *automatic assessment* (Figure 4) and c) learners who work on the EET and subsequently perform a *self-assessment without receiving an automatic assessment*. In the following we will elaborate on the new EET-task that we designed for this endeavour.

Designing an EET addressing the positional relationships of two linear functions

A well-known problem is that learners often have problems with changing representations of functions (Duval, 2006; Schoenfeld et al., 1993). In addition, learners need to understand how the parameters of two linear functions (slope and y-intercept) relate to their positional relationship (parallel, identical, crossing) and the number of intersection points (zero, one, infinitely many). This is a complex issue. Figure 6 illustrates the association of these three areas.

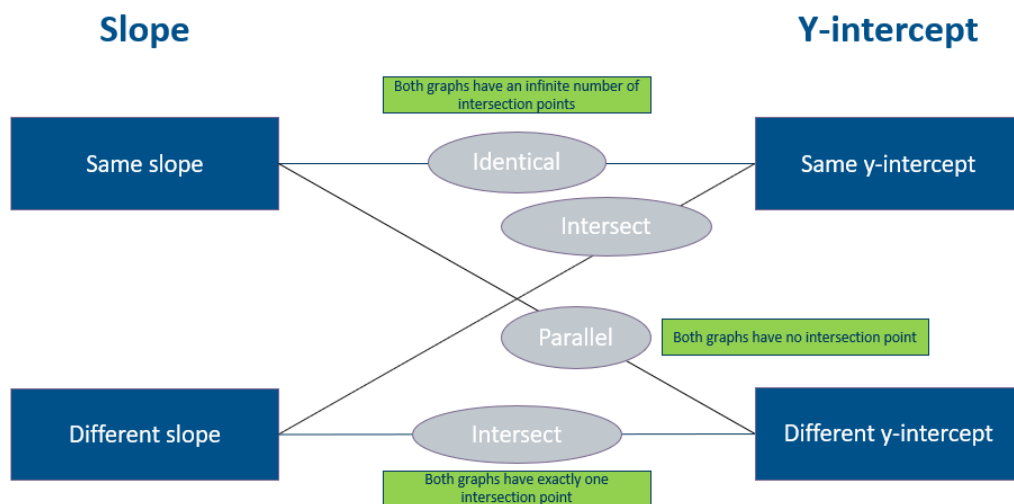


Figure 6. Association of the positional relationships (grey), parameters (blue) and number of intersection points (green)

Figure 7 shows the EET that we designed to address the aforementioned aspects. The learners are asked to write a conjecture explaining the association between the parameters of the two linear functions, the positional relationship, and the number of intersection points. Because there are three possible positional relationships, the learners are asked to create three examples. The learners can create examples by dragging the points on the linear functions, using the sliders, or manually entering/altering the algebraic expressions. Changes in one form of representation are transferred to the other. The dynamic visualization allows learners to quickly generate multiple examples and directly observe links between the symbolic and graphical representation. In the GeoGebra-Applet, the learners can zoom in and out and move the coordinate grid.

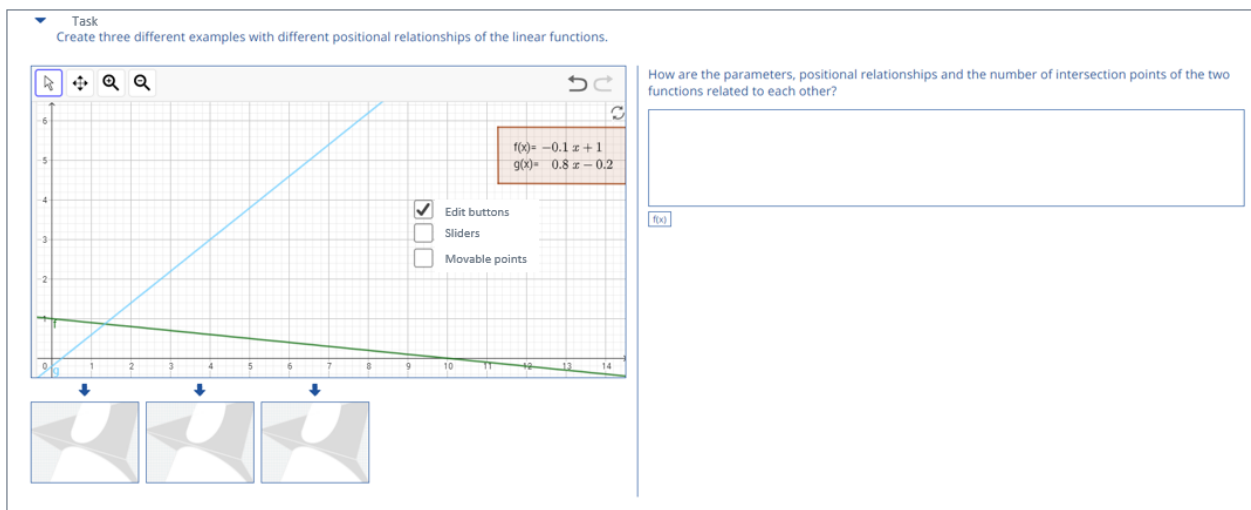


Figure 7. EET addressing the association between the positional relationship, the parameters and the number of intersection points of two linear functions

Designing characteristics for the EET

Designing characteristics for EETs that support the iterative and autonomous processes of conjecturing is a complex issue (Harel et al., 2022). To guide this process, Harel et al. (2022) proposed to generate characteristics representing potential mathematical perspectives and distinguish between three types of characteristics: basic expected characteristics (BEC), integrative expected characteristics (IEC) and challenging characteristics (CC). Building on the work of Harel et al. (2002), we conceptualize BECs as characteristics for which it is expected that the initially

generated examples of the students have these characteristics. IECs are a “combination of basic characteristics or of initial directions for conjectures” (Harel et al., p. 5) or those that are deliberately generated by the learners. CCs have the “potential to challenge the students’ thoughts and to inject new ideas” (ibid, p. 5). Since students should investigate the association between parameters, the number of intersection points and the positional relationship, we aimed to construct characteristics that address each aspect.

We predicted that the students would begin by exploring linear functions that have a negative slope and those that have a positive slope with different y-intercepts, which lead to the following BECs:

- Both graphs have a positive slope
- Both graphs have a negative slope
- Both graphs have a different y-intercept
- Both graphs have exactly one intersection point
- Both graphs intersect at exactly one point

Since we expected students might refine their initial conjecture in a way that it also focuses on the more complex case of identical functions (where both slope and intercept must be equal) we considered the following IECs:

- Both graphs have exactly the same slope
- Both graphs have the same y-intercept
- Both graphs have an infinite number of intersection points
- Both graphs have no intersection point
- Both graphs are parallel to each other
- Both graphs are identical

We point out that we included "Both graphs have exactly the same slope" and not "Both graphs have a different slope". The reason is that these two characteristics are equivalent. However, the first one may support students in investigating the special case of equal slopes.

The case of identical functions is particularly challenging as these functions are also parallel. Hence in the case of identical functions, all characteristics (without both graphs have no intersection point) will be fulfilled, while for parallel non-identical functions, only the “same slope”-, the “no intersection point”- and the “parallel”-characteristic is fulfilled.

Finally, we added the following challenging characteristic, which cannot be fulfilled by any example, to activate learners to make new conjectures or to check their existing conjectures:

- Both graphs have exactly two intersection points.

It is important to note that it could be a part of the task process that learners may generate examples with different parameters but the same positional relationship. We hypothesize that in this case, the characteristics help them refine their conjectures and examples.

METHODOLOGY

As described, the study pursues the research question of whether the combination of self-assessment and automatic assessment activates learners to engage with their examples and conjectures

metacognitively and content-wise more than self-assessment or automatic assessment alone. The study will be conducted with three intervention groups. Each group receives the same EET (Figure 7) but will receive different assessment forms (self-assessment module combining self- and automatic assessment, only automatic assessment, only self-assessment). To ensure that all intervention groups have the same knowledge when they start the intervention (e.g., working on the EET), all three groups will receive the same introductory phase in which already learned technical terms are repeated (i.e., parameters, slope, y-intercept). Subsequently, they are introduced to the different positional relationships of linear functions. In the introductory phase, no explicit connections are made between the parameters of linear functions, the number of intersection points, and positional relationships since they are to explore this in the context of the developed task. Outcome measures to capture differences between the intervention groups comprise the written conjectures of the students, the generated examples, and a pre and post-test assessing students' conceptual knowledge concerning the parameters of two linear functions, their positional relationship, and their number of intersection points. In addition, a subset of learners in each intervention group will be video graphed to capture differences in metacognitive processes.

EXPECTED RESULTS

We do not necessarily expect one of the interventions will provide the best results for all outcome measures. For example, the combination of automatic- and self-assessment may particularly support conceptual knowledge by pointing out conflicts between self- and automatic assessment. However, investigating and resolving the conflicts may carry the risk of students losing track of writing an elaborated conjecture. Hence dealing with the information about the conflicts, generating examples, and writing an elaborate conjecture may be a cognitive overload for students. Overall, we expect to gain nuanced insight into learning processes that can help design formative assessment environments.

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ROLE OF INTERACTIVE TASKS AND A COMPREHENSION-ORIENTED DESIGN IN INSTRUCTIONAL VIDEOS: RESULTS OF A PILOT STUDY

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Instructional mathematics videos that can be found e.g. on YouTube are very popular with students as support for doing homework or learning for an exam. However, from a didactical point of view, these videos can often be criticised for only conveying the superficial use of procedures and for leaving students as passive recipients of knowledge. In a randomised experimental study, we aim to find out how interactive tasks and a comprehension-oriented design can support not only the acquisition of procedural but also of conceptual knowledge. This paper reports on the results of a first pilot study with $n = 85$ participants that will be repeated in summer 2023 in a larger scale. For the acquisition of procedural knowledge, no differences between groups can be reported. For the acquisition of conceptual knowledge, interactive tasks seem to be favourable.

Keywords: *Instructional mathematics videos, interactive tasks, procedural knowledge, conceptual knowledge, systems of linear equations*

INTRODUCTION

Digital technology plays an important role in students' everyday lives and also gains more and more relevance in educational practice. In mathematics educations, "[t]echnology no longer refers solely to computers, mathematical software (e.g., dynamic geometry software platforms) or graphing calculators" (Moore-Russo et al., 2015, p. 108) but encompasses many different kinds of digital tools such as mobile apps and other media. One type of such tools are instructional videos, sometimes also referred to as educational videos or explanation videos, which are available as free learning resources on YouTube and other platforms. Teachers can integrate these videos into their educational practice; however, students also use these videos on their own outside of lessons to prepare for an exam or as support when completing homework assignments (Bersch et al., 2020). Fiorella (2021) defines instructional videos as "dynamic audiovisual presentations in which an instructor delivers oral explanations while presenting corresponding visual information" (p. 487). This definition mainly focuses on the simultaneous delivery of content in an auditive and visual way which is one of the main advantages attributed to instructional videos according to the cognitive theory of multimedia learning (Mayer, 2022). However, what this definition does not specify is who delivers the content and, even more importantly, how. Another definition by Wolf (2018, p. 4) even focuses on the fact that such videos are usually produced by amateurs who are neither professional media content producers nor designated experts on the topic of the video. In combination with the fact that there are usually no control mechanisms regarding the correctness and didactical substance, the quality of available videos varies. One of the main critical points usually discussed in the rather young research field concerned with instructional mathematics videos is that many of these videos only convey the use of procedures instead of also focusing on concepts and relationships (see e.g. Korntreff & Prediger, 2022; Lobato et al., 2019). Another problem can be the way in which students watch instructional videos. If students are only passive recipients of knowledge and not actively engage with them, they will not be able to actively construct meaningful mathematics knowledge (Bersch et al., 2020). However, as there are also many positive aspects of instructional videos such as the possibility to learn with them anywhere and anytime at one's own individual pace choosing the topics and instructors fitting the own needs and preferences (see e.g. Lobato et al., 2019; Wetzel & Ludwig, 2021a), it is important to dedicate research on how to overcome these problems or at least reduce their impact. In this paper, we present the results of a first study on the effect of

interactive tasks and a comprehension-oriented design on the procedural and conceptual knowledge acquired through watching a video on the topic of systems of linear equations.

THEORETICAL FRAMEWORK AND RESEARCH QUESTIONS

It seems desirable to create videos that not only teach students the superficial use and application of procedures (also referred to as procedural knowledge) but also convey conceptual knowledge. Conceptual knowledge can be defined as the “explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain” (Rittle-Johnson & Alibali, 1999, p. 175). However, to convey conceptual knowledge, videos must actually contain corresponding “concept elements” which many of the existing videos on YouTube do not do (Korntruff & Prediger, 2022). Kulgemeyer (2020) argues that for acquiring conceptual knowledge that can be transferred to new learning situations, further learning activities are needed such as a follow-up task at the end of or right after the video. However, such a learning activity cannot only be situated at the end of or after the completion of a video but also during it, e.g. by the means of interactive tasks (Wetzel & Ludwig, 2021b). Interactive tasks during a video can also potentially increase students’ engagement when watching the video which a task at the end or after the video cannot accomplish. To our knowledge, there are no empirical results regarding the acquisition of conceptual knowledge with instructional mathematics videos with the target group of high school students, neither for videos with nor without interactive tasks. We thus want to examine the acquisition of conceptual knowledge more closely, also when compared to procedural knowledge, and the influence of interactive tasks on both knowledge types. As many available videos online focus on procedures only (e.g. Bersch et al., 2020), we are also interested in the different impact on procedural and conceptual knowledge when a video focuses on procedures versus when it is designed in a comprehension-oriented way. By the latter, we mean a video that not only focuses on the “What?” and “How?” but also on the “Why?” and the connections between single pieces of knowledge. We want to answer two main research questions:

RQ1: What impact does a *comprehension-oriented design* of an instructional mathematics video have on the acquisition of procedural and conceptual knowledge?

RQ2: What impact do *interactive tasks* in an instructional mathematics video have on the acquisition of procedural and conceptual knowledge?

Regarding RQ1, it is likely that a comprehension-oriented design positively influences the acquisition of conceptual knowledge, if conceptual knowledge can be acquired by means of a video alone. It is unclear, how interactive tasks influence the acquisition of the two knowledge types, however we assume that students with interactive tasks have a higher cognitive engagement and thus get higher test scores, especially regarding the acquisition of conceptual knowledge which needs additional learning activities according to Kulgemeyer (2020). Our main research interest are videos viewed by students on their own at home which are usually videos on a topic the students already know which they watch as support for homework completion or exam preparation (Bersch et al., 2020). Therefore, the videos used in our study are videos on a topic the students already know.

METHODS

Study setup

To answer the research questions, we conduct a randomised experimental study in a pretest posttest design with four different treatment groups which will each view a different video:

- Group 1: Video is *not* comprehension-oriented⁶ and does *not* contain interactive tasks

⁶ Not comprehension-oriented means that these videos focus mainly on procedures

- Group 2: Video is *not* comprehension-oriented and contains interactive tasks
- Group 3: Video is comprehension-oriented and does *not* contain interactive tasks
- Group 4: Video is comprehension-oriented and contains interactive tasks

The whole study is conducted in one sitting. At the beginning, each student receives a code snippet randomly assigning them to one of the four treatment groups. Students then write a pretest in 25 minutes, watch the videos on tablets with headphones at their own pace in a maximum of 20 minutes and write a posttest in 25 minutes. Watching the video on their own pace means that students can pause, fast forward and rewind whenever they so desire. A first pilot study was conducted in January 2023 on the results of which this paper reports. The main study with a larger sample size will be conducted in summer 2023.

Creation of videos

To create the videos, we relied on criteria for good instructional videos in general and for mathematics videos in particular. Many general criteria are linked to the cognitive theory of multimedia learning which includes recommendations such as no distracting elements, a corresponding presentation of visual and auditive content and a visual highlighting of the most important points in the video (Mayer, 2022; Brame, 2016). Then there are criteria related to the length of videos. One recommendation by Guo et al. (2014) based on the analysis of about seven million video watching sessions is that videos should not be longer than six minutes to guarantee a maximum level of engagement. From a didactical point of view, criteria often mentioned are the use of a positive language when talking about mathematics (Bersch et al., 2020), the use of an interesting introductory example (Bersch et al. 2020; Kulgemeyer, 2020), a discussion of necessary pre-knowledge (Ratnayake et al., 2019) and common misconceptions and a summary of the content in the end (Kulgemeyer, 2020).

As a topic for the videos, we decided on systems of linear equations which are usually taught in grade 9 in Germany with two equations and two variables. We chose this topic, since there are different algorithmic rule-based approaches to determine the set of solutions (e.g. method of substitution and method of elimination) so the topic can be taught focusing on procedures alone but also there are many opportunities to teach concepts such as the meaning of a certain result for the set of solutions. The whole topic of systems of linear equation is too large to be covered by just one video. Therefore, the videos focus on the subtopic “determining and interpreting the set of solutions”. To determine the set of solutions, there are different procedures available. Again, all of them would be too much content so we decided to focus on how to solve systems of linear equations graphically and on the substitution method. For the interpretation part, we focus on the relation between the symbolic and graphical representation of systems of linear equations and how inferences can be made from the set of solutions on the corresponding system of linear equations and vice versa.

On the technical side, the videos were created as screencasts of animated PowerPoint presentations. At first, we created videos of the type “Khan-style tablet drawing” (Guo et al., 2014, p. 42), however, it took longer to write equations and calculations in handwriting which was problematic related to the recommendation of a maximum video length of six minutes by the same authors. Guo et al. (2014) propose to divide a video in several videos in such a case which might make sense for a YouTube channel owner but was not applicable to our study where the video is meant to be watched in one sitting. This aspect is problematic in general and requires further research if a video is meant to cover more than just a procedure while at the same time following the design guidelines recommended in literature. To keep our videos at a maximum length of ten minutes, like e.g. proposed by Biehler et al. (2023) for instructional mathematics videos, we still had to drop some

criteria such as the discussion of necessary pre-knowledge which was shortened to a quick mentioning at the beginning.

For the comprehension-oriented video, we added an extensive discussion of the relation between the geometrical location of two lines and the corresponding system of linear equations and set of solutions. For the video with focus on procedures, we only quickly mentioned this aspect and instead dedicated time to a further example for the method of substitution. The interactive tasks were added after the videos had been created using the software H5P (<https://h5p.org>) with the content type “interactive video” which allows to add different interactive elements to an input video. We used the interactive elements “Single Choice set”, “Text” and “Fill in the Blanks” (the latter for the input of numbers, e.g. “Find the value of x .”) and created six short tasks. As the purpose of this study is not to find out what kind of interactive tasks are most effective, we used similar kinds of tasks throughout the whole interactive video. The main purpose of these tasks is to cognitively activate the learners so that they pay more attention and engage more deeply with the mathematical content in the video. The tasks and their solutions are also discussed within the video itself right after the interactive prompt. Therefore, in the non-interactive videos without the prompts, the content of the tasks is also available to the learner, just not as part of an interaction.

Test instrument

To our knowledge, no publicly available test exists targeting systems of linear equation divided in items for procedural and conceptual knowledge. The study thus also aims at piloting the developed test instrument and to analyse the items within it. Scanning several textbooks for grade 9, we identified typical types of tasks focussing on systems of linear equations. Some task types could not be considered for the test if they required both procedural and conceptual knowledge to be completed. Based on these preliminary considerations, we created 34 tasks which were administered to five professors for the didactics of mathematics for an expert review procedure. Each item was rated on a six-point linear scale with the left end representing tasks only requiring procedural knowledge and the right end tasks only requiring conceptual knowledge in reference to Rittle-Johnson and Alibali (1999) stating that “[t]hese two types of knowledge lie on a continuum and cannot always be separated; however, the two ends of the continuum represent two different types of knowledge” (p. 175). Items with a mean result between 1 and 3 were then considered as requiring rather procedural knowledge while items with a mean value between 4 and 6 as requiring rather conceptual knowledge. Based on this, we accepted items with a mean within these two limits if their standard deviation is less than 1. Applying this rule, seven items were eliminated, nine were identified as procedural and 18 were identified as conceptual knowledge items. Twelve of the 18 conceptual items were chosen for the test and the nine procedural items were supplemented by three further items with similar assignments but different numbers. Procedural items include arithmetical problems such as algorithmically determining the set of solutions, checking whether a value pair is a solution or ordering the steps of the substitution method. Conceptual items mostly rely on the relation between the symbolic and the graphical interpretation of systems of linear equations and the set of solutions, e.g. for a given linear equation students must find another linear equation such that the set of solution meets certain conditions. Both pretest and posttest consist of twelve items (six items rather requiring procedural and six rather requiring conceptual knowledge). However, not all participants are administered the same items following a multi-matrix design.

Participants

The study was conducted in January 2023 in one German high school in three grade 9 classes and one grade 10 class. A total of $n = 85$ students participated in the study. Each student was given a code randomly at the beginning assigning them to one of the four treatment groups and one of the test booklets. Due to missing ill students in each class, not all codes were distributed resulting in

$n_1 = 24$ participants in group 1, $n_2 = 21$ participants in group 2, $n_3 = 18$ participants in group 3 and $n_4 = 22$ participants in group 4.

Data analysis

Items were rated dichotomously as correct (1) or false (0). Raw scores were processed further in R using the TAM package. Due to the use of different test booklets following a multi-matrix design, item scores were equated first using the method of joint calibration (Wu et al., 2016). With the equated data set, a two-dimensional Rasch model analysis (first dimension: procedural items, second dimension: conceptual items) was conducted using the results of the pretest and posttest as if they were results of different participants to estimate the item difficulty parameters. In a second step, a four-dimensional Rasch model analysis (dimensions: procedural items pretest, procedural items posttest, conceptual items pretest, conceptual items posttest) was conducted to estimate the person ability parameters using the item difficulty parameters from the first analysis as input. Based on these person ability parameters, classical test theory tests were conducted. To answer RQ1, group 1 and group 2 are analysed together as group NCO (not comprehension-oriented) and groups 3 and 4 are analysed together as group CO (comprehension-oriented). Similarly, for RQ2, group 1 and 3 together form group NIT (no interactive tasks) and group 2 and 4 form group IT (interactive tasks). For group differences, we performed one-way ANOVAs having checked for normality and equal variance.

RESULTS

After running the Rasch model, a fit analysis of the 24 test items yielded infit-t values between -1.45 and 1.61 which is inside the range of [-1.96, 1.96] so there is no indication that the items do not fit the Rasch model (Wu et al., 2016). Therefore, all items were used for the estimation of person ability parameters. In the pretest, the mean person ability parameter is -0.38 (SD = 0.86) for the procedural items and -0.52 (SD = 0.95) for the conceptual items.

A one-way ANOVA analysis was performed to check whether equal preconditions among the four treatment groups can be assumed. There was no statistically significant difference in the pretest scores between the four groups ($F(1, 82) = 0.66$, $p = 0.418$) for conceptual items. For procedural items, however, there was a statistically significant small difference between at least two groups ($F(1, 82) = 4.56$, $p = 0.036$). To further examine this difference, we performed two ANOVAs for procedural items to compare groups NCO and CO ($F(1, 82) = 7.19$, $p = 0.009$) as well as groups NIT and IT ($F(1, 82) = 0.10$, $p = 0.748$). As there is a statistical difference in procedural items scores between groups NCO and CO, we only report on results for groups NIT and IT for procedural items since only for these groups we can assume equal preconditions.

A summary of the changes of person ability parameters from pretest to posttest can be found in Table 1. In all groups, the mean person ability parameter increased. A one sample t-test was computed for all reported increases in Table 1 to determine whether they are different from zero. All reported increases are statistically significant from zero ($p < 0.01$).

Groups	NCO	CO	NIT	IT	All
Procedural items	-	-	0.84 (0.48)	0.67 (0.48)	0.76 (0.46)
Conceptual items	1.37 (0.44)	1.42 (0.50)	1.24 (0.47)	1.55 (0.42)	1.39 (0.47)

Table 1. Mean and standard deviation (in brackets) for differences of person ability parameter from pretest to posttest

To answer our first research question, we compare person ability parameter increases for conceptual items for groups NCO and CO. Even though the group CO has a slightly higher mean increase than

the group NCO (1.42 compared to 1.37), an ANOVA analysis yielded that this difference is not statistically different ($F(1, 82) = 0.18, p = 0.674$). We can therefore not conclusively give an answer to our research question based on this data.

For the second research question, we can compare both, differences for procedural and conceptual items, between the groups NIT and IT. Contrarily to what we expected, the increase of person ability parameters from pretest to posttest is higher for videos with no interactive tasks ($M = 0.84$) than for videos with interactive tasks ($M = 0.67$) for procedural items. However, this difference is not statistically significant based on the results of an ANOVA ($F(1, 82) = 2.81, p = 0.098$). For conceptual items, we can report a higher increase for group IT ($M = 1.55$) than group NIT ($M = 1.24$). Again, an ANOVA was performed and this difference was statistically significant ($F(1, 82) = 10.82, p = 0.001$). This effect is also visualised in Figure 1. So, to answer our second research question, we can state that interactive tasks in instructional mathematics videos can have a positive effect on the acquisition of conceptual knowledge.

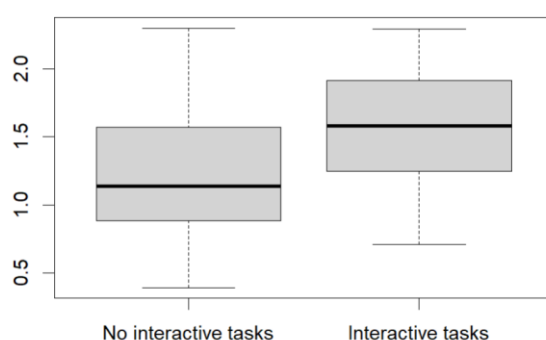


Figure 1. Comparison of differences of person ability parameters for conceptual items from pretest to posttest of the groups NIT and IT

DISCUSSION AND OUTLOOK

For procedural items, we cannot report any statistically significant differences between groups NIT and IT (and we had to exclude NCO and CO from the analysis, see results section). One might expect that interactive tasks which are meant to lead to more cognitive engagement also lead to more procedural knowledge. Especially, since interactive tasks in our study focused mainly on procedures, for example, in an equation with just one variable x , participants were prompted to calculate and enter the correct value for x . However, at least for our small sample size, we cannot report any effects on procedural knowledge. That leads to the question whether for the acquisition of procedural knowledge the interactive design of the video might not be as relevant. The larger scale main study in summer 2023 will hopefully contribute to the answer of this question.

For conceptual knowledge, the first important result to report is that all four groups significantly improved from pretest to posttest. This is also interesting considering that participants in group NCO received a video that focused mainly on procedures. There is not even a statistically significant difference between groups NCO and CO. However, mathematical relations were not omitted completely from the videos in group NCO but rather mentioned shortly without any detail. It would be an interesting insight if such short mentions can already lead to the acquisition of conceptual knowledge. In the main study, the explicit relationship between the video content and the results must therefore be examined more closely.

Regarding the difference between the groups NIT and IT, the groups with interactive tasks had a significantly higher increase from pretest to posttest for conceptual items than the group without interactive tasks. This is in line with Kulgemeyer (2020) stating that follow-up learning activities are necessary for a conceptual knowledge increase and our assumption that interactive tasks during

the video can have the same function. However, even if interactive tasks lead to more conceptual knowledge, they were not a prerequisite, as all groups increased their conceptual knowledge.

CONCLUSION AND OUTLOOK

The biggest limiting factor of the presented results is the small sample size. Therefore, we cannot conclusively answer our research questions and need to rely on the results of the main study which will be conducted in summer 2023. However, we can still report that interactive tasks seem to positively influence the acquisition of conceptual knowledge.

In the main study, there will also be an extensive analysis of a corresponding questionnaire with items on topics such as engagement during the video which will also give insights on the participants' level of cognitive activation reached through the interactive tasks. Furthermore, data will be collected regarding the way the participants interact with the video such as pausing, rewinding, or jumping forwards and how much time was spent on each interactive task (for group IT) and if the answers were correct. These additional data will provide more information based on which the research questions can be answered.

One general observation is the complex interplay between the length of a video and its didactical value which proved to be a real practical challenge for us. Further research must be dedicated to investigate the impact didactical criteria and the video length have on student learning and how these two factors interrelate.

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Making sense of educational practice in a transforming society

THE EDUCATIONAL GAME CHOICO AS DIGITAL LEARNING ENVIRONMENT FOR SUPPORTING STRATEGIES FOR CAREERS DECISION MAKING PROCESS,

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The transition from education to the labor market, through the choices of tertiary education institutions, is an important stage in the life of young people. Problem-solving skills, rooted in mathematics education are a basic component of young people's career readiness. In the digital age of the 21st century, how could educators support students in developing employability skills? Can digital tools support the design of strategies for a smooth and safe transition from education to the labor market? This paper presents a case study of the implementation of the Educational Game Choico (Choices with Consequences), as a digital tool for integrating computational thinking in the educational process. By simulating roles as co-designers in learning technologies, students actively participate in building knowledge and developing problem-solving skills and strategies that will support them in the career decision-making process. Under the framework of digital transformation of education, in terms of the development of employment skills, we consider that good teaching practices, such as this one, could be integrated into innovation centers, for strengthening the digital potential of education and modernization of vocational education and training.

Keywords: computational thinking, careers decision-making process, problem-solving skills, digital tool, Choico Game.

INTRODUCTION

The rapidly evolving modern society makes the role of school vital in preparing young people for the transition to the labor market and above all to equip them with the knowledge, attitudes and problem-solving skills to play an active role as future citizens in actions and decisions that will have a positive impact in their personal and social life (Argyri & Smyrnaiou, 2021). The development of mathematical thinking is a necessary condition for developing problem-solving skills, which also deals with such other skills as adaptability, decision-making, flexibility, creativity, and even innovation (Geisinger, 2016). Analysis and problem-solving are included in the list of transversal skills related to career paths and be considered for young people as a requirement for coping with the challenges of the modern labor market and the need to reach a successful school-to-work transition (Caena & Punie, 2019). Computational thinking involves the application of mathematical skills to solve problems and has been seeking its place in the educational field in the new society dominated by new technologies (Wu & Yang, 2022). Computational thinking is recognized as part of the mathematical problem-solving process and its application is not limited only to mathematical problems it could be extended to educational processes and relationships in the arts and sciences (Shute, et . al, 2017). Moreover, digital tools could provide powerful new techniques for deepening the learning and experience of mathematics as a basis to model complex phenomena (Rycroft-Smith & Connolly, 2019). This means that on one hand educators have to transform teaching and learning practices in mathematics under the terms of school-to-work transition and on the other hand one of the challenges of the Digital Transformation Era is the use of digital tools for the development of students' problem-solving skills. This paper is based on the issue of students' career readiness and focuses on the processes of computational thinking as part of mathematical problem-solving to engage students in decision-making for socio-scientific problems. In that way, this paper addresses

the affordances of the Educational Game 'Choico', as a digital tool that integrates computational learning environments into the educational process for making decisions careers.

COMPUTATIONAL THINKING AND PROBLEM-SOLVING SKILLS IN THE EDUCATIONAL PROCESS

Computational thinking can be integrated into the educational process within frameworks that recognize learning (1) as the acquisition of knowledge and skills, (2) as identity formation through creative expression and social engagement in digital media information and (3) as developing an understanding of how social-cultural reality is shaped, emphasizing strategies for solving problems (Kafai, et. al, 2019 cited in Kafai & Proctor, 2022). Computational thinking is re-conceptualized as a 'thinking model', in a way that makes it possible to integrate it into all subjects of the curriculum and it could be considered a way of thinking to solve real problems (Li, et.al, 2020). Such an approach advocates for the removal of computer literacy from the narrow limits of the possibilities of using computers and can be considered a powerful tool for students to realize, discuss and deal with complex socio-scientific issues and problems (Kafai & Fields, 2013). "Computational participation" is included to describe the different dimensions of integration of computational thinking in the educational process, where planning and application systems are not only the function of the algorithmic way of thinking but of the practices, which are necessary to join, within wider social networks and understand the cultural and social nature of a networked society (Kafai & Burke, 2014). The cognitive structure of computational thinking provides a structured, conceptual and strategic way of thinking about the process of solving problems through decomposition, the process of pattern recognition and pattern matching, the formulation of a possible solution by various processes such as abstraction, evaluation, and debugging, as well as algorithmic thinking with the presentation of solutions (Einhorn, 2012). Computational thinking as a way of thinking and solving problems is a multidisciplinary and interdisciplinary core skill of the 21st century, that contributes to the transformation of school systems into the digital age (Labusch, et.al, 2019). In terms of digital transformation of education for developing employability skills, such as computational thinking, problem- solving skills, the Computer Technology Institute and Press "Diophantus" focusing on research and development in Information and Communication Technologies (ICT) will create and manage thirteen innovation centers for organizing training courses, in order to meet the unique and distinct educational needs for integrated digital learning environments. Under this framework, what educational materials and/or training services could be involved in training courses of innovation centers for educators in order to support integrating computational thinking in the educational process, so that they, in turn, support the students to create their personal meaning and knowledge of how to learn for those?

EDUCATIONAL GAMES AS DIGITAL TOOLS FOR INTEGRATING COMPUTATIONAL THINKING

Children's ability to learn based on their own thoughts, to modify digital games when game functions are not as expected, and to engage in error-finding strategies for improvement helps them understand the ever-changing nature of digital media, and in the development of problem-solving skills (Gee, 2003; Grimes & Fields, 2015). By using digital tools, students can become capable of discovering and expressing scientific ideas through the construction, modification, and sharing of structures, such as models, programs and simulations (Smyrniou & Moustaki, 2012). For this reason, the focus of interest in game creation is system thinking as a framework for approaching science learning and computational thinking as a design system. "Designer mindset" refers to the development of educational game design skills, the synthetic and analytical nature of design or system thinking, and less to the use of programming language (Games, 2010). To integrate digital tools as systems thinking tools, it is necessary for teachers to be able to harness the power of using digital technology and put it at the service of the educational process. Teaching is recognized as an

art because it requires creativity and imagination. Teaching is not a theoretical science that describes and explains some aspect of the physical or social world. It is closer to the kind of science, such as engineering, computer science, or architecture aimed at improving social reality: a design science (Laurillard, 2013). The use of digital tools is placed at the service of the educational process for students to interact with multiple representations to construct digital structures (Kynigos, 2004; Kynigos 2015) under the terms of 'microworlds'.

MICROWORLDS AS COMPUTATIONAL EDUCATIONAL GAME ENVIRONMENTS

Papert (1980), used the term 'Microworld' in order to describe a computational learning environment rich in meaning-making opportunities involving dynamic representations (Healy & Kynigos, 2010). The concept of the "half-baked microworld", has been proposed by Kynigo (2007a, 2007b), serves to engage students by assimilating ideas and representations, but where they are asked to modify, expand, but also improve with the aim of creating a new version of the original microworld, which will reflect their own ideas (Kynigos & Yiannoutsou, 2018). Microworlds can function as digital learning environments of educational games (Grizioti & Kynigos, 2018), in order to allow students: i) The dynamic manipulation of objects, to discover their properties and the relationships that govern their behavior, ii) To make assumptions and use feedback from the digital environment to check them, in turn to correct his own ideas about the field formed by processing and debugging the microworld.

RESEARCH FRAMEWORK

In the framework of the doctoral study 'Connecting Secondary Education with Workplaces' at the Department Educational Studies of National Kapodistrian University of Athens, have been researched worked based learning strategies and methodologies that could enhance the career readiness of young people in terms of development skills, knowledge and attitudes in order to support them in transition from education to the labor market. This study focuses on research of preparing students for the transition from secondary education to workplaces in the specific context of integrating computational thinking for the development of problem-solving skills by utilizing the digital learning environment of the educational game Choico terms.

The selection points in the educational game is based on the career's readiness indicators (Covacevich, et al., 2021; OECD. 2021) (educational visits to workplaces, assessments of personal skills/interests, entrepreneurship competitions, volunteering, writing research scientific essays, job fairs, internship at workplaces, career counselor, job shadowing, careers talks with guest speakers) and the consequences of the choices have a positive or negative impact on the academic knowledge, self-esteem, communication skills and stress. Career readiness depends on academic knowledge and skills, but also refers to the ability to transfer them in specific situations. Future careers also, require the development of critical thinking, the ability to solve problems, the ability to adapt to changes, but also social and communication skills, which are called 'transversal skills' (Argyri, 2019; Smyrniou, et. et al., 2017). Among the personal learning skills that will support young people to manage the challenges and changes in their personal lives in the changing modern world, self-confidence has been selected as a value of optimism, hope, resilience, self-efficacy and a sense of purpose. Stress could be considered as a value of well-being that is a search for life satisfaction, care for physical, mental and social health and adoption of a sustainable lifestyle (Sala, et.al, 2020). The Choico digital learning environment enables the reproduction, modification and creation of "choice-driven simulation games" (Kynigos, & Grizioti, 2020). Choices, consequences, rules and relationships can easily be modified by the user, who could create different versions of the represented game system (Grizioti, & Kynigos, 2021).

Methodology

The study lasted 6 hours with 14 students of the STEM club of Evangeliki Model High School of Smyrna in Athens. Through Lee et.al 3-stage model, "Use-Modify-Create" (UMC) (cited in Grizioti & Kynigos, 2021; Lytle et. al,2019) students used the first version of the educational game, which contained errors, they exchanged views and discussed modifying it and finally attempted to create their own versions.

In each stage of the model, indicators as a measure of assessment of students' way of thinking in the problem-solving process were integrated (Annizar, et.al, 2020), as shown in Figure 1.

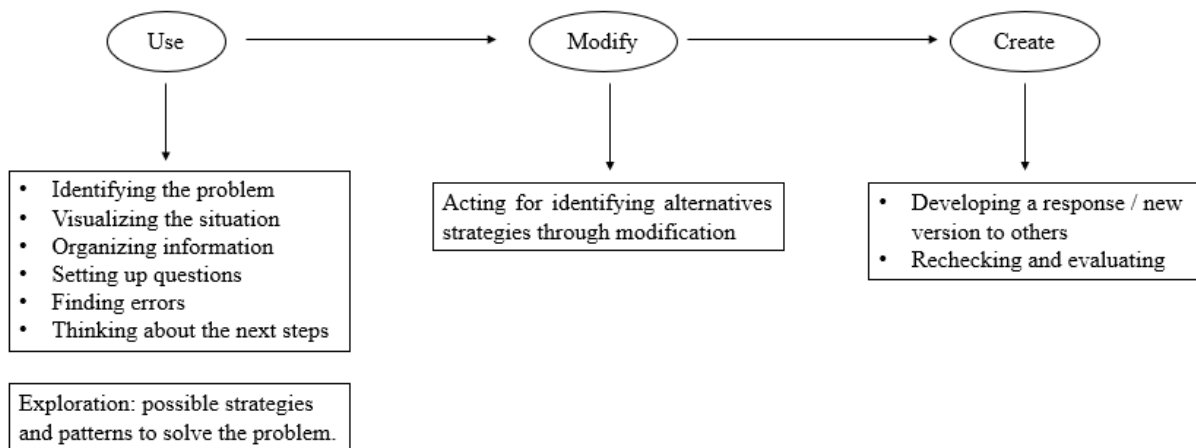


Figure 1. Indicators of mathematical thinking strategies integrated into Lee et.al's 3-stage model

Data collection

Participants students were familiar with the activities that provided indicators of career readiness, as they are also the sample of doctoral study. The educator participated as a researcher and she had the role of supporter and facilitator of the implementation of 3 stage phases. She guided the learning process through inquiry questions oriented to the components of the cognitive structure of computational thinking to assess problem-solving strategies, but also to the contribution of the digital learning environment of the educational game Choico for the development of problem-solving skills for making decisions for finding alternative ways (solutions) that support students' career readiness. Educator-researcher collected data while keeping notes (statements), by discussions of participants in analysis and reflection on the information generated by the research process.

Data Analysis

The assessment of the research process is based on the analysis of indicators that describe mathematical problem strategies used by participants through the collected statements of educators' notes provided by replies to inquiry questions. In particular: i) Identifying and representing the problem: Do you agree with the career guidance stations (the selection points in the main scene) and their values? What are the possible ways to support your career readiness for a safe transition from education to the workplaces?, ii) Exploring possible strategies and acting on strategies to modify the educational game: Do you suggest modifications? If you suggest modifications, explain in which parts (e.g.to the selected points or/and of them, to the termination rules, iii) Rechecking and evaluating based on students' new versions. For assessment of the impact of participation in the implementation of the Choice Educational Game, replies to the open question of what are the positive or negative features that they recognize are compared.

Results

In the first stage of the model, which includes the identification and representation of the issue/problem, most of them tried to exchange ideas for the strategy of winning and termination rules. In this stage, they mentioned that there were 3 points that if select them, then they directly lost, so they understood that the object of the game for the player is to stay in the game for as long as possible and the player who manages to select points to stay the longest is considered the best. While students interact with the game they try to discover the structure and discover patterns in the effect of their choices, they express different views of the positive or negative impact of selected points. In their discussions:

Student: Why does choosing a career counselor have a negative impact on stress?

Student: Why do I select career talks with guest speakers that haven't had a positive impact on my communication skills?

Student: The impact of volunteering on communication skills is very low.

In the second stage of modification of the model, some of them agree with the selected points and fields that selected points had consequences, but different views were also expressed.

Student: Yes, I think that as we have mentioned in our educational visits to workplaces the requirement for employment is a combination of academic skills, but also we need management skills and critical thinking. So, instead of communication skills, there are more skills that we could add.

Student: Unfortunately, stress is present in all stages of our decision-making careers, so needed to add the same arithmetic value for all selected points.

From the above discussions, we could highlight that they tried to find alternative strategies to support their career readiness, as they tried to modify by adding more skills as selected points and changing the arithmetic values.

In the third stage of creating a new version, they were very enthusiastic, as they could easily use the specialized programming language. Most of them follow their views of the phase of modification, and according to their personal interests, they put different impacts on the selected points. Additionally, they tried to change the end rules, but as they tried to test their new versions playing in teams, they ascertained that they have to be careful in order to select strategies based on new numbers that they have selected. They created a matrix, put the values, and sum each column to find the max and the min of each of them. Then based on the sums, they tried to put new end rules, but they weren't sure of these selections, so they decided to test the new ones with their classmates. Unfortunately, the time in the school classroom is limited for this pilot activity, but we hope it will be held in the future.

Students did not recognize any negative aspect of their participation in the Choico Game, but they expressed positive comments. They consider that the digital learning environment was very supportive for thinking creatively about strategies, while they modified or rechecked their actions.

Student: It is very interesting that for the same goal (career readiness), starting for the same points you have many different possibilities that you could follow and you never lost.

The additional value of this case study was the combination of digital tools in the procedure of developing students' problem skills as they try to discover different strategies for career guidance.

Student: I have never participated in activities such as this...it was amazing using digital tools for investigating the guidance of my future career.

Participants analyzed the information (points and their impact on the values of career readiness) from different perspectives and tried to categorize them into manageable to find the strategy for winning (strategy for the solution). They developed the ability of analytical critical thinking as a basic element of the computational thinking process, as they tried to find patterns with the organization of the arithmetic data in a matrix, for modification alternative strategies.

CONCLUSIONS

Conceptual approach to learning with representation of a real situation, that is the ways of transition from education to workplaces, supported by digital tools, so as students create their personal meaning through the process 'learning by doing' making sense of educational practices in a transforming digital society. The Educational Game Choico could be an effective digital tool for the integration of computational thinking and constructivism learning theory, for the development of problem-solving methodologies, and critical and analytical thinking skills. The student's ability to learn based on their own thoughts, to modify digital games when game functions are not as expected, and to engage in error-finding strategies for improvement support mathematical ways of thinking.

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ADDRESSING THE CONGRUENCE AND SIMILARITY OF FIGURES WITH TECHNOLOGY: A CROSS-NATIONAL COMPARISON

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In this paper, we briefly examine how digital technologies are included in the curriculum guidelines of two countries in Europe (Italy and Portugal) and two in Latin America (Colombia and Mexico) and present four examples of how technology can be very differently used to address the topic of congruence and similarity of figures at middle school level. This work is part of a cross-national study in which we focused on the factors affecting digital technology (DT) integration in mathematics education. The aim of this paper, which is not a research paper and is far to be comprehensive or generalizable, is to provide insights into the challenges of DT integration in mathematics education, drawing the attention of the readers to the impact and limitations that various factors can have on it.

Keywords: *Technology integration, Cross-national comparison, Congruence and Similarity of Figures*

INTRODUCTION

Although national policies and curricular recommendations promote the use of technology in mathematics education, digital technology (DT) resources are often underutilized in practice, and rarely considered for their mathematics-enhancing potential. Research (e.g. Julie et al., 2009; Sacristán, 2017; Sacristán et al., 2021; Faggiano et al., 2021) has also shown that in many countries, technology tends to be used as an add-on to traditional paper-and-pencil tasks and/or for presentation and communication purposes (which was exacerbated by the COVID-19 pandemic).

The integration of DT into mathematics education is influenced by a range of factors at various levels, including societal and governmental policies, regional and school-level regulations, and teacher practices. At the teacher level, for example, challenges can arise due to the absence of clear guidelines in official documents and curricula regarding the appropriate utilization of recommended resources or how to determine when to employ them.

In this work, we aim to show very different approaches to how technologies are conceived and used. Specifically, through a cross-national analysis (Jablonka et al., 2018), we examine how digital technologies are included in the curriculum guidelines of two countries in Europe (Italy and Portugal) and two in Latin America (Colombia and Mexico), and present examples from each country of how one middle-school mathematics topic is addressed using DT resources.

We selected a topic that is present in all of our middle school curricula (ranging from grade 7 to grade 9), which we deemed suitable for being taught using digital tools, specifically dynamic geometry. The topic we chose was *Congruence and Similarity of Figures*. Surprisingly, we found very few examples of digital approaches to this subject in materials designed for teachers, with the most notable ones originating from Italy and Mexico, the latter being quite dated. We begin by exploring the Italian approach, which we regard as the most thorough.

THE CASE OF ITALY

In the introduction of the Italian National Guidelines for basic school (MIUR, 2007) that state the goals and aims for mathematics, there are some general considerations regarding the use of technology. The guidelines underline that it is important to encourage the deliberate and purposeful use of calculators and computers beginning in the early years of primary school. This may include using these tools to verify the accuracy of mental and written calculations, as well as to explore the world of numbers and shapes. The use of technology is one of the learning aims to be achieved by the end of middle school, specifically in relation to the themes of “numbers” and “space and figures”. Students should learn to: perform basic mathematical operations between different types of numbers (natural numbers, whole numbers, fractions, and decimal numbers) using mental math, standard written algorithms, calculators, and spreadsheets, while carefully considering which tool is most appropriate for the task at hand; reproduce geometric figures and drawings with precision and accuracy, utilizing appropriate tools such as rulers, squares, compasses, protractors, and geometry software. However, it is worth noting that, although there are official ministerial projects on coding and computational thinking in Italy, these are not curricular and mathematics is not mentioned at all (e.g., see <https://www.indire.it/en/progetto/computational-thinking/>).

Additionally, there are some valuable, yet often overlooked, resources available on official websites such as from the Italian Mathematics Union (UMI-CIIM: <https://umi.dm.unibo.it/materiali-umi-ciim/>) or in the final reports of national projects like PQM (2015) and *mat@bel* (<http://www.scuolavalore.indire.it/superguida/matabel/>). These resources have been developed by experienced teachers to support the improvement of learning outcomes in the logic-mathematics field. Most of these resources are designed for middle and secondary school students and sought to harness the potential of new technologies. The following example comes from the repository of the PQM ministerial project.

A teaching activity on proportionality with dynamic geometry (8th grade)

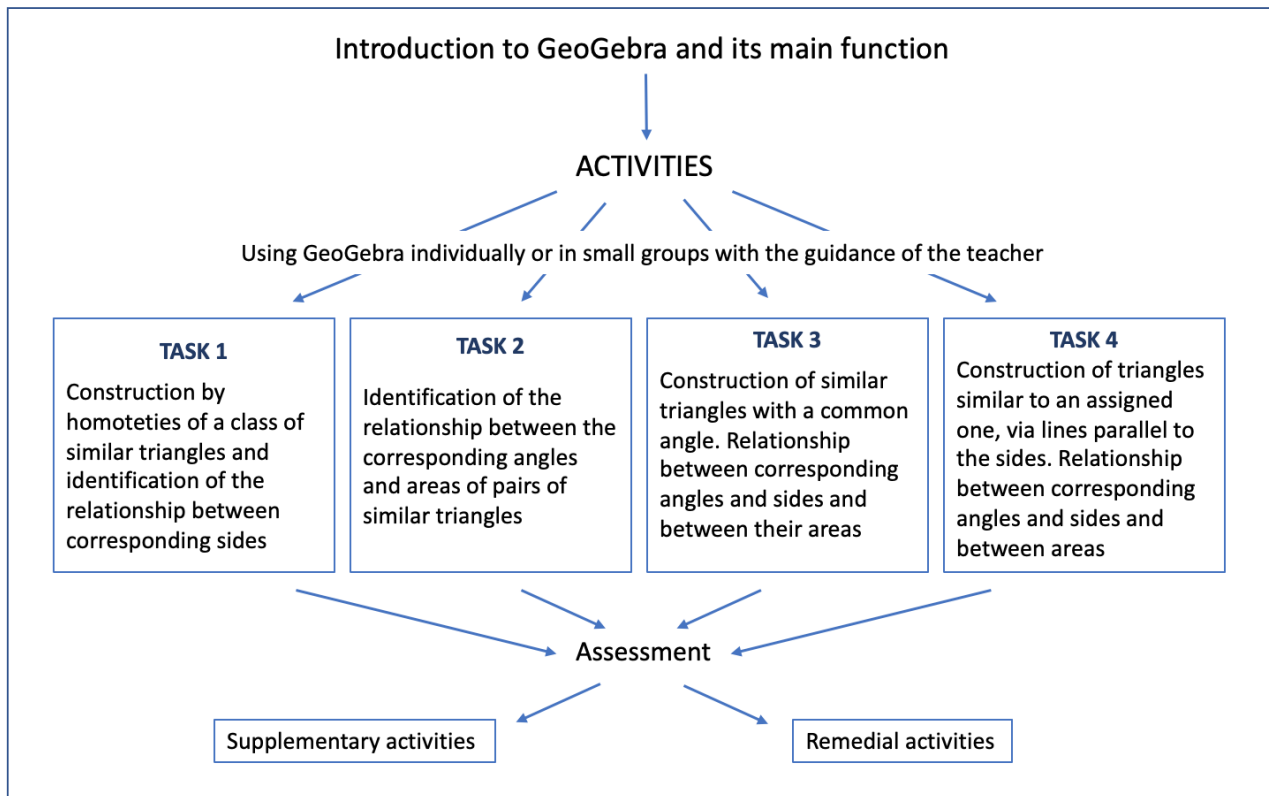
The teaching activity we present concerns proportionality in Geometry and is based on the use of dynamic geometry (GeoGebra) to introduce 8th-grade students to the concept of Similar Triangles and their properties (http://www.scuolavalore.indire.it/nuove_risorse/proporzionalita-in-geometria-con-geogebra/). Figure 1 shows the structure of the teaching proposal that is designed to be developed through five activities/tasks (including the brief introduction to some relevant GeoGebra functionalities) and to last around twelve hours (including the final assessment and a further remedial or supplementary activity). Methodologically it is suggested that pupils should work in pairs using one worksheet per pair in order to encourage initial peer discussion on the proposed activities. These activities are primarily heuristic and generally follow the following sequence: first, analyzing or reproducing a drawing, followed by structured reflection on the assigned task, then discussion with the teacher on proposed solutions, and finally, the teacher providing a definition for the introduced concepts. The teacher plays a crucial role in managing student reflections and comments, effectively facilitating discussion and ultimately arriving at a shared definition of the concept to be introduced.

The primary goal of the initial activity is to create a set of similar triangles that share a common angle and to establish a relationship between their corresponding sides. To begin, students will be asked to use the “dilation” tool and to make conjectures about how it works. Next, students will participate in an exploratory activity that encourages them to create pairs of triangles and formulate their own conjectures regarding patterns that may exist between their side measurements. Through a teacher-mediated collective discussion, students will work together to formulate an accurate definition of the property they had previously conjectured. The teacher will introduce the term “similar” to describe the pairs of triangles that were constructed. Finally, the activity will culminate

with the discovery of the properties of similar triangles, which will help to establish the concept of proportionality in geometry and provide a basis for exploring Thales's theorem.

Figure 1. Structure of the teaching proposal on Proportionality in Geometry, with Geogebra.
(Translated and adapted from PQM, 2015)

With GeoGebra, pupils can manipulate the constituent elements of constructed figures through



drag-and-drop activities, resulting in both direct and indirect movements. The production of a conjecture is founded on the interpretation of invariants and the relationships between them. This makes it an excellent example to demonstrate how the dynamic nature of the software can play a critical role in enabling pupils to formulate and strengthen their conjectures, thanks to the drag-and-drop function.

THE CASE OF PORTUGAL

In Portugal, the official curricular documents clearly value the use of DT. They consider DT as unavoidable and powerful resources to promote more meaningful learning and expand the contexts in which the student's action takes place and the diversity of perspectives on the mathematical objects studied. And, since 2021, there have been curricular changes that include Computational Thinking as part of the mathematics curriculum (implemented in 2022, although many teachers do not yet know how to address it). The curricular document (ME, 2021) values mathematical literacy, and defines the aims that all students must be able to achieve. The digital literacy of students ought to encompass a variety of skills, such as conducting calculations, creating graphs, running simulations, gathering and organizing data, engaging in mathematical experimentation, researching, modelling, and sharing ideas. The curricular document (ME, 2021) also includes more specific information about DT use for each mathematical topic. For instance, the emphasis on the use of technology is reinforced in the context of geometry teaching –“The study of geometric objects must be accompanied by experience (where technology plays a key role)” (pp. 10-11)– and there are

several references to technology use in relation to the teaching and learning of the concept of similarity.

At present, the Ministry of Education has a collection of tasks available on its website (<http://aem.dge.mec.pt/pt/recursos>) designed to aid teachers. However, the majority of these tasks are in a traditional paper-and-pencil format, and only a limited number suggest utilizing DT. Currently, publishing companies are creating educational materials for teachers, which comprise books, also centred on computational thinking, featuring exercises that utilize tools like GeoGebra, Excel and Scratch. The following example comes from an official textbook (Almeida, 2022).

A task on similar triangles with dynamic geometry (7th grade)

In Figure 2, we present a task for 7th-grade students that requires them to create sets of triangles that meet specific criteria. The students are then prompted to scrutinize the pairs of triangles to identify similarities and experiment with different combinations. Based on their findings, the students are expected to develop an understanding of the conditions that two triangles must meet to be considered similar.

1. Using a dynamic geometry program (such as GeoGebra), construct non-congruent pairs of triangles [ABC] and [DEF], such that:
 - 1.1. the sides of the triangle [ABC] measure three times the sides of the triangle [DEF]
 - 1.2. $\widehat{BAC} = \widehat{EDF}$, $\overline{ED} = \frac{\overline{BA}}{2}$ and $\overline{DF} = \frac{\overline{AC}}{2}$
 - 1.3. $\widehat{BAC} = \widehat{EDF}$, and $\widehat{CBA} = \widehat{FED}$
 - 1.4. $\widehat{BAC} = \widehat{EDF}$, $\overline{ED} = 4\overline{BA}$ and $\overline{EF} = 4\overline{BC}$
2. On each pair of triangles drawn, if necessary, measure sides and angle amplitudes. Determine the ratio between the lengths of the sides of each pair of triangles presented. Are the triangles similar? Experiment with different triangles under the same conditions.
3. Based on the results obtained in the previous question, what condition do you suggest should be verified when one wants to recognize whether two triangles are similar?

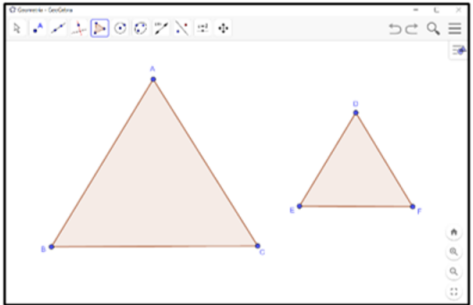


Figure 2. Task on similar triangles from a 7th-grade approved textbook (Translated and adapted from Almeida, 2022, p. 130)

This task challenges students to consider the relationships between the length of the sides and the angle measurements of triangles, allowing them to determine whether or not the triangles are similar. From there, the students are expected to identify the criteria that determine triangle similarity. The dynamic geometry software can be used to experiment with a variety of triangles, enabling the students to explore and formulate conjectures.

THE CASE OF MEXICO

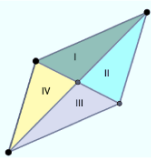
In Mexico, there have been few official government initiatives for integrating digital technologies to middle-school mathematics education. The most significant was the Teaching Mathematics with Technology programme (EMAT), launched in 1997 by the Mexican Ministry of Education (SEP) and which lasted for 11 years (see Sacristán et al., 2021). In the current (but soon to be changed) 2017 national mathematics guidelines (SEP, 2017) for middle-school, the use of digital resources are recommended “to encourage students to explore mathematical ideas and concepts, as well analyse and model problematic phenomena and situations”. Among the DT tools suggested are spreadsheets, symbolic manipulators, graphing tools, and Geogebra. However, the guidelines do make it clear that this can happen if schools have the necessary resources available. Hardly anything else is stated regarding technology in the printed guidelines, but in the corresponding website (<https://www.planprogramasdestudio.sep.gob.mx/sec-ae-pensamiento-mate1.html>), for middle-school there are a few examples of technology-based activities (called “ICT use”). In particular, for

the topic of “Patterns, geometric figures and equivalent expressions”, there are a few references to tasks from the EMAT programme (no other technology-based activities are presented). And for the topic of “Figures and geometric bodies” for all three middle-school grades, the website states that it “is suggested to work with dynamic geometry programs, such as GeoGebra, and with activities in LOGO”. All these references are very interesting and show the impact that the EMAT programme had; on the other hand, it is striking that no links are given to any of the referenced resources, other than printed page numbers from the EMAT activity books.

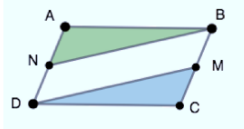
Congruences of triangles and quadrilaterals with dynamic geometry at middle-school

Triangles and quadrilateral: Figures directly or inversely congruent
 Aim: Distinguish when two figures are directly congruent or inversely congruent.
 How are the triangles formed by the diagonals that cross the rhombus shown?
 Some are directly congruent, while others are inversely congruent.

- If the point of intersection of the diagonals is the common vertex of the four triangles, what is the value of the angle, at this common vertex, in each of the four triangles?
- Therefore, to classify triangles as directly or inversely congruent, a respective rotation or reflection will suffice.
- Which triangles are directly congruent?
- Show the above using the ROTATE command and describe what happens.
- Which are the inversely congruent triangles?
- Show the above using the REFLECTION command and describe.



Triangles and quadrilateral: How to check the congruence of figures
 Aim: To use translation and rotation to verify congruence between two figures.
 The drawing shows the parallelogram ABCD and the midpoints of the opposite sides BC and AD.



- How are triangles ABN and CDM related to each other?
- Can there be a rotation that puts one triangle on top of the other?
- What point could be the centre and what is the value of the angle?
- To place one triangle on top of another, you could also perform a translation and then a rotation (or vice versa); in this situation it would be necessary to indicate with a vector the direction of the translation.
- Could you point out that vector in the drawing?
- Then, you would have to choose a point as the centre and the value of the angle for that rotation. What would it be?
- Try to do it with both methods and describe what happened.

Figure 3. Mexican middle-school dynamic geometry tasks on congruence of triangles and quadrilaterals (Translated and adapted from the EMAT book by Zubieta et al., 2000)

For grade 9 students, the Mexican 2017 guidelines state as learning aim that the student should “build similar polygons [and] determine and use similarity criteria of triangles” (SEP, 2017, p. 324), and suggests (as stated above, the use of GeoGebra or LOGO). However, we couldn’t find any recent activities on this topic. There are, however, two from the EMAT middle-school Dynamic Geometry activity book (Zubieta et al., 2000), that we present in Figure 3. As we can see, these are guided tasks to explore congruent figures with Dynamic Geometry –through reflection questions, and the rotation, reflect and translate commands.

THE CASE OF COLOMBIA

In Colombia, in the national guidelines (MEN, 2006), the use of digital technology is recommended but not mandatory. It is recommended to use technologies to favour the solution of problems and the communication of mathematical ideas in class. Concerning resources, in the official textbooks the use of digital technologies is also recommended. For example, in the textbook for grade 8 (MEN, 2015), GeoGebra is used to study different topics (e.g., function) but its use is scarce in the entire geometry unit in general. On the other hand, the Colombian national government promotes an official teaching repository (www.colombiaaprende.edu.co) created, with the support of South

Korea, for a project on ICT training. These resources include different kinds of resources for teaching with didactical suggestions and complementary activities for students: e.g., lessons planned by teachers and shared with colleagues; other resources like videos, tests and specific Apps. However, it does not include specific math-oriented software.

Interactive apps for the teaching of similarity (8th grade)

In the Colombian mathematics curriculum, the teaching and learning of similarity is proposed as a fundamental teaching content in spatial thinking from the first grades of primary school to the last grade of secondary school. In middle school, for grades 8-9, the guidelines (MEN, 2006) state as learning aims: “I conjecture and verify properties of congruence and similarity between two-dimensional figures and between three-dimensional objects in solving problems” and it is intended that students apply and justify the use of congruence and similarity criteria between triangles. Looking for teaching resources with DT for such topic, it was noteworthy that the national official repository (<https://www.colombiaaprende.edu.co/en/node/91283>) does not include anything using dynamic geometry (nor for any other topic).

Although the repository does provide 8th-grade interactive apps for the topic (see Figure 4), these could be considered as computer-aided instruction (CAI) tasks, that translate what could be solved with paper-and-pencil, rather than provide any technology-enhanced mathematical thinking.



Figure 4. A task on similarity of figures from the Colombian national digital resources' repository.

A CONCLUDING REFLECTION ON THE EXAMPLES

A reference to DT integration is present in the guidelines of all four considered countries. However, it is done in different ways. Firstly, there is a difference concerning the compulsory status of DT integration: in the case of Mexico and Colombia, DT use is recommended but is not mandatory, and depends on individual teachers and/or schools, although in the Colombian case, some repositories with DT apps are provided for teachers; in the case of Italy, it is highly recommended; and in the case of Portugal, it is mandatory. However, it seems that being compulsory, or not, does not necessarily affect the opportunities given to the teachers to be prepared and trained for the integration of DT in their practices. In all the four countries, the official guidelines tend to specify or suggest DT that can be used. Sometimes only some DTs are suggested, while in some other cases, longer lists are proposed. But in all our countries, dynamic geometry software, calculators and spreadsheets seem to be highlighted. A further important aspect that can make a difference is the extent of the methodological suggestions for DT integration in classroom activities.

Some repositories are also available in each of our countries. Even if they are not always developed by official institutions and their use is not mandatory, they can constitute important resources for teachers, especially when they are equipped with comments and suggestions for the implementation in teaching practices.

The examples in previous sections show that the use of DT to address the congruence and similarities of figures can be conceived in different ways depending also on how much DTs are

considered to be mathematics-enhancing resources. The Colombian task, in particular, doesn't use any affordances for exploring the congruence of geometrical figures; it is a task that could as easily have been presented on paper. On the other hand, the Italian and Colombian examples are resources for teachers that include didactical suggestions, although the Italian one highlights a complete methodology for technology integration. In contrast, Portugal and Mexico's examples are tasks for students only.

Interestingly, three of the examples propose explorations with DG, but with very different approaches. In the case of Mexico, it is a guided task with questions directed towards finding or confirming congruence. In the Portugal example, the approach is more open and exploratory, and the questions are not only for identifying the conditions needed for triangles to be similar, but on creating an awareness of why they are true. Finally, the case of Italy also promotes constructions and from the constructions the identification of geometric properties; but the example stands out because it includes heuristics and explicitly promotes a working methodology (including collaborative work and discussion).

With this non-research paper, we intended to provide some insights into the challenges of digital technology integration in mathematics education in various countries. Further ongoing work seeks to develop a cross-national analysis focussing on the impact and limitations that various factors can have on digital technology integration in mathematics education.

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STUDENTS' MEANING-MAKING PROCESSES IN THE DIGITAL ERA: HOW CAN THE TEACHER FOSTER MATHEMATICAL DISCOURSE?

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The aim of this work is to study how the teacher can prompt students' meaning-making processes in a technology-rich context. With this purpose, we present a teaching activity on the notion of rotation as an isometry of the Euclidean plane involving a dynamic geometry environment. The experimentation of the teaching activity in a 7th-grade class has been analysed and discussed taking into account the notion of teacher instrumental orchestration and elements of Sfard's commognitive framework to show how the teacher prompted the evolution of the students' mathematical discourse. Although the paper presents and discusses only an example of mathematical discourse during a collective discussion orchestrated by the teacher, it aims to contribute to the reflection on the value of educational practices in an era of intense digital transformations.

Keywords: Mathematical discourse, Instrumental Orchestration, Commognition, Dynamic Geometry

INTRODUCTION

Technologies have recently assumed an increasingly important role in everyday life, as well as in the school context, and their use has influenced the way teaching activities are carried out in the classroom. Hence, growing relevance has been assumed by the teachers' need to rethink their practices in order to effectively and consciously make sense of the integration of technologies into teaching activities. As technologies can offer teachers opportunities to create appropriate learning environments in which students are involved in the construction of mathematical meanings, their integration has become one of the main research topics in mathematics education (Trgalová et al., 2018). This issue is approached by taking into account several perspectives: the design and development of resources; the development of the mathematics curriculum with an appropriate design of activities and/or sequence of activities; and the benefits that can be gained by exploiting the potential offered by technologically rich environments in a way that fosters students' learning. In particular, following the discursive approaches to research in mathematics education (Sfard et al., 2001), the use of technology in the teaching-learning processes can be investigated for its role in fostering the production of mathematical discourses rich in conjectures and in the need to verify them.

In this paper, we are interested in reflecting on how the evolution of students' mathematical discourse in the meaning-making processes can be supported through appropriate activities involving technologies and conscious teacher behaviour. To this end, we present and discuss the results of a teaching activity on rotation involving a dynamic geometry environment with the aim to study the students' mathematical discourse and the way the teacher can prompt its evolution. The notion of teacher instrumental orchestration developed in the field of research in mathematics education and particularly in the case of the use of technologies in the classroom (Trouche, 2004; Drijvers et al., 2010) will be our reference to analyse the role of the teacher. The analysis of the data gathered (video recordings, transcripts and students' protocols) has been developed taking into account Sfard's (2001, 2008) commognitive framework concerning the role of interpersonal communication in mathematical thinking and learning.

Through an example of the analysis of the evolution of the mathematical discourses, results will show how the teacher's orchestration of the activity led the students to collectively construct the meaning of rotation by exploiting the potential of dynamic geometry.

THEORETICAL FRAMEWORK

The Instrumental Orchestration

Starting from the metaphor of the orchestra as a harmonic composition of different instruments, Trouche (2004) offered a theoretical lens to describe how the teacher can coordinate different coherent sets of artefacts within the classroom, with the aim of guiding the students' instrumental genesis (Artigue, 2002) thus improving the teaching-learning process. Drijvers and colleagues (2010) then paid their attention to the teacher's intentional and systematic organisation and use of the various artefacts available in a learning environment in a given mathematical task situation, distinguishing three main elements: a didactical configuration, an exploitation mode and a didactical performance. Referring to the metaphor of musical orchestrations: the didactical configuration can be compared to the choice of musical instruments with which to compose the group and the choices about their arrangement in space so that the different sounds produce polyphonic music; the exploitation mode can be compared to the determination of the partitioning of the music for each of the musical instruments involved, taking into account the expected harmonies that will emerge; the didactical performance can be compared to the musical performance, in which the actual interaction between the conductor and musicians reveals the feasibility of the intentions and the success of their realisation (ibidem, p. 215). In the next sections, to describe how the teaching activity analysed in our study was orchestrated by the teacher, we will present the didactical configuration, the exploitation mode and the didactical performance.

The Commognitive approach

Extending the concept of thinking from the individual sphere to the interpersonal sphere, Sfard (2008) defines thinking as the individualised form of communication activity. Thus, thinking stops being separate from any communicative act and becomes, together with cognitive processes, the same representation of a specific phenomenon. Therefore, Sfard (2008) combines the terms cognition and communication, producing the new term *commognition* and illustrating how this approach can be applied to mathematical thinking: unlike other scientific or colloquial discourses, mathematical discourse is characterised by objects that are discursive constructs and part of the mathematical discourse itself. Accordingly, mathematics is seen as an autopoietic system, comprising both the discourse and its objects, which is therefore capable of growing incessantly from within as new objects are added (Sfard 2008, p. 129). Therefore, mathematical discourses are described through the *words* (i.e., the key terms used to describe their characteristics) and the *visual mediators* (i.e. the artefacts on which the communication process operates). Indeed, various objects are involved in mathematical discourse, such as signifiers (i.e. the word or symbol that functions as a noun in the discourse participants' utterances), realizations, primary objects and discursive objects. Realizations can take the form of written or spoken words, algebraic symbols, drawings, or even gestures. While a signifier can be considered as a "root" of a "tree" of realizations, which Sfard defines as a "*hierarchically organised set of all the realizations of the given signifier, together with the realizations of these realizations, as well as the realizations of the latter realizations and so on*" (Sfard, 2008, p. 300). Realization trees and, thus, mathematical objects are personal constructs and provide important information on the evolution of a given person's mathematical discourse. Hence, taking into account the form and results of their processes, mathematical discourses are characterised by the *endorsed narratives* they produce and the mathematical *routines* (Sfard, 2008, p. 245) which follow the process of creating and validating the narratives. From this perspective, mathematical routines can be regarded as explorations in that they end with narratives

that contribute to the creation of a mathematical theory. Moreover, the *ritual* is also an important component of learning mathematical discourse and is realised as a willingness to be part of a certain community and to speak the same language. In other words, ritual represents the unconscious production of actions that can only simulate a certain acquisition of knowledge and skills by students. Therefore, the teacher's purpose is to transform ritual into exploration in such a way that students become aware of their own mathematical activities and discourses, thus evolving them towards constructing mathematical meanings.

Research question

With the aim to contribute to the reflection on the value of educational practices in an era of intense digital transformations, in this work, we analyse the results of a teaching activity on rotation involving a dynamic geometry environment attempting to answer the following research question: how can the evolution of students' mathematical discourse in the meaning-making processes be supported through the teacher's orchestration?

METHODS

This study is based on the analysis of the video recordings of a teaching activity (consisting of four phases), developed and experimented with by an expert teacher in her 7th-grade class, aiming at the construction of the notion of rotation as an isometry of the Euclidean plane. To investigate how the evolution of students' mathematical discourse in the meaning-making processes can be supported through the teacher's orchestration, we first read the development of the teaching activity in terms of didactical configuration, exploitation mode and didactical performance. Moreover, according to the learning aim of the designed activity, and with reference to Sfard's framework of Commognition, in order to answer our research question, we created an example of a rotation signifier realization tree. Then, the transcripts of the videos were analyzed, using our realization tree, attempting to study the evolution of the students' mathematical discourse on the rotation signifier. In this paper, we discuss an example of such analysis. In what follows we start presenting an overview of the four phases of the teaching activity focusing on the main elements of the teacher's orchestration.

Instrumental orchestration of the teaching activity

The *didactical configuration*, characterised by the involved artefacts and the teaching setting, is defined by: the synergic use of a manipulative artefact and a dynamic geometry environment (GeoGebra); the students' individual use of a digital device (iPad); and finally the use of an Apple TV that allowed the sharing of any of the participants' screens. The teacher appreciated the idea of synergically combining the use of different kinds of artefacts (Faggiano et al., 2018), and more precisely to introduce the topic by means of a manipulative tool. She then made her own choices to adapt the sequence (that was designed by pre-service teachers and researchers (Faggiano & Mennuni, 2020)) to the context of her class. In particular, she made some changes in the choices of the artefact to be used in the second phase (that was previously designed to be accomplished using paper, pencil, ruler and compass), asking students to realise the requested construction of a rotated figure using their iPads and the tools in GeoGebra. This was due to the students' familiarity with GeoGebra and her usual mode to exploit the affordances of the Apple TV to share students' work during the discussions.

The *exploitation mode* is described by the design of the teaching activity with its four phases. In each of the four phases of the activity, students are asked to accomplish a task using the given artefacts. Then a collective discussion is orchestrated by the teacher to focus students' attention on the specific aims of the phase. During the first phase, students are involved in characterising rotation as a rigid movement dependent on an angle and a fixed point called the centre, through the use of a

manipulative tool. In this phase, students are prompted to identify an initial idea of the conservation of distances between the centre and each pair of corresponding points on the two flags. In the second phase, GeoGebra is used with the aim of pushing students to build rotated figures by exploiting the dynamic geometry potentials and the observations done during the previous phase. In the third phase, students are given a GeoGebra file containing a flag, a point P and a slider α representing the angle. They are asked to use GeoGebra's Rotation tool to construct the rotated flag, with respect to P and the angle α (see Figure 1a). Then, students are asked to describe what happens, and to explain the reasons, if they change: (a) the angle by moving the slider, (b) the point P, (c) the position of the initial flag. Finally, in the fourth phase, students are given in GeoGebra two rotated flags and they are asked to find the hidden centre of rotation. The aim of this phase is to characterise the centre as the unique point obtained by intersecting any two perpendicular bisectors of every segment joining each pair of corresponding points. This requires the use of the property, as it arose in the previous phase, regarding the preservation of the distances between the centre and every pair of corresponding points.

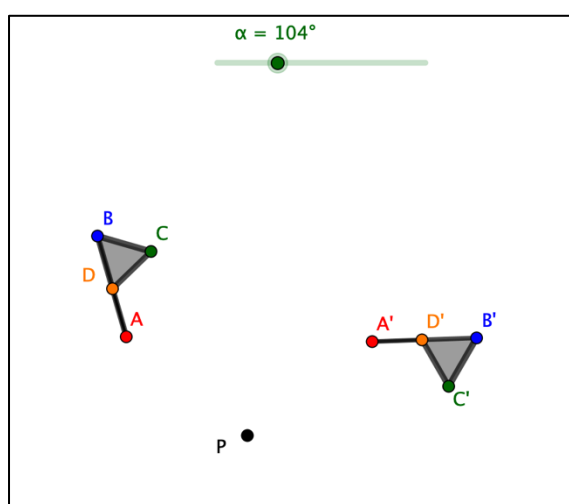


Figure 1a. GeoGebra file for the task in phase 3

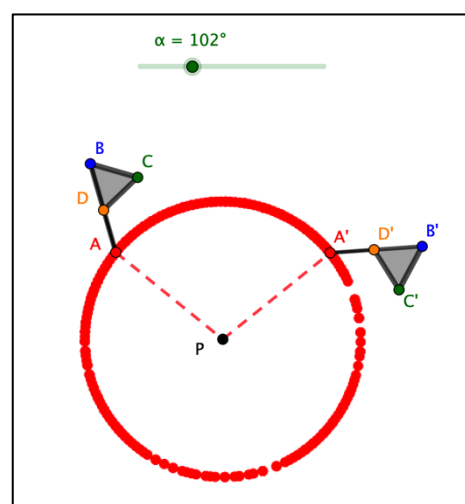


Figure 1b. Using the "trace" on spot

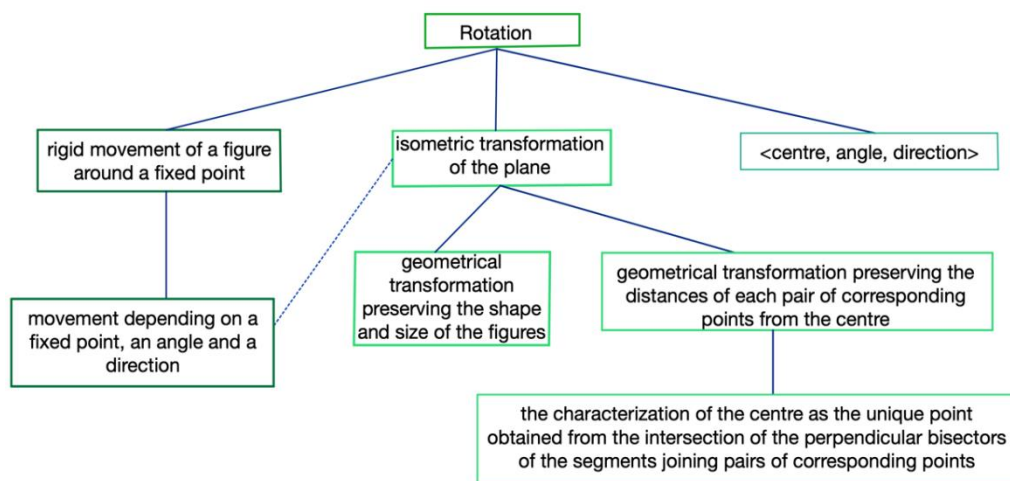
The *didactical performance*, which is characterized by the interplay among the teacher, the students and the artefacts, reveals the students' construction of the meanings. The entire teaching activity required four hours and involved a 7th-grade class. At the beginning of the teaching activity, the teacher explained to the students that they would have to use a manipulative tool and their iPads to accomplish four different tasks (related to the four phases described above), to be discussed collectively at the end of each phase. The GeoGebra files and worksheets of the tasks have been shared with the students through the AirDrop function on their devices. During each phase, students were given 20 minutes to accomplish the different tasks, and the teacher used the remaining 40 minutes for the collective discussion. We do not have enough space to report the exact details of the tasks given to the students, but the teacher's requests were in tune with the exploitation mode. Herein, we illustrate an example of the ad hoc decisions taken by the teacher while she was teaching, focussing on the discussion that followed at the end of the third phase of the activity. To help the students construct the properties of the rotation, the teacher asked them to share their GeoGebra file on the Apple TV in the classroom and explain their observations. To guide the students in constructing the realization of the rotation signifier related to the property of preservation of the distances of the corresponding points from the centre P, they were pushed to interact directly with the tools on GeoGebra. In particular, the teacher on the spot asked the students to activate the trace on a couple of points A and A' and describe what they observed when changing the values of the angle of rotation α with the slider. To achieve her aim, the teacher asked the students to draw the segments AP and A'P and pushed them to observe that the trajectories described by the traces of

the points were circumferences with the centre in P and the distances between P and the points A and A' had to be equal since they were the radius of the circumferences (see Figure 1b).

In this paper, to bring to the fore the role of the teacher's instrumental orchestration, we analyse the meaning-making process during the collective discussion conducted by the teacher at the end of the third phase of the teaching activity.

RESULTS AND DISCUSSION

As a reference to investigate the evolution of the students' mathematical discourse, considering the learning aims of the teaching activity, we have built the realization tree in Figure 2. In the leftmost branch, we started with the idea of rotation as a rigid movement that is the focus of the first two phases of the activity with the manipulative tool. This first realization can be expanded by pushing the students to observe that with this movement, the flags do not change their shape and size and that the movement only depends on a point, an angle and a direction of rotation. The most important part of this realization tree consists of the idea of rotation as isometry. In tune with the aims of the third phase of the activity (on GeoGebra), this realization can be expanded considering the property that characterises the centre of rotation, i.e. the preservation of the distances of each pair of corresponding points from the centre. This last realization can also evolve, exploiting the potential of GeoGebra's tools as it is in the design of the fourth activity, in a further realization of the characterization of the centre as the unique point obtained from the intersection of the perpendicular bisectors of the segments joining pairs of corresponding points. Finally, for the sake of completeness, we have also included an essential realization of the rotation signifier that only considers rotation as a plane transformation that can be described by the triad (centre, angle,



direction of rotation).

Figure 2. Our realization tree of “rotation”

In this paper, we focus on the realization of the rotation signifier by considering the construction of the narratives concerning the preservation of the distances between the centre and the pairs of corresponding points. To show the evolution of the students' mathematical discourse on the rotation signifier as it was fostered by the teacher's orchestration of the collective discussion, we analyse two excerpts taken from the third phase of the activity. These excerpts concern the discussion on what happened to the rotated flag when changing the angle of rotation through the slider.

- 1 Teacher: I would like to make you think about how A and A' vary, how B and B' move between them. I don't know if you saw those red dots which were there. *[With her fingers she traces in the air the trajectory of points]*

- 2 Teacher: I am interested in whether they move at will or they are in some way constrained. Maurizio? *[She moves her hands randomly]*
- 3 Maurizio: They move with respect to the distance from point P
- 4 Teacher: They definitely move according to distance, and how are these distances? *[She sharply points out “definitely”. To talk about the distance she fixes a segment with her fingers]*
- 5 Maurizio: The two distances of the real flag and the clone flag from point P are the same
[He moves his right forearm downwards. And at the same time, the teacher marks the distances PA and PA' on the screen]

This first excerpt refers to the moment in which the teacher decided to push the students to think about and reflect on the movement of the pairs of corresponding points. In her intervention reported in [1], the teacher intentionally led students' attention to the GeoGebra trace of the points of the rotated flag. Her intervention came with a small gesture made with her fingers to describe the movement that the red dots drew on GeoGebra. At this point, the student Maurizio answered (in [3]) the teacher by creating his own narrative to be validated by the teacher: he started with the idea of the distance from the centre of rotation, which is a new realization of the rotation signifier. The teacher felt that Maurizio's mathematical discourse could be guided to evolve towards the identification of the trajectory of the movement as a circumference. Accordingly, the teacher approved his narrative (in [4]) also with a specific gesture: she fixed a segment with her fingers and moved it back and forth twice with respect to a fixed point. When the teacher asked him what these two distances looked like, she decided to share the GeoGebra file with her iPad on the Apple TV – in which the measures appeared along the distances considered. She wanted to understand if Maurizio's answer was due to visual observation alone - and in this case, according to Sfard's theoretical framework from his answer we find a ritual - or if it was a new narrative originating from some exploratory mathematical routine that therefore also needed to be approved. The answer in [5] given by Maurizio resolved the teacher's doubt. Maurizio's intervention, “*The two distances of the true flag and the clone flag from point P are the same*”, shows how he has abandoned the passive use of the word rotation introduced by the teacher from the beginning. Indeed, this intervention makes it clear that this student constructed his own realization of the signifier rotation: the rotated flag is realized with the expression “*clone flag*”. So from this point, it can be seen how Maurizio's mathematical discourse on the rotation signifier was evolving as he was constructing his personal realization of the rotation. Consequently, it is possible to find a link between the intervention of the same student reported in [3] and that in [5] insofar as through the word “*clone*”, Maurizio emphasized the idea of the dependence of the rotated flag on the initial flag, but also on the center of rotation. Maurizio's evolution of his mathematical discourse depended on the teacher's orchestration of the collective discussion. In fact, the teacher's attention was attracted by the word “*clone*” and the linked expression “*the two distances between...*” so she decided to interact with the artefact marking the distances on the screen and pointing her index fingers at the two extremes of the segments (in [5]).

Thanks to the orchestration and the teacher's interventions, the other students in the class also seemed to have enriched the rotation signifier with the new realization linked to the preservation of the distances of the corresponding pairs of points from the centre.

- 6 Teacher: Let's see Ilaria
- 7 Ilaria: I have joined A and its correspondent [A'] and they trace a trajectory, a circumference centred on P *[She joins her palms together]*

8 Teacher: They draw a circle centred on P. Is this always true?... do you agree with Ilaria?

9 Ilaria: I have done the same thing for point D.

What is shown in this second excerpt is the narrative that Ilaria exhibited to the teacher after a brief confrontation with her classmate Maria (who traced with the pen in the air a circumference). In fact, from the analysis of the video recordings it was possible to observe that the evolution of Ilaria's mathematical discourse took place from a narrative between the two students, characterised mainly by glances and gestures. This confrontation between the two students arose from the stimuli given by the teacher during the discussion. Invited (in [6]) by the teacher (who has noticed the interaction with Maria), Ilaria presented her narrative reported in [7], accompanied by the gesture of joining the palms, to have it validated by the classmates and the teacher. As Ilaria took a single pair of corresponding points and built the circumference centred in P, with her intervention reported in [8], the teacher repeated the construction made by Ilaria (with Maria) and tried to stimulate all the other students, and in particular Ilaria, to go over. Indeed, with the intervention in [9], it is clear that Ilaria was able to generalise the property for the other pairs of corresponding points. This excerpt can also be interpreted by exploiting the aspects of mathematical discourse introduced by Sfard. Indeed, in this case, we are dealing with a mathematical routine that can be characterised as follows. There is first a *ritual* in which Maria and Ilaria must confront each other before exposing their narratives to the teacher. This example can be seen as a ritual in that this behavior of the two students was found several times during the activity and especially during the collective discussions, demonstrating a certain iterated character in itself. In addition, this behavior highlights the two students' need to create a mathematical discourse in order to discover the object under investigation, which must pass through validation of their own peer narrative. Then, it becomes a mathematical *routine* of an exploratory nature in that the narratives of the two students are subsequently endorsed by the teacher, contributing decisively to the discovery of the new realization of the signifier.

Guided by the teacher, the students needed to describe the movement of the flag's points as the position of the initial flag moves with a circumference with center P to which both the points of the initial flag and those of the rotated flag belong. This additional realization of the rotation signifier is fundamental in the evolution of the students' mathematical discourse. It will allow them to enrich the realization of the rotation constructed up to that point. This enrichment is essentially given by associating the distance between the center of rotation and the corresponding pairs of points on the flags with the radius of the circumference that has just been identified.

CONCLUSIONS

With the attempt to investigate how, in a technology-rich context, the teacher can foster the evolution of students' mathematical discourse in the meaning-making processes, this study is framed by the notion of the Instrumental Orchestration and the Commognitive approach. Through the analysis of our teaching experiment, we have shown that the teacher's orchestration of the activity and the related collective discussion led the students to collectively construct the realization of the rotation signifier related to the conservation of the distances of the points of the two flags from the center of rotation and consequently pushed the students towards an evolution of their mathematical discourse. Although the paper presents and discusses only an example of mathematical discourse during a collective discussion orchestrated by the teacher, it aims to offer a contribution to reflect on the value of educational practices in an era of intense digital transformations. Indeed, results are representative of how the teacher's orchestration can guide the students' development and evolution of mathematical discourse, thus fostering the students' meaning-making process.

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SINE, YOU THINK, IT CAN DANCE? AN AESTHETICALLY DRIVEN MATHEMATICAL ACTIVITY FOR MEANING MAKING ON TRIGONOMETRIC FUNCTIONS

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This paper discusses the role of mathematical aesthetic experiences in the shaping of two 11th grade students' meaning making on trigonometric functions, while they engage in coding dancing figural models as music visuals for a specific song. MaLT2, the main digital resource used, is a programmable 3D Turtle Geometry software affording dynamic manipulation of variable values. GeoGebra was also used as a tool for plotting parametric functions. Students freely explored and manipulated functions of sine and cosine for animating their digital artifacts, starting from their simplest forms and building up to Fourier series approximations. They connected these functions to figures' length or angle temporal change, based on their periodic and symmetric dynamic features. Findings, though limited in scale, provide a glance into how an aesthetically driven mathematical activity in such digital environments supports students' meaning making and mathematical inquiry.

Keywords: Educational technology, programming, dance, trigonometry, aesthetics.

INTRODUCTION

Researchers have highlighted the importance of (re)considering the aesthetic aspects of school mathematics, based on the affordances of digital technologies (Papert, 1978; Sinclair, 2018; Bu & Hohenwarter, 2015). They argued that their expressive and sensory power has allowed students to experience mathematical aesthetics in a novel and accessible way, by integrating the factors of subjectivity and personal sensibility within its traditional autonomous, elitist perception (De Freitas & Sinclair, 2014). Nonetheless, aesthetic considerations related to mathematics learning remain marginalised and understudied in this research community (Sinclair, 2018). Following a humanistic perspective on mathematical aesthetics, as conceptualised by Sinclair (2004, 2018), we designed a learning activity that privileges aesthetics by ingraining mathematics in personal artistic creation. In this study, we traced and analysed students' aesthetic learning experiences (Papert, 1978; 1980) from their engagement with expressive digital media, where trigonometric functions were used and explored for the evaluation and creation of animated shapes, matched and tuned to a specific song.

MATHEMATICAL AESTHETIC LEARNING EXPERIENCES

Traditional approaches connect the mathematical aesthetic with objective values of mathematical beauty, mainly based on its rationality and purity of logical deduction within a platonic view. In this perspective, mathematical aesthetic experiences, that result from the ability to appreciate these aesthetic values, are accessible only to a small minority of people. This epistemological discussion has mainly focused on the fundamental role of aesthetics in mathematicians' practices during their creative processes of developing new mathematical content. Three main aspects emerged from this discussion; the role of aesthetics in 1) guiding the mathematician to discovery, 2) motivating the selection of certain problems and 3) helping a mathematician evaluate a certain result (Sinclair, 2004). A growing number of researchers perceived the aesthetic as a mode of cognition and highlighted the importance of cultivating it in the mathematics classroom (Papert, 1978; Sinclair, 2018; Jasien & Horn, 2022). However, the elitist traditional perspective eliminates opportunities for most students from experiencing it. With traditional learning practices dealing with mathematics as

a body of isolated knowledge detached from human senses, governed by deductive logic, where truths are already true, aesthetic experiences are completely neglected in schools (Sinclair, 2018; Jasien & Horn, 2022). Papert (1978) was strongly opposed to this objective view of mathematical aesthetics, that deprives students of experiencing it, by pointing out, both theoretically and practically, ways of designing aesthetically rich learning environments that allowed drawing, exploring and making of mathematics. He linked the notion of aesthetics in mathematics to aesthetics in the arts, through adding the subjective component of the ‘sensible’ (Sinclair, 2004). The embodied sensory engagement in mathematical inquiry, i.e. what is perceived with one’s eyes, ears, skin, emotions and senses in general, introduces students to extra-logical facets in mathematical thinking. This type of engagement, that we define here as mathematical aesthetic learning experiences, connects mathematical meaning making to aesthetic practices and affective processes. According to Sinclair (2004), such experiences shape mathematical sense making in three ways, as adapted from considerations on the role of aesthetics in mathematicians’ inquiry; through 1) a generative, 2) a motivational and 3) an evaluative role. The generative role involves the guiding process of gaining insight connected to both problem posing and problem solving. It is physically driven by feelings of wonder and curiosity that give rise to ideas on the formation of a particular problem or on the way to proceed with its solution. The motivational role refers to the development of personal interests that attract learners to engage in mathematics in particular ways. Having the freedom to select mathematical concepts, problems and strategies based on inner motivational mechanisms can lead students to develop a personal taste and agenda on mathematical inquiry. It is connected to emotions of interest and desire. Finally, the evaluative role concerns the learners’ engagement in the process of deciding whether a specific result of mathematical inquiry is good or beautiful enough, following a socially shared or a personal set of criteria. It is connected to emotions of surprise, amusement, anger, confusion and disappointment.

DESIGN RESEARCH

The task and the digital resources

Two digital resources were used for the design of the task for different, complementary purposes; MaLT2 (<http://etl.ppp.uoa.gr/malt2/>) and GeoGebra (<https://www.geogebra.org/calculator>). On the one hand, MaLT2 was the main expressive medium for the creation of the animated artefacts. This software integrates a UCB-inspired Logo procedural language with Turtle Geometry in 3D and dynamic manipulation of parameter values through sliders (Grizioti & Kynigos, 2021; Kynigos & Karavakou, 2022). Users can construct figural models through programming for the movement of an avatar (turtle = hummingbird in 3D) that leaves a coloured trace behind. They can also animate them by defining a parametric procedure (e.g. ‘*TO shape :t*’) whose parameter (:t) is included as input in a logo command (e.g. ‘*left :t*’ or ‘*right 2*:t*’ or ‘*forward 30*sin(:t)*’) or in a sub-procedure (e.g. *dancer1 :t*). By dragging a slider, the values of the corresponding parameter change and the figural transformations of the avatar’s trace are shown in the 3D scene (Fig. 1). By constantly pressing the keyboard’s right arrow for moving its slider, a parameter can conventionally represent time, embedding the concept of motion in time. Based on its expressive, graphical and dynamic affordances, we considered MaLT2 as an appropriate medium for the creation of periodically animated figural models, matched both chronically and qualitatively to a specific musical piece. On the other hand, GeoGebra had a supportive role throughout the task. Two GeoGebra files were designed, used as graphing calculators for plotting i) trigonometric functions of the form $a_1\sin(b_1t)+c_1$ and $a_2\cos(b_2t)+c_2$ and ii) approximations of Fourier series of the form $a_1\sin(t)+b_1\cos(t)+a_2\sin(2t)+b_2\cos(2t)+a_3\sin(3t)+b_3\cos(3t)+a_4\sin(4t)+b_4\cos(4t)$, where parameter values a_i , b_i could be manipulated through sliders.

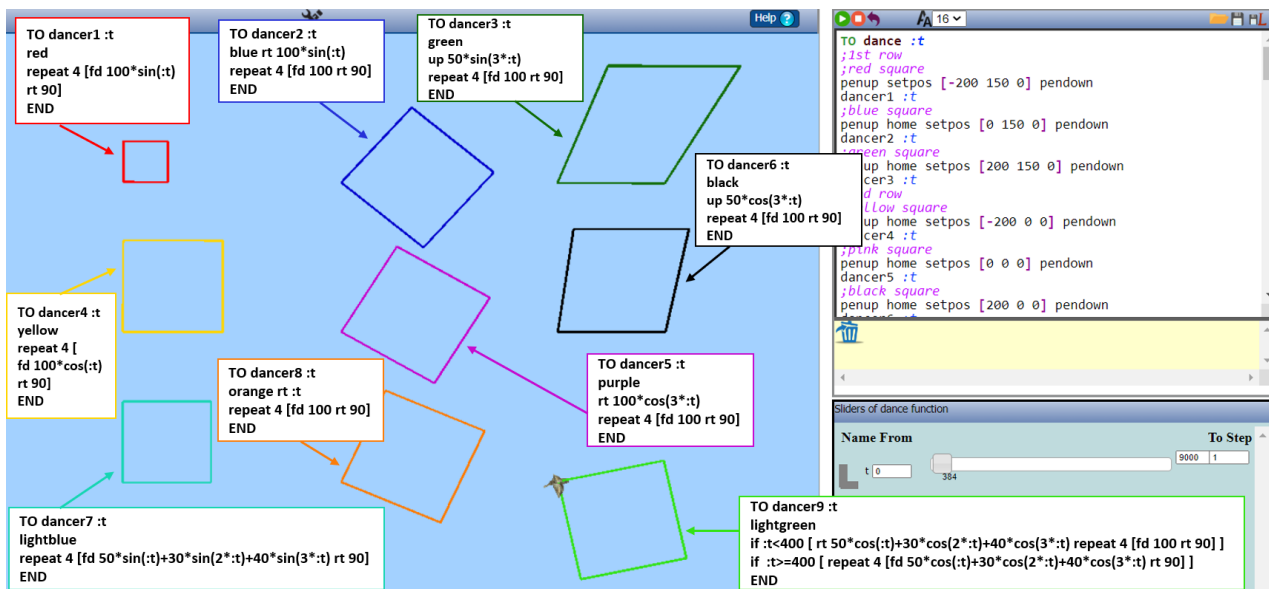


Figure 1: Screenshot from the constructed artefacts in MaLT2 for Phase 1

For the 1st phase, that had an introductory-preparatory status, we designed a ‘dance competition’ simulation in MaLT2. It consisted of nine squares moving simultaneously in a different periodic way through arrow-key-dragging of the slider for the parameter t of the ‘dance’ procedure (Fig. 1). As the values of t were steadily increasing, each square was moving periodically either by rotating or by fluctuating its sides’ length, with their period and way of moving varying depending on the trigonometric functions included in either a turn or a moving-forward command. For example, the red square (Fig.1, dancer1) was moving in a simple fluctuating way based on the command ‘*forward 100*sin(:t)*’ with a period of 360 (in t -values); the purple one (Fig.1, dancer5) was making a rotating motion based on the command ‘*right 100*cos(3*:t)*’ with the period of 120 and the light-blue one (Fig.1, dancer7) was fluctuating in a more complicated periodic way, based on a Fourier approximation of ‘*forward 50*sin(:t)+30*sin(2*:t)+40*sin(3*:t)*’, with a period of 360. A specific song with stable repetitive rhythm was chosen to accompany the ‘dancing’. Finally, a task sheet was designed, where all nine procedure codes were given nameless, in random order. The task included students working together on rating the nine different procedure codes with grades from 1 to 10, based on the corresponding square’s motion they believed was created by each one of them, as ‘dancing move’ matching (or not) to the song. For the 2nd and main phase, students were free to create dancing figural animations in MaLT2 for another given song with various rhythmic units.

Data collection and analysis

This study is part of a design research project, following its final adaptation phase before its main pilot classroom implementation. Two students of 11th grade, Petros and Maria, who had some previous experience with MaLT2 and GeoGebra, participated in a six-hour, divided into two days, out-of-school setting. The collected data included screen and voice recordings from one shared laptop, written task sheets and gestures noted down by the attending researcher during students’ activity. Data was analysed using coding while tracing for the generative, the motivational and the evaluative role of aesthetic experiences in mathematical meaning making, as defined in the previous section. At the next section, some concrete examples of situations where such aesthetic roles are evident in students’ activity and discourse are described.

RESULTS

Phase 1: Evaluating the aesthetic code

During the first phase, students worked together in order to mutually agree on a grade for the evaluation of the code of each square's 'dancing move'. Even though this task had an introductory purpose, there were some interesting cases where aesthetic experiences played an evaluative and a motivational role to their mathematical inquiry. For example, while arguing on the evaluation of the first two rows of squares (dancer1-dancer6), students were led to comparison, argumentation, conjecture formation and testing in GeoGebra. During their inquiry, meanings on the period and input-output values of trigonometric functions came up:

- Petros: I like sine! You get it? It goes right and the to the left because it gets negative values! (...) But it's unfair to put a higher grade here (dancer1). The green (dancer3) is better matching its tempo.
- Maria: Yes, but the red one (dancer1) is making a nicer move. Even though it is too slow. I really like that when the music tone gets higher, the square is to the right and highest point and when the tone gets lower, the square is to the left.
- Petros: Ok, but still. The green is so perfectly synchronized to the song. It's like circling together. We need to give a higher grade to the green one. Maybe 5 or 6 and 7?
- Maria: The black square is not as synchronized as the green one. It seems though like the black one is a bit slower. (...) These two squares move in 3D and correspond to the commands $up\ 50 \cdot \sin(3t)$ and $up\ 50 \cdot \cos(3t)$. I think they have the same period.

Students' aesthetics played an evaluative role for the judgement of each square's movement. They chose to use the issue of synchronicity as a main criterion for grading the codes. They started a process of comparing the periods of each animation in order to determine which codes are better matching to the song's rhythm. The motivational role of aesthetics led them to a series of negotiation and mathematical investigation serving their aim to be fair. In order to test his conjecture about the period of the first two rows of squares (dancer1-6), Petros turned to the first GeoGebra file and set the parameters in order to plot and compare the functions $10\sin(t)$ and $5\sin(3t)$, then $5\sin(3t)$ and $5\cos(3t)$ (Fig. 2.1.) and then $5\sin(5t)$ and $10\cos(5t)$.

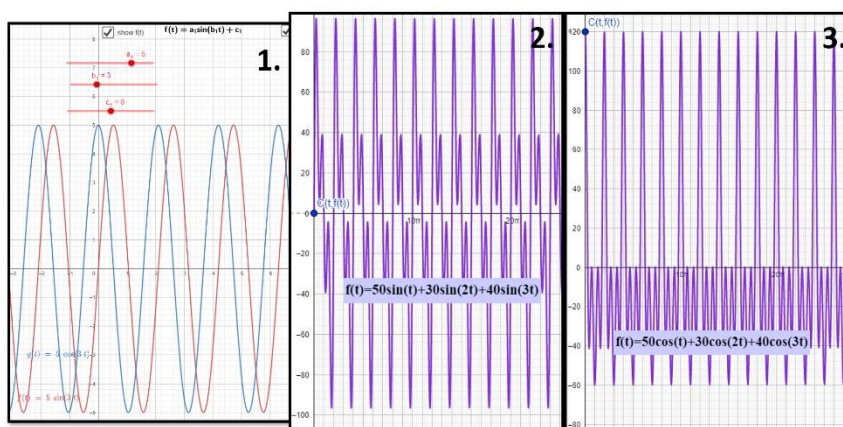


Figure 2: Screenshots from Petros's investigation in GeoGebra

- Petros: Ok, it makes sense. When this $b=1$, the red and the blue squares are much slower, because, you see? The higher the number is, the thicker the graph gets. More ups and downs means quicker motion. The green and the black are 3 times faster. A whole period of the red corresponds to 3 periods of the black.

Maria: Between the green and the black, I think the green is better. Because it starts from being 2D, at position zero, and then leans forward and backwards, while the black starts by leaning. The starting point is important; it also matches the song better. And I think this applies to all codes including cosine, since it starts at the highest point, not at zero. So not at the neutral position as it happens with sine.

The evaluative role of her aesthetics led Maria to pay attention on squares' starting position. That motivated her to find a general evaluative rule for squares' starting position, that she based mathematically on the phase difference between sine and cosine and on their output values when their input value is zero. She made these last remarks while observing both the graph in GeoGebra and the figures in MaLT2 while setting the slider value to $t=0$. The evaluative and motivational aspects of aesthetic mathematical learning experiences were also evident during their evaluation of the third row of squares (dancer7, dancer9). The complexity of the squares constructed by dancer7 and dancer9 codes, caused by the approximation of Fourier trigonometric series as input of a logo command, led them to feelings of curiosity and wonder and further aesthetically driven discussion:

Maria: The light-blue (dancer7) is perfect. It should get a clear 10.

Petros: Are you sure? It is for sure a nicer move than the previous ones, but it's not perfectly synchronized with the song. It is not exactly sine and cosine, but it seems like having the same period as these ones (showing dancer1 and dancer4).

Maria: It is this one! Forward $50\sin(t)+30\sin(2t)+40\sin(3t)$. Because the light-green one (dancer9) changes after $t=400$. So, its code is the one with 'if'. (...) It is not as symmetrical as the light-blue. I'd give 10 to the light-blue and 8 to the green. It seems to me completely out of rhythm, even though its move is nice.

Petros: Even though one is sum of sines and the other one sum of cosines with the same numbers, the first is symmetrical to the x-axis making a smoother move.

In order to support his decision, Petros got motivated to plot and compare the graphs of each trigonometric sum given in the task sheet in the second GeoGebra file (Fig. 2.2; 2.3.) and agreed on the rating of dancer7 to be 10 as the most symmetrical and smoothing one, with aesthetics having a motivational and evaluative role to his mathematical inquiry.

Phase 2: Making the aesthetic code

During the 2nd main phase, students carefully listened to the song they would make the dancing animation for. Their initial discussion included how each one pictured it, supported by reasons mainly based on emotions that the music brought them. During this brainstorming phase, the aesthetics had a generative and motivational role for setting up and pursuing goals related to features of trigonometric functions. For example, for the beginning of the song (first 24 seconds), one idea given by Maria defined their final creation:

Maria: What if we make a square fluctuating like the red one each time this part of the song is repeated? I think of a square going up and down and then at the second repetition another one, of different colour, appearing at another position, moving exactly the same. And then two others, following the same pattern. It will slowly raise the tension, at the same way the music does.

Petros: Yes! And when the beat drops, more shapes can appear all together! (...) The music is like constantly escalating, so more stuff can be added to render that.

Maria wrote the code of the *square* procedure and used it as sub-procedure in the *dance* procedure (Fig. 3.1.). She used the function $\sin(t)$ as an input to the *square* procedure, so that she would imitate

the red square's (dancer1) dancing move from Phase 1. However, the result disappointed them, with their aesthetics having an evaluative impact on reconsidering the function in use.

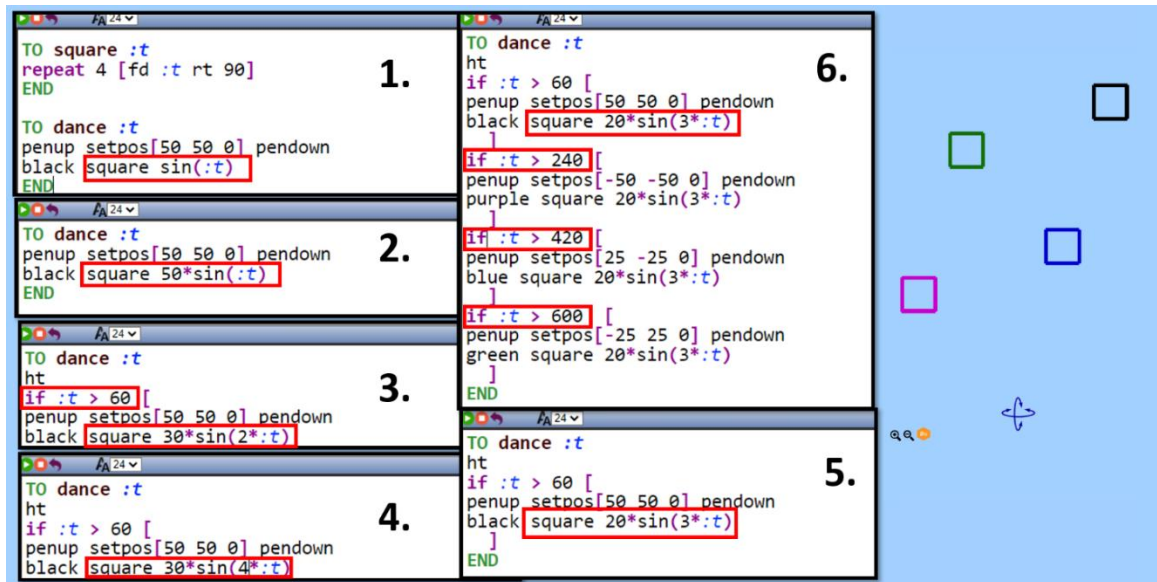


Figure 3: Screenshots from students' programming in MaLT2 in Phase 2 (Part 1)

- Petros: Why doesn't it move? We put $\sin(t)$, it was supposed to go like this, up and down.
- Maria: Well, the red square (from Phase 1) was $100\sin(t)$. We need this number in the front. But let's make it 50. The squares must be smaller to match the music, because at first it sounds very calm. (She changed the input function to $50\sin(t)$ as shown in Figure 3.2.)
- Petros: No, but it is completely out of rhythm. It's too slow. We need to make it quicker.
- Maria: We can change the number inside! To effect the density. But I want the square to be even smaller. Let's make it $30\sin(2t)$? (She changed the input function to $30\sin(2t)$, $30\sin(4t)$ and finally to $30\sin(3t)$ as shown in Figure 3.3.; 3.4; 3.5.)
- Petros: Finally! No, the rhythm matches perfectly! Let's add the next square. Set its position at -50 -50 0, so to be symmetrical. (...) The second starts at 240 and the third at 420. It is a clear period, so at every 180 steps something is happening.
- Maria: This means that the fourth square appears at $420+180=600$? Replay the song.
- Petros: Perfect! That way we can add this tension that the music creates! (...) After $t=780$ they can also be a bit quicker, or make a complex move, during this intense part.

As shown by the above parts of their discussion, the students went through a series of mathematical inquiry, guided by cycles of aesthetic experiences. It started with Maria shaping the idea of adding a small moving square every time a musical pattern is repeated, in order to slowly raise the tension, where her aesthetics played a generative role. Then it continued with both students motivational and evaluative aesthetic mechanisms used to fix the square's motion in order to be synchronized and match the song rhythm and tone, which led them to a circle of conjecture making and testing for the period and the dynamic representation of the sine function in MaLT2. Finally, the generative role of Petros's aesthetic experiences, boosted by the motivational and evaluative aspect of both students' ones, started a new circle of mathematical inquiry for reflecting the growing tension of the song, where they investigated, the use of sum of trigonometric functions in order to create more intense moves. Another example, where aesthetic experiences caused by the song played a key role

for their mathematical meaning making process, was for the last part of the song, as partly shown below:

- Petros: It gives me the feeling of stress. It has a growing tension, even more than before.
- Maria: What if we'd make a hexagon growing and shrinking and then after only a few seconds another one doing the same move? As if it the second is chasing the first.
- Petros: Yes, great idea! But how? (...) We can try $t+10$ inside each function, instead of t .

Students' aesthetic experiences originated from listening to the song had a generative role on the formation of the idea of 'chasing' as representing the feeling of stress. The motivational and evaluative aspects of their experiences led them to a lot of experimentation in MaLT2 and Geogebra, which ended up in the final part of the code (Fig. 4). Petros's following remark indicates the aesthetic influence over their choice of Fourier series approximation as input function:

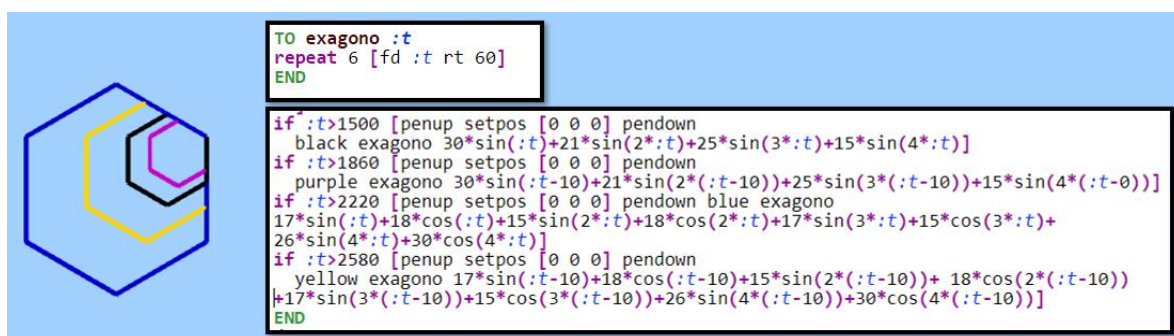


Figure 4: Screenshots from students' programming in MaLT2 in Phase 2 (Part 2)

- Petros: At first we use a sum only of sines, starting in a smoother dancing move. Then as the song gets more chaotic, we tried a not very symmetrical one. So the 3rd and the 4th hexagons move with a more weird sum, including both cosine and sine.

Petros tested different combinations of sum parameters graphically in GeoGebra and then dynamically in MaLT2. He concluded that sum of sines of the form $a_1\sin(t)+a_2\sin(2t)+a_3\sin(3t)+a_4\sin(4t)$ leads to a symmetrical move and intentionally avoid it by adding cosine factors of the form $b_i\cos(it)$ for creating the chaotic effect. This type of mathematical inquiry of setting out goals, posing problems and exploring their solutions using mathematical properties was guided by his motivational and evaluative aesthetic experiences. Throughout this process, he developed meanings on elements of Fourier trigonometric series approximation, such as symmetry, periodicity and its practical utilisation, as being able to represent different types of periodic variation.

CONCLUSIONS

The two students that participated in this study engaged in rich mathematical inquiry through cycles of problem posing, argumentation, conjecture formation and testing within both digital resources, appreciating the results and reconsidering the solution. These cycles were boosted by their aesthetic experiences, evident when they were listening to the song or observing the figural dancing animations in MaLT2. Throughout the six hour activity, we traced many cases in which their aesthetics played either a generative, a motivational or a evaluative role on their mathematical engagement. From these cases, that were viewed as mathematical learning experiences meanings, they developed mathematical meanings on trigonometric functions and Fourier trigonometric series approximation regarding periodicity, symmetry, input and output values, and intervals of monotonicity. They made sense on these concepts and relations and used them either to make a fair evaluation (Phase 1) or to create an aesthetically satisfying dancing animation (Phase 2). Both ways,

mathematical meanings were generated naturally and gradually evolved through their expression and manipulation in the digital resources. Even though this paper involves a case study, it provides an example where students' aesthetics guided their mathematical learning experience, that was both aesthetically and mathematically rich in a proportional way. Our next research steps involve the adjustment of this study to the classroom level in order to gain a clearer general image on the role of aesthetics and expressive digital resources on mathematics learning.

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SYNCHRONOUS DISTANCE VS ON-CAMPUS LEARNING – EFFECTS ON INTEREST AND SELF-EFFICACY

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Synchronous distance learning is still a very new and little-researched learning environment. Using the example of a preparatory course, this paper investigates the effectiveness of synchronous distance learning tutorials and compares them with on-campus tutorials. We focus on the affective characteristics of interest in mathematics and mathematical self-efficacy, both of which are considered predictive of study success and retention. A study with 159 prospective teachers shows that synchronous distance learning and on-campus tutorials are comparable with respect to the development of interest and self-efficacy in general. However, interest in mathematics is less affected in distance courses than in on-campus courses. This paper discusses the implications for offering synchronous distance learning on affective characteristics in higher education.

Keywords: *Preparatory course, Distance learning, Interest in mathematics, Mathematical self-efficacy*

INTRODUCTION

The transition from school to university is particularly difficult for many students in disciplines with a high mathematics component (Hoyles et al., 2001). As a result, many students in STEM education drop out or change their major. Many universities therefore offer voluntary mathematics preparatory courses that are designed to facilitate the transition process by strengthening the content-related and affective prerequisites of prospective students. Accordingly, preparatory courses often have several goals: On the one hand, they refresh and improve the mathematical basics learned in school. On the other hand, affective characteristics such as interest in mathematics or mathematical self-efficacy are to be strengthened (Hochmuth et al., 2018).

Unlike traditional courses, individual universities have already been offering preparatory courses in different variants with e-learning elements for several years (Derr et al., 2018; Greefrath et al., 2017). These ranged from pure distance courses for self-study to pure on-campus courses with online learning materials. Other universities have only recently gained experience with distance learning in the context of pandemic emergency remote teaching (Büchle et al., 2021). The COVID-19-induced shift to distance learning represents a completely new approach to teaching, as distance learning in this context is often offered synchronously via videoconferencing and is strongly oriented toward on-campus instruction in its design. We therefore refer to this type of learning as *synchronous* distance learning to capture the main differences to more traditional learning environments. However, the extent to which synchronous distance learning courses are suitable as complementary or alternative options for entering students has hardly been investigated to date. This contribution therefore examines the effectiveness of synchronous distance learning courses and compares them with on-campus courses (Kirsten & Greefrath, 2023). Doing this, we focus on the development of students' affective characteristics, namely their interest and self-efficacy, as these characteristics are considered to be predictive of study success and retention (Geisler et al., 2023).

DISTANCE LEARNING PREPARATORY COURSES

Since preparatory courses are voluntary and take place a few weeks before the start of studies, for many students' participation depends on compatibility with other commitments. For this reason, some universities have been using distance and blended learning alongside on-campus courses for

many years (Derr et al., 2018). While distance learning in this context is often implemented as an asynchronous online course with materials available on a learning platform, blended learning courses also include single face-to-face events on campus. Due to the high degree of self-study, such courses allow prospective students to work at their own pace and intensity without being tied to the university location. In particular, this allows students who live far away, study on a dual or part-time basis, or cannot attend on campus for other reasons to participate (Fischer, 2014).

Because independent learning requires a high degree of self-regulatory skills, the effectiveness of distance and blended learning courses is often questioned from an educational perspective. Therefore, studies comparing distance or blended learning courses with on-campus courses have been conducted at a few universities. These studies focus largely on mathematics performance and measure the effectiveness of a course variant in terms of student learning gains. For example, Greefrath et al. (2017) reported that students who chose a distance learning preparatory course performed significantly better in the subsequent mathematics exam than students who attended the on-campus course. Similar findings were reported by Fischer (2014), who compared blended learning and on-campus preparatory courses. Again, students in blended learning courses performed better at the end of the preparatory course than their peers in on-campus courses, even when controlling for pre-test scores. However, it should be noted that students with stronger prerequisites, such as a better overall school performance or an advanced mathematics course at school, were more likely to choose a blended learning course in this study (Fischer, 2014). To our knowledge, the influence of course modality on the development of *affective* characteristics has hardly been studied so far. Initial indications are provided by Fischer (2014), who found no differences between blended learning and on-campus preparatory courses in terms of students' self-efficacy or attitudes toward mathematics.

To what extent the results described here are transferable to synchronous distance learning courses remains questionable. Since these courses, unlike those described above, typically involve fixed schedules and video-based face-to-face interactions, it is reasonable to assume that they place different demands on students in terms of self-directed learning and social interaction. This may affect their effectiveness. Although Büchele et al. (2021) report that overall performance in 2020 pandemic preparation courses was better than in previous years, other studies suggest that pandemic distance learning may have a negative impact, particularly on students' affective characteristics (Händel et al., 2020; Kempen & Liebendörfer, 2021; Reinhold et al., 2021).

INTEREST AND SELF-EFFICACY

Both, interest in mathematics and mathematical self-efficacy are considered relevant predictors for academic learning and thus for study success. In person-object theory, *interest* is understood as a special relationship of a person to an object (Krapp, 2005). A positive relationship promotes engagement with the object and can implicitly aim at expanding one's knowledge and skills in this field. Therefore, interest in mathematics is considered to predict learning success since interested learners pay more attention to the object of study (Hidi & Renninger, 2006). Conversely, in terms of retention, dropouts often report a declining interest in mathematics (Geisler et al., 2023). Accordingly, the importance of interest in academic decisions can be emphasised (Köller et al., 2001).

Because mathematics in school and university is often very different, students' interest in mathematics often changes at the beginning of their studies. First-year university students experience a mismatch between what is expected and what is experienced, leading to a decline in interest (Kosiol et al., 2019; Rach & Heinze, 2017). The development described here can already be observed during preparatory courses when university content is taught in addition to school content (Hochmuth et al., 2018). The trend described here could be exacerbated by distance

learning, as it has been shown in the context of the COVID-19 pandemic that individuals with a higher interest in mathematics have a higher need for face-to-face instruction (Reinhold et al., 2021).

The term self-efficacy refers to the confidence in one's own abilities to perform certain actions successfully (Bandura, 1997). Thus, from a discipline-specific perspective, *mathematical* self-efficacy describes a person's confidence in his or her abilities to successfully perform *mathematical* actions (Pajares & Miller, 1994). This can include solving specific tasks, constructing proofs or, more generally, mastering a mathematics major. Empirical findings show that students' self-efficacy initially declines during the transition from school to university (Schunk & Meece, 2006). This may be explained by a variety of challenges that accompany the start of studies: The rapid progression of lectures, a high amount of self-study, and failures in weekly exercises often result in a low experience of competence. However, a decline in self-efficacy appears problematic as self-efficacy can have a direct or indirect influence on performance (Schunk & Pajares, 2002). Furthermore, confidence in one's abilities also influences the decision for or against retention: Dropouts, especially if they leave their studies early, have a lower mathematical self-concept than other students, which is a more general but related construct of student self-assessment (Geisler et al., 2023).

Empirical findings show that students' mathematical self-efficacy can be increased both in blended learning courses and in on-campus courses (Fischer, 2014; Hochmuth et al., 2018). To what extent this finding is transferable to synchronous distance learning remains an open question. However, a study by Kempen and Liebendörfer (2021) on the first pandemic semester suggests that distance learning may have a negative impact on self-efficacy, at least for certain student groups.

RESEARCH QUESTIONS

Synchronous distance learning is still a very new and under-researched learning environment. Using the example of a preparatory course, this contribution therefore examines the effectiveness of synchronous distance learning courses and compares them with on-campus courses. More precisely, the study is based on a preparatory course held at the University of Münster in September 2021, including lectures for the entire group and parallel tutorials for active participation in smaller groups. In terms of experimental variation, this course was offered in two variants: In both variants, lectures were video-recorded and provided asynchronously on the university's learning platform. The daily tutorials, on the other hand, were offered synchronously, either on campus or via videoconferencing. Since the development of affective characteristics can be particularly influenced by a change in modality, we focus on these aspects. In particular, we examine the development of students' interest in mathematics (Krapp, 2005) and their mathematical self-efficacy (Bandura, 1997). Both characteristics are related to academic success and retention (Geisler et al., 2023) but are possibly not addressed to any great extent in distance learning (Kempen & Liebendörfer, 2021). More specifically, research is guided by the following question:

How do interest in mathematics and mathematical self-efficacy develop during a synchronous distance learning preparatory course and how does this development differ in an on-campus course?

METHODS

Design and sample

The preparatory course examined is designed for entering student teachers at the primary and lower secondary level. Because both degree programs contain a comparable amount of mathematics there is a joint course for both programs. The aim of the preparatory course is both to deepen mathematical knowledge from lower secondary school and introduce students to university mathematics. Therefore, the course teaches basic math skills such as fractions, transformations, and linear equations, as well as strategies for mathematical reasoning and problem-solving. The course

included eight lectures and seven tutorial sessions. For each tutorial session, there was a worksheet with tasks that address the content of the lecture and encourage practice and application.

To compare synchronous distance learning with on-campus learning, both tutorial options were designed to be as similar as possible in terms of time slots, tutors, group size, and instruction. The tutors were guided by predefined course plans in which the teaching methods and expected learning outcomes were described over time. To keep teaching methods in both tutorial options comparable, elements of on-campus teaching, such as group work and writing on the blackboard, were transferred to the digital learning environment using breakout sessions, integrated whiteboards, and split screens. For a detailed description of the course design and associated challenges in the two tutorial options, see Kirsten and Greefrath (accepted).

The sample consists of $N = 159$ first-year students, of whom $n = 133$ were pursuing a teaching degree in primary education (127 female, 5 male, 1 not specified) and $n = 26$ a teaching degree in secondary education (16 female, 10 male). Students could independently sign up for a course option, resulting in $n = 71$ students in the on-campus course and $n = 88$ in the distance learning course. In both groups, data collection took place at the beginning and the end of the course, more precisely during the first tutorial session (T1) and the last tutorial session (T2). The sample considered here only includes those students from whom data is available at both measuring points.

Instruments and procedure

Since interest in mathematics and self-efficacy have already received much attention in preparatory course research, we made use of existing instruments (see Table 1).

Construct	# Items	Sample item	α	Source
Interest in mathematics	5	I simply enjoy puzzling over a mathematical problem.	.77/.76	Köller et al. (2000)
Mathematical self-efficacy	4	I am confident that I can perform well in homework and exams in mathematics.	.82/.76	Hochmuth et al. (2018)

Table 1. Overview of the instruments

Following the original scales, a four-point response format was used for interest and self-efficacy, ranging from "strongly disagree" (1) to "strongly agree" (4) (Hochmuth et al., 2018). Due to possible discrepancies between school and university mathematics, we did not include institutional references in both scales. However, it can be assumed that students related the questions to school mathematics, at least at T1. The reliabilities of both scales were satisfactory for group comparisons. In addition to affective characteristics, we collected data on students' age and gender, as well as their grade point average (1.0 to 4.0) and their last mathematics grade at school (0 to 15 points).

RESULTS

Although students independently assigned themselves to a tutorial option, an even distribution was achieved in terms of degree programmes (80.3% and 86.4% primary school teaching), gender (88.7% and 92.0% female), age ($M_{OC} = 19.53, SD = 2.16$ and $M_{DL} = 19.39, SD = 2.01$), grade point average ($M_{OC} = 1.93, SD = 0.49$ and $M_{DL} = 1.94, SD = 0.45$), and last mathematics grade at school ($M_{OC} = 10.63, SD = 2.56$ und $M_{DL} = 10.68, SD = 2.21$).

Total		On-campus (OC)		Distance learning (DL)	
T1	T2	T1	T2	T1	T2

Interest	2.48 (0.59)	2.37 (0.53)	2.60 (0.58)	2.41 (0.52)	2.39 (0.59)	2.36 (0.53)
Self-efficacy	2.56 (0.52)	2.61 (0.43)	2.70 (0.49)	2.72 (0.39)	2.44 (0.52)	2.52 (0.45)

Table 2. Mean values (and standard deviations) of the affective characteristics separated according to tutorial variants and measuring points (T1/T2)

Table 2 provides an overview of the mean values achieved at the individual measuring points. Descriptive data suggest that interest decreased over time, while overall self-efficacy increased slightly across measurement time points. To examine the development of affect in more detail, mixed ANOVAs were conducted with interest and self-efficacy as dependent variables. The factor *time* entered with two measuring points as the within-between factor, while *tutorial variation* entered as a two-level subject-between factor. The interaction between these factors is represented by the factor *time x tutorial*. In checking the preconditions, Levene's tests showed that the variances were homogeneous for both measures ($p > 0.05$). A normal distribution, as well as sphericity, could be assumed due to the sample size and the two-stage design. The F-statistics are presented in Table 3.

	Interest				Self-efficacy			
	F	df	p	η^2	F	df	p	η^2
Time	16.54	1, 156	.001	.096	2.61	1, 156	.108	-
Tutorial variation	2.30	1, 156	.123	-	11.44	1, 156	.001	.068
Time x tutorial	8.45	1, 156	.004	.052	0.97	1, 156	.326	-

Table 3. Results of the mixed ANOVA

For *interest*, the ANOVA showed that there was a statistically significant main effect of time, indicating that the students' interest decreased significantly from the pre-test to the post-test. Additionally, there was a statistically significant but small interaction effect between time and tutorial variation. This effect suggests that interest in mathematics declines slightly stronger among students in on-campus courses than among their peers in distance learning courses. For *self-efficacy*, there was neither a statistically significant main effect of time nor an interaction effect between time and tutorial. Only the subject-between factor of tutorial variation proved to be statistically significant, meaning that students in distance learning tutorials had significantly lower confidence in their own mathematical abilities than their peers in on-campus tutorials.

DISCUSSION

Although preparatory courses, unlike other university courses, have been offered as distance or blended learning courses for several years (Derr et al., 2018; Fischer, 2014), *synchronous* distance learning is still a fairly new learning environment that is only becoming established due to the COVID-19 pandemic. Therefore, this contribution aimed to compare synchronous distance tutorials with traditional on-campus tutorials concerning the development of affective characteristics. Overall, we observed little or no difference between course options, indicating that on-campus and distance learning tutorials can address interest in mathematics and mathematical self-efficacy similarly.

About *interest in mathematics*, a decline in interest during the preparatory course was observed in this study for the entire group. This finding is consistent with previous studies (Hochmuth et al., 2018) and can be explained by the fact that elements of university mathematics were already covered in the preparatory course (Kosiol et al., 2019; Rach & Heinze, 2017). However, the intensity of the development differs in the two tutorial variants: While in on-campus tutorials interest decreases more strongly, synchronous distance learning seems to affect interest less. If the declining interest is attributed to discrepancies between school and university mathematics, we can assume that cognitive dissonance is perceived more intensely in on-campus classes. While students in distance learning can consciously or unconsciously retreat, take breaks, or do other work in parallel, such avoidance strategies are hardly possible in on-campus classes. In this case, the decline in interest for students in distance learning tutorials might merely be delayed. This should be considered when students of different tutorial options take the same freshman course. If synchronous distance learning courses are to be offered as an equivalent alternative to preparatory courses, more attention should be given to discontinuities that occur, for example, by explicitly discussing the differences between school and university mathematics. However, when interpreting the results, it should also be noted that students in on-campus courses showed a greater interest in mathematics at the beginning of their studies. This could reinforce the experience of divergence so that not only the tutorial variant but also the initial interest is relevant.

With regard to *mathematical self-efficacy*, a slight, non-significant increase can be observed in both tutorial variants in line with Fischer (2014) and Hochmuth et al. (2018). There was no difference in the development of students' self-efficacy in the two tutorial variants. However, the results suggested group differences, as students who chose a distance learning course generally had lower self-efficacy. This indicates selection or choice effects, consistent with previous findings that students with a higher self-concept show a greater preference for on-campus learning (Reinhold et al., 2021). With this in mind, distance learning courses should also target students with low self-efficacy and enable or reward even small experiences of success.

Overall, the results indicate that variation in tutorials is not crucial for the development of selected predictors of retention. Thus, under certain conditions, synchronous distance learning tutorials may comparably address affective characteristics to traditional on-campus tutorials. It should be noted, however, that students in this study were not randomly assigned to tutorial variants. On the contrary, slight selection and choice effects could be observed, which may affect participation in the preparatory course. Moreover, only those students who participated in both measuring points were included in the sample. Including students who dropped out of the preparatory course early could provide deeper insights into the development of affective characteristics.

CONCLUSION

The study presented in this paper provides initial evidence that distance learning preparatory courses, i.e., with an asynchronous lecture and a synchronous tutorial via videoconferencing, can offer an equivalent alternative to traditional on-campus courses with regard to the development of affective characteristics. With this in mind, it is likely that synchronous distance learning courses could be offered in addition to asynchronous distance and blended learning courses in the future. This may promote social interaction in addition to location flexibility. However, the results also suggest that students with weaker prerequisites – in particular, lower interest and lower self-efficacy – are more likely to choose a distance learning course. This should be reflected in the design of courses, for example, by aligning distance and on-campus courses more closely to the respective target group and its characteristics.

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PROGRAMMING AND COMPUTATIONAL THINKING IN MATHEMATICAL SUBJECT AREAS

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In England, computing is a subject with no formal relation to mathematics. However, several national projects have explored such a possible relationship and shared concrete materials, which has contributed to a general awareness of the potential for programming and computational thinking (PCT) in mathematics. In this paper, we study whether and how English teachers include programming and computational thinking (PCT) as part of their mathematics teaching in a context where there are no legislative curricular requirements to do so. The study finds that English teachers mostly teach PCT as analogue computational thinking and that they are more likely to do so in the mathematical subject areas of functions, statistics, and algebra. We argue that the high frequency of statistics and the low frequency of geometry are interesting as they conflict with what prev research has found. To address this, we discuss the potential impact of the prevalence of analogue teaching of PCT on the subject matters it is integrated into.

Keywords: Subject Areas, Programming, Computational thinking

INTRODUCTION

Over the last 50 years, the idea that programming and computational thinking (PCT) hold the potential for students and practitioners to engage with mathematics in new ways has been expressed in various studies (e.g., Benton et al., 2018; Papert, 1982). More recently, PCT has also gained momentum in curricular revisions, where it has become part of the mathematics curriculum in countries such as France and Sweden (Bocconi et al., 2022). There are notable differences between countries that include PCT in their curricular revisions, both in terms of what PCT content is emphasized and with what mathematical content it is coupled. As found in the study by Tamborg (2022), these decisions are not only the product of carefully thought-out considerations but also somewhat influenced by contingency in the policy-making process. While previous overview studies have documented the beneficial effects of PCT in specific mathematical subject areas, such as geometry, proportion, ratio, and arithmetic (Clements & Sarama, 1997), we still lack insights into the connections mathematics teachers make between PCT and mathematics, and how they experience them. It is highly likely that curricular decisions on what PCT areas to combine with what mathematical content will influence the connections pursued by teachers. In that respect, England provides an interesting case, since computing has been introduced as a distinct subject with its own curriculum (that closely relates to PCT). This implies that English mathematics teachers are not required to teach PCT as part of their mathematics teaching. In this paper, we study English teachers' current practices of teaching PCT as part of their mathematics teaching. Empirically, we rely on a survey sent to teachers in Denmark, Sweden and England, but in this paper, we exclusively report on data from respondents in England. Studying their approaches to teaching PCT and mathematics allows us insights into the extent to which teachers in England find PCT relevant to teach and, if so, in what ways they teach it. This leads us to ask the following research question: *What mathematical subject areas have been outlined as prominent to couple with PCT in research*

on mathematics education, and how does this correspond to how mathematics teachers in England include PCT in their teaching?

To answer this question, we begin by summarizing existing literature on which mathematical topics have historically been found to benefit from PCT. We then describe how different goals in different contexts and teacher-specific teaching choices seem to have an impact on how PCT and mathematics are taught. We then describe what underlying construct of PCT we relied on for the survey data that informs this study. From this, we present our results regarding the use of PCT in mathematical subject areas by teachers in England. In the conclusion, we discuss the findings and reflect on the limitations of this study.

RELATED WORK

The idea of a close connection between PCT and mathematics is far from new in mathematics education research. In the 1970s and 1980s, Papert (1982) linked computational thinking and mathematics education, arguing that computers and programming can provide useful ways for students to interact with and learn mathematics. Concepts that were perceived as abstract and difficult to understand could, by drawing on computational thinking, be taught and understood in new, concrete, and systematic ways (Papert, 1982). Later, Wing (2006) emphasized the close relationship between computer science and general education, suggesting that computational thinking should be thought of as a general and basic skill similar to reading and writing.

Research on the Potentials of PCT

While both Wing and Papert's work led to an interest in the combination of PCT and mathematics, the connection has been drawn to different mathematical topics. In particular, research found PCT to be beneficial in the teaching and learning of geometry, numbers, algebra, and arithmetic (Benton et al., 2018; Clements & Sarama, 1997; Noss, 1986; Papert, 1982). Prior to the work by Wing (2006), functions have also been thought to form a tight connection between programming and mathematics, as formulated in APOS theory (Arnon et al., 2014; Ayers et al., 1988; Dubinsky, 1984). APOS is an acronym for Action, Process, Object, and Schema. Actions are concrete, physical, or mental engagements with mathematical concepts and problems. Processes refer to mental operations or algorithms that students use to solve problems and understand concepts. Objects refer to mathematical concepts and entities that students encounter and learn about, such as equations, functions, or geometric shapes. Schemes refer to the organization and structure that students give to the concepts and processes they have learned. For example, understanding how to represent linear equations as functions in a coordinate system would be a schema. Using APOS theory, Dubinsky and colleagues (Ayers et al., 1988) explored how students develop their conceptions based on specific actions in a programming language and the ability to encapsulate mathematical processes with a programming language.

Research on the Role of Teachers in Exploiting Potentials of PCT

As the implementation of PCT into mathematics has become more common, interest in how it is implemented in the classroom has increased. The importance of the role of the teacher as a facilitator to bridge the connection between PCT and mathematics is evident from the ScratchMaths project, which consisted of a 2-year intervention for students aged 9 - 11 in England with a distinct focus on algorithms. In this project, Benton et al. (2018) found differences in the students' learning from the intervention related to differences in whether teachers discussed algorithms as a general phenomenon or as different, comparable "strategies" throughout the intervention.

Misfeldt and colleagues have investigated Swedish mathematics teachers' conceptions of the relation between mathematics and programming through a survey of 133 Swedish teachers following an in-service training initiative (Misfeldt et al., 2019). Misfeldt and colleagues found that

the teachers identified a strong – but not very strong – link from programming to mathematics, that they were interested in working with programming, but that the teachers did not feel ready to do so.

These studies illustrate that the way teachers choose to implement PCT in their mathematics teaching is important for what their students get out of it, and that teachers do not necessarily feel ready to teach programming, even if they do see a link between that and mathematics. Thus, it shows the need for understanding not only how connections between programming and mathematics are identified theoretically, but also how these two are connected (in the classrooms) by teachers. In this paper we take up the mantle of such an approach by conducting a descriptive analysis of English mathematics teachers' answers to a survey about how they apply PCT in their teaching and in relation to different mathematical topics. Again, teachers in England provide an interesting case, due to the national implementation of PCT in a subject separate from mathematics, which in principle should leave teachers to only implement it in mathematics when they find good reasons to do so.

THEORETICAL FRAMEWORK INFORMING THE DATA COLLECTION

As argued above, our main interest in this paper was the teachers choices in whether and how they integrated PCT in mathematics. Our study therefore sought to allow the respondents themselves to report in which subject areas of mathematics they teach PCT. This design of our survey is informed by two theoretical sources: a definition of PCT, on which we have based the survey, and a conceptualization of mathematical topics.

First, we defined computational thinking for the respondents of the survey to ensure a common understanding of what it entails. For this, we relied on the definition by Kallia et al. (2021) of a structured problem-solving approach, namely: “A structured problem-solving approach in which one is able to solve and/or transfer the solution of a mathematical problem to other people or a machine by employing thinking processes that include abstraction, decomposition, pattern recognition, algorithmic thinking, modelling, logical and analytical thinking, generalisation and evaluation of solutions and strategies” (Kallia et al., 2021, p. 28). This allowed computational thinking to be understood in relation to how it could be integrated in mathematics.

Second, we wanted to describe distinct subject areas in which the teachers could use PCT to teach mathematics. Here, we relied on the 10 subject areas described in the KOM framework developed by Niss and Højgaard (Niss & Jensen, 2002; Tamborg et al., 2022). These areas include numbers, arithmetic, algebra, geometry, functions, infinitesimal calculus, probability, statistics, discrete mathematics, and optimization. By drawing on this categorization and combining it with the definition offered by Kallia et al. (2021), we are able to examine the relationship between PCT and its use by the respondents. While the potential of combining specific PCT and mathematical topics can fluctuate over time and tend to dwell on specific themes and subjects, looking at what the English teachers do in their own practice might enable us to see what the teachers view as feasible connections between PCT and mathematics.

DATA & METHOD

The analysis builds on data from a comparative study of how PCT is integrated with mathematics teaching at the compulsory level in three countries with design experiments and the development of exemplars (Tamborg et al., 2022). In this article, we only include data from teachers in England. The survey was initially distributed to a random selection of 100 schools from the English government's website. It was sent out to the school leaders, who were contacted multiple times over a three-week period. As there was a limited response from the schools, we obtained additional answers by sending a link out to 300 teachers through a central gatekeeper in England, who urged these teachers to share it with their colleagues. This network is closely affiliated with initiatives to

use technology when teaching mathematics in England. This is likely to introduce a selection-bias, with the teachers who answered our survey being more inclined to use technology (and potentially PCT) in their teaching than their average colleague.

Who Answered the Survey

108 teachers in England answered the survey, and 62 of them provided answers to questions about their use of PCT. The teachers taught classes with students between 12 and 16 years old, which translates to lower secondary school. As mentioned in the introduction, England provides an interesting case as the computing curriculum in this context is separate from mathematics. There are thus no curricular requirements for the respondents to cover PCT as part of their mathematics teaching. Despite the fact that the English school system has included a computing curriculum since 2013, only half of the teachers reported that their students have a course in computer programming or computational thinking. 25 percent reported that such a subject is not taught at their school, while another 25 percent were unsure, but we do not know whether the respondents' students have been taught computing at another point during their schooling. Furthermore, only nine of the 62 teachers allow their students to use computers in their classes, and six of the teachers work at schools that do not allow computers at all.

The teachers partook to a very limited extent in formal professional development on both computer programming and computational thinking. Only six teachers thus received training in computer programming over the last two years, while 20 teachers participated in some professional development in computational thinking. 15 of the teachers have informally (e.g., through YouTube videos) trained themselves on computer programming, and 23 of the teachers have done the same for computational thinking.

Asking About Subject Areas and PCT

In order to see the connections made by teachers between subject areas and PCT, we first show the share of teachers who are employing PCT and then what subject areas they are covering with PCT. The teachers' use of PCT comes from the question: "Are there elements of computer programming or computational thinking in your mathematics teaching? (select all that apply)". Here, the survey distinguishes between computer programming and computational thinking, which were defined for the respondents as described in the prior section. Furthermore, the survey included a distinction between computational thinking being taught by digital means (e.g., by using a computer programme) and computational thinking being taught by analogue means (e.g., by making the students draw operations). The teachers could also respond that they did neither or that they did not know whether they did so.

The teachers who do include PCT in their teaching are asked in what subject areas they use PCT by answering the question, "In which of the following mathematical subject areas do you include computer programming in your teaching?" The teachers were presented with the 10 subject areas described in the section on the theoretical framework. In the survey, the respondents were also asked to report to what extent they use PCT to cover the subject. Due to the limited number of respondents compared to the complexity of our analysis, we only compare whether it was used or not to cover the subject. Furthermore, we investigate in what mathematical subject area the respondents taught PCT.

RESULTS

Of the 62 teachers who answered the questions about using PCT in their teaching, 25 do not use PCT in their teaching, while six were unsure whether they used PCT in their teaching (Figure 1).

23 of the teachers indicated that they use analogue computational thinking in relation to their teaching, the largest group indicating use of PCT by far, while only 10 and 6 teachers use computer programming and digital computational thinking, respectively. Analogue computational thinking thus seems to be the most common PCT element.

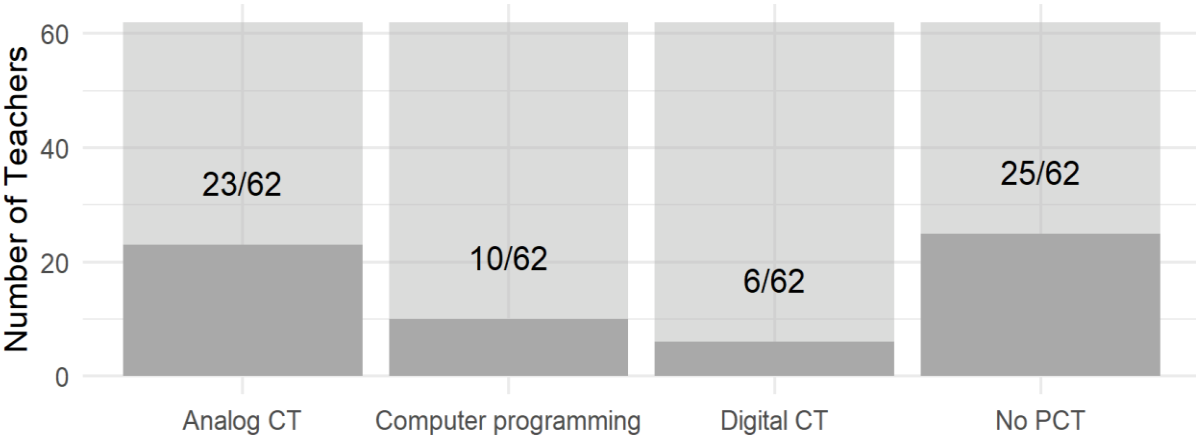


Figure 1. Teachers’ Use of PCT in Mathematics

Figure 2 shows the distribution of teachers who use PCT in relation to the ten different areas of mathematics. Again, it seems useful to look at this by compartmentalizing the different subject areas into groups dependent on the number of teachers who answer that they use PCT in relation to them.

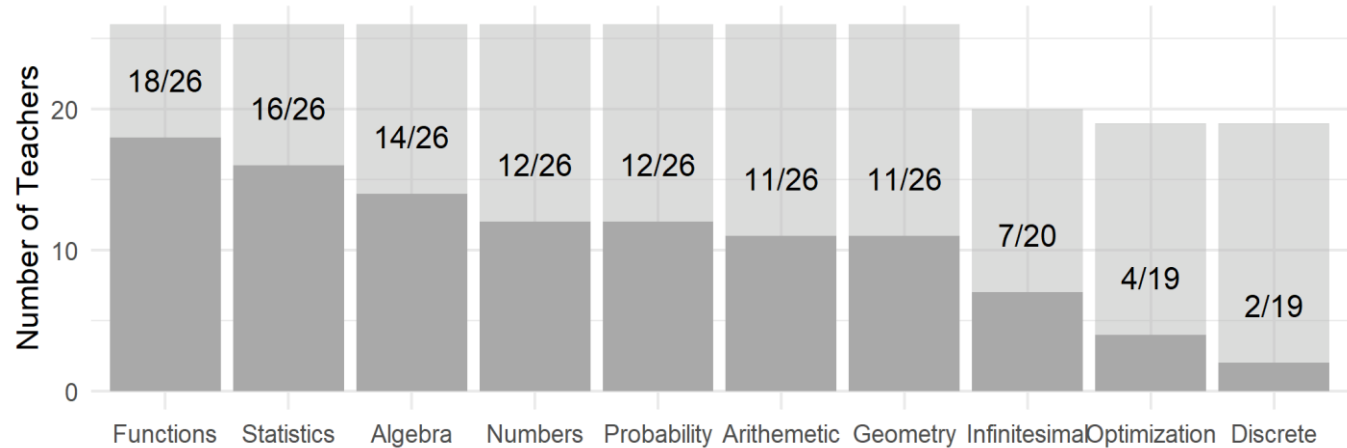


Figure 2. PCT in Subject Areas

We can thus locate three different ‘groups’ of subject areas, with a different number of teachers indicating use of PCT in relation to each. The subject areas to which most teachers apply PCT include functions, statistics, and algebra (18, 16, and 14 teachers, respectively). The areas that the second-most teachers indicate using PCT in relation to are arithmetic, geometry, numbers, and probability (12, 12, 11, and 11 teachers, respectively). Infinitesimal, optimization, and discrete mathematics has the most limited connection to PCT as counted by teachers’ use (7, 4, and 2 teachers). Multiple teachers furthermore indicate that these final three subject areas are not part of the mathematics curriculum.

Finally, figure 3 shows which combinations of subject areas teachers indicate they use PCT in relation to. Though we cannot know for sure whether the teachers are actively combining these areas, it might point towards trends in that regard. The model shows the top ten most frequent subject area combinations, and the number of times respondents noted that they had covered both

by PCT. The most common combinations of subjects are between statistics and functions, and between algebra and functions (13 teachers). Next are the combinations of functions and arithmetic, functions and probability, and probability and statistics (11 teachers).

Besides being the most commonly used ‘code’, functions are also most closely connected to the other subfields, and specifically to algebra and statistics. Statistics are interesting, as they are less well-described as having a strong relation to computational thinking as e.g., geometry or numbers would have. Geometry is surprising due to its significant role in the literature and its absence both as a highly integrated field and as a less common combination with the other subject areas.

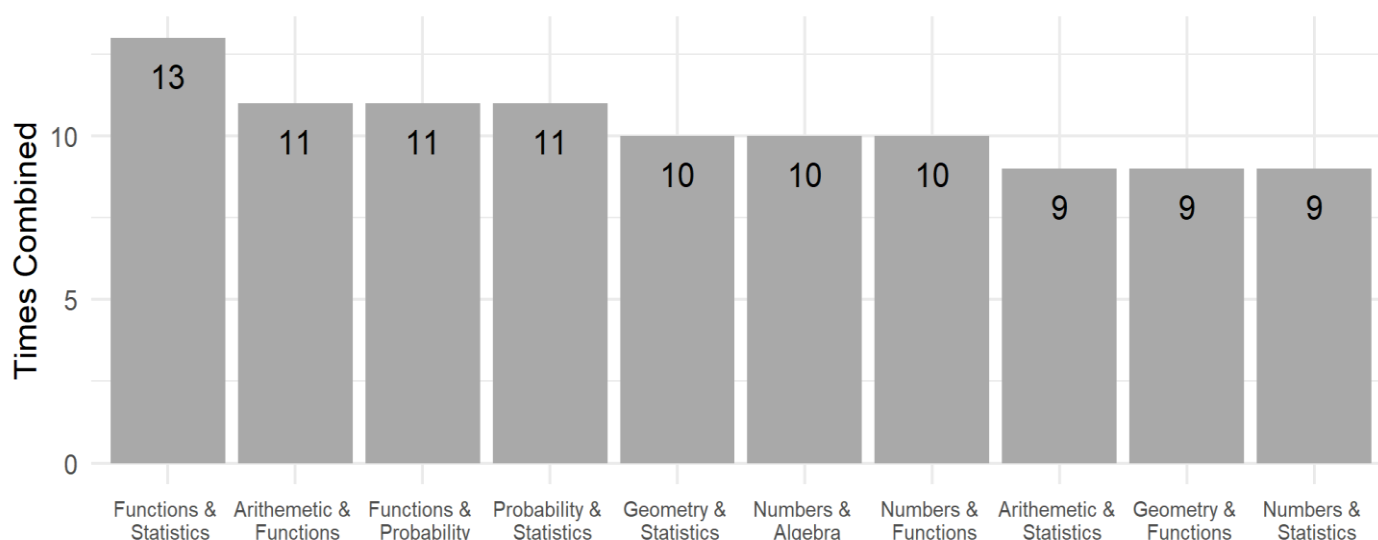


Figure 3. Subject Areas Combinations Covered by PCT

DISCUSSION AND CONCLUSION

In this paper, we studied to what extent mathematics teachers in England teach PCT and in what mathematical subject areas they do so. Thus, we try to bridge studies of the mathematical potentials inherent in including PCT with studies of how teachers are employing PCT in their classrooms. Looking at responses from 62 mathematics teachers in England, we found that about half of them are applying one or more components of PCT to their teaching. A majority of this is done by analogue computational thinking, which reflects the little use of computers in English classrooms. This is also shown in our sample, in which only nine teachers report to use computers in their classrooms, despite potential selection-biases towards more tech-prone teachers.

The teachers who teach PCT as part of their mathematics teaching most often do so in the subject areas of functions, algebra, and statistics. While algebra and functions are highlighted in the state-of-the-art, statistics is less commonly associated with PCT. When comparing the teachers’ choice of subject areas to teach PCT, there are two surprises in comparison with the state-of-the-art. First, the most common connection between PCT and functions has been more or less forgotten (or at least left out) in recent research. Second, while geometry is commonly described in the state-of-the-art as being much used with PCT, only 11 of 26 teachers report it as part of their practice. A reason for this discrepancy could be the focus on analogue uses of computational thinking in England, which is in opposition to the focus on digital platforms and digitally enforced computational thinking in the literature. Thus, if there is a digital precedence for including PCT in geometry, then the learning materials might include digital remedies. Furthermore, it might also reflect the fact that geometry plays a less central role in the English mathematics curriculum, which could lead teachers to spend less time on integrating it with other subjects.

The limited number of respondents does not allow us to test the statistical significance of the findings. However, we believe that outlining what subject areas teachers are applying PCT to is an important first step toward understanding where teachers see potential for such integration and raises some important questions. One of these is whether the distinction between digital and analogue ways of engaging with PCT is impacting the potentials in relation to engaging students with different mathematical subject areas.

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INTEGRATING DIGITAL GAMES IN THE CLASSROOM: A CASE STUDY WITH GAME PROBCHALLENGE

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This paper reports how a mathematics teacher uses a digital game for learning and teaching in the classroom. We utilize a case study approach to examine the pedagogical activities present in the lesson. The results of the study indicate that not all of the lesson objectives were met. Even though the teacher planned the lesson and the activities for the students, his lack of knowledge about game-based pedagogy had an impact on the learning outcomes.

Keywords: digital game, DGBL, mathematics teacher, pedagogical activities

INTRODUCTION

Digital games are played in digital settings that permit interaction with electronic systems that provide visual feedback (Moore-Russo et al., 2015). Digital game-based learning (DGBL) refers to any educational activity that incorporates digital games. Research on the use of digital games has demonstrated positive outcomes not only for K-12 students but also for college students in terms of motivating them to learn mathematics, increasing their engagement, fostering active participation, and enhancing their mathematical knowledge (e.g., Avraamidou et al., 2015; Go et al., 2022). However, in spite of its potential for students' learning, DGBL has a relatively low adoption rate by teachers (Rüth et al., 2022). Teachers play a crucial role in the implementation of this pedagogical strategy. The research on DGBL demonstrates that it is important that teachers use various pedagogical activities when they implement DGBL in classrooms (Bado, 2012; Foster & Shah, 2020). Furthermore, the literature review reveals a dearth of studies that examine the teacher's pedagogical moves during DGBL mathematics instruction. Therefore, the primary objective of this paper is to examine the implementation of DGBL in mathematics classrooms and analyze the teacher's pedagogical moves during this process. The following research question has been posed: How does the teacher incorporate digital games into the classroom instruction?

THEORETICAL BACKGROUND

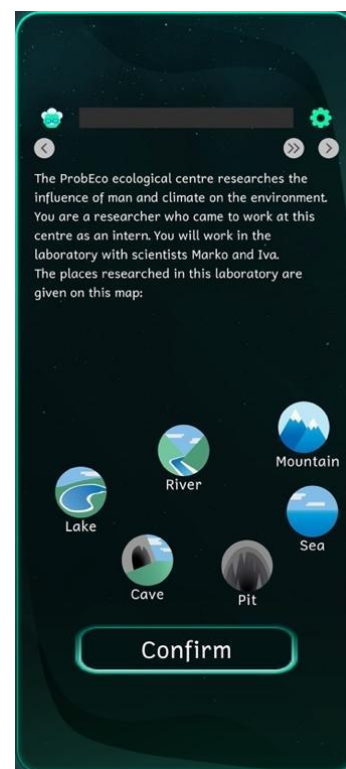
Teachers who want to use DGBL in their practice face numerous challenges. For instance, when attempting to incorporate games into instruction, games quite often do not correspond with the required curriculum or the traditional classroom routine. The effort that is needed to adapt the instruction may explain the gap between their desire and actual use of games in the classroom (Foster & Shah, 2020). In some cases, teachers learn how to teach using games through informal means, such as trial and error or by asking their peers for advice (Takeuchi & Vaala, 2014). Foster & Shah (2020) note that such learning is not unexpected given the lack of progress in the field to develop and test systematic models that capture teachers' knowledge of DGBL and to support pre-service and in-service teachers with appropriate professional development opportunities. To realize the full potential of DGBL as an educational strategy, teachers must comprehend the pedagogical aspects of game implementation and develop the appropriate DGBL competencies.

The study of teacher DGBL competencies and game-based pedagogy is currently underway. However, some researchers offered frameworks that describe what these concepts should entail. Nousiainen et al. (2018) consider competences as a multi-layered concept comprising cognitive, skill-based, and affective components of knowledge, skills, attitudes, values, and ethics. They

identified four areas of teacher DGBL competences: *pedagogical* (curriculum-based planning, in-game tutoring, and assessment), *technological* (overcoming technological barriers and analyzing technological tools), *collaborative* (teachers' capacity and willingness to share and communicate content, ideas, and practices), and *creative* (ability to take a playful stance, explore, and improvise). When it comes to educational games, pedagogical competence entails activities specific to DGBL and the educational context such as framing the game within the curriculum and in-game tutoring. Kangas et al. (2017) refer to planning, orientation, playing, and elaboration activities. Developing a pedagogical framework for game-based learning requires planning. During the *planning* phase, a teacher may wonder how the game relates to the rest of the instruction and how gaming situations are structured. During *orientation*, the teacher introduces the game, the gaming process, and the pedagogical objectives of the game. During *playing*, the teacher serves as an active tutor and guide, facilitating and scaffolding student learning. Activity evaluation and reflection are required for *elaboration*. Elaboration, the final step of the DGBL process, is crucial for the learning process because it allows teachers and students to discuss the game, clarify, and reflect on what they've learned (Bado, 2022).

GAME PROBCHALLENGE

There is a wide range of commercial games aimed primarily at the entertainment market, but there is also a growing body of educational digital games. These can serve as a platform for solving problems, for exploring a model of some facet of our world, for fostering collaboration and role play, and so on. Complex game designs typically require sizable development budgets, which are out of reach for most elementary schools and educational researchers. We followed Es-Sajjade & Plaas (2018) stance that a simple game designed specifically for mathematics and informed by established educational theories could produce the desired educational effect. Therefore, within Erasmus+ project called GAMMA (GAMe-based learning in Mathematics), we designed a game called ProbChallenge. ProbChallenge game was developed as a collaborative initiative in which researchers, educators and teachers teamed up with one game-designer to build a game intended for upper-secondary mathematics. The intention behind ProbChallenge was to design a digital game that is intended for learning new mathematical concepts, and not only for practicing already learnt mathematical content. The development of the game was framed in Allevén et al. (2010) model for educational-game design. The model combines the MDA-framework for game-design (Mechanics, Dynamics and Aesthetics) with learning objectives and Instructional Design Principles. We describe the game within this framework. The game ProbChallenge is a single player narrative game. According to the story of the game (*aesthetics*) the player is an intern who comes to work in the laboratory (Figure 1) and needs to learn about probabilities to become an equal member of the laboratory. During the game the player engages in game mechanics that are aimed at supporting the player in getting acquainted with basic concepts related to probability such as and calculates probabilities. The game has five levels. The player can choose which of the first four levels he wants to play, while for the fifth level he must achieve a certain result in the fourth level. The first level is the easiest and the player is introduced to the basic concepts needed in the following levels. Each level is more difficult than the previous. The game gives feedback for the entered answer (*dynamics*). After the level is played, the player receives feedback on his performance. The game also has a glossary.



The following learning objectives can be achieved with the GAMMA ProbChallenge: determine the sample space and possible outcomes, describe certain and impossible events, calculate the probability of an event, use the classical definition of probability, calculate the probability of intersection, union, and complement. Levels 1-4 are required for learning, while Level 5 is for practicing. Instructional design principles involved include findings from cognitive load theory such as employing worked examples, avoiding animations because of negative effect on working memory and different types of information in combination—a picture plus narrative (Sweller et al., 2019). The game can be downloaded from Google Play and the Apple Store.

METHODOLOGY

In this research we utilize case study design. The participant is a Croatian high school mathematics teacher who will be referred to as Mr. T. He is open to professional development and participates in a variety of national and international projects for teachers, such as Erasmus projects. Mr. T has over ten years of professional experience. He got a job at the school as soon as he completed his studies, and he is young enough that technology and its use are not a problem for him in both his personal and professional life. Mr. T joined the GAMMA project because he had previously used playful learning and teaching in classes, primarily non-digital games or gamification using existing applications such as Kahoot for student competition, Genially for escape rooms, and Quizizz for independent practice. He participated in the development of the GAMMA ProbChallenge game but has not yet been involved in professional development related to game-based learning and its classroom implementation. He was chosen as the participant of the study because he decided to use the game GAMMA ProbChallenge with his students.

Observation was chosen as the method of data collection because we wanted to collect information regarding the practical application of digital games in classroom settings. A lesson observation form was used to this end. The lesson, which lasted 40 minutes, was video recorded which allowed for a more detailed analysis on the teachers' pedagogical activities. During the lesson the first author wrote field notes, which consisted of both descriptive information and reflective information (Tenzek, 2017). Description information consists of factual data, such as time, date, setting, and descriptions of situations, while reflective information features the researcher's reflections about the observation. During the video analysis a two-minute observation period was used. The following descriptive information was recorded: (i) whether the teacher interacts with the students; (ii) whether he gives instructions to the entire class or to only some students; (iii) what kind explanations he gives (related to the game or mathematics); (iv) whether he answers the students' questions, and/of monitors the students to see what they are doing.

The observation was conducted openly; the participants were informed of the objectives to build trust, prevent a sense of deception, and make it more acceptable for the researcher to record the students' work and discussions during the observation. During the observation, the students were completely at ease, and their behavior did not indicate any discomfort due to the camera presence. They also did not hesitate to ask the teacher some questions when they encountered obstacles. Therefore, we believe that the researcher did not influence the behavior of the participants, and that the classroom situation was genuine.

The observational data were coded and analyzed in relation to Kangas' et al. (2017) framework on pedagogical activities. Therefore, each assigned code was examined and clustered into either planning, orientation, playing, or elaboration category:

Planning:

Teacher wonders how the game is linked to the rest of the instruction and how gaming situations are organized. He/she determines when, how, and why to use digital games, as

well as how knowledge is shared, created, and validated among students. This includes organizational aspects, such as determining the learning, and gaming groups.

Orientation:

The teacher begins the gaming session with an introduction and provides context regarding the goals of gaming. This phase entails introspection and familiarity with the topic and anticipated learning outcomes.

Playing:

The teacher serves as an active tutor and guide who supports and scaffolds student learning. This can include the following: teacher stops the game for a while and initiates conversation about the topic; asks specific questions during the game aimed at focusing the students' attention on the concepts they were to learn by playing the game; the teacher stops the game to discuss with students the choices they had made during the game.

Elaboration: reflection and evaluation of activities

The teacher discusses the game content after the gaming situations and in this way connects the learning experiences gained in the games with the curriculum and learning. It can include activities where students use elements of what they learned in the game and expand and transform them by relating other information to it.

Example of data analysis is given in the table below (Table 1).

Field notes	Lesson minute	Codes	Category
Teacher talks to the class, announces goal of the lesson	2	Lesson goal	Orientation
Teacher asks students if they used probability in their life. Fours students give examples	4	Lesson goal	Orientation
Teacher announces to the whole class that they will play the game related to probability and shows how the game is played to the whole class.	6	Lesson goal Game instruction	Orientation
...			
Teachers walks around classroom; students are playing Level 1	16	Playing	Playing

Table 1. Data analysis

RESULTS

In this part, we present the results of the lesson observation in which we observed how the teacher implements a digital game in the teaching of mathematics. We have identified following game-based pedagogical activities within the lesson:

Pedagogical activity Planning:

Mr. T decided to use the digital game GAMMA ProbChallenge when it was time for the Probability topic in the curriculum. He intended to teach the following concepts: sample space, outcome, equally likely events, possible event, impossible event, certain event, and calculating the probability

of simple events The teacher installed the game on the students' computers prior to the lesson. Students sat in groups of four during the lesson, with two students sharing a single computer. It was expected that students would collaborate to solve game-related tasks during game play. This seating arrangement was designed prior to the actual class, as the students were greeted by that arrangement when they entered the classroom. According to Kangas et al. (2017), this pertains to the pedagogical task of planning. During the lesson preparation, Mr. T also created a digital worksheet for students' reflection after game play and paper worksheets for assessing students' understanding of learned concepts. Also during planning, the teacher decided that students should solve the digital worksheets collaboratively, whereas the paper worksheet should serve as a formative evaluation of their knowledge at the conclusion of the lesson and required independent work.

Pedagogical activity Orientation:

At the beginning of the lesson, Mr. T introduced students to the lesson objectives. First, he asked students if they ever heard about probability and about its application. Several students gave answers related to use of probability: weather, gaming, sporting bets. Then the teacher announced they will play a game with two levels. The teacher gave instructions about gameplay: how the game should be played, demonstrated navigation through the game and the activities that follow each level.

Pedagogical activity Playing:

While students played Level 1 of the GAMMA Probability Game, Mr. T circulated throughout the classroom, available to assist anyone who needed it. Students discussed the task with one another or within the group but did not ask the teacher for clarification or assistance.

The teacher also circulated throughout the classroom as students played Level 2 of the game. Students at this level posed questions and initiated conversation by asking questions. Some questions were of a technical nature, such as why the game does not offer more than two chances to answer the same question after an incorrect response has been given. Some questions in the game concerned mathematics, such as whether probability could be greater than 1. Each group of students had questions, and the teacher visited each group and spoke with them individually (for example see Figure 2).



Pedagogical activity Elaboration:

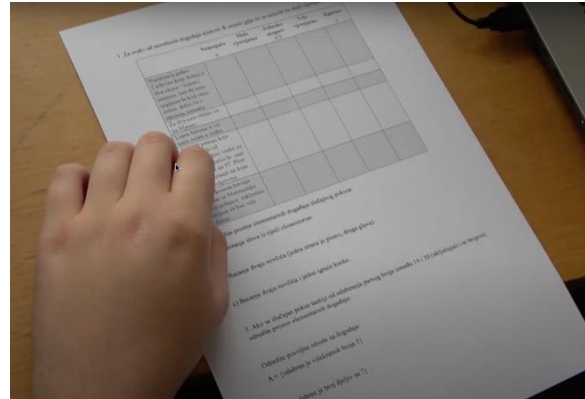
After each level was played (game play happened two times) students had to fill out a digital worksheet in pairs. The worksheet contained the task 3-2-1, which is meant for reflection. The students must write *three key terms* they learnt in the game, *two concepts* they think they have mastered and *one skill or concept* they would like to master. Both times we observed students calling the teacher and asking him what they should write. Some students commented among themselves: “What else should I write?”, “What should I learn better?”, “What is the third key concept?”. The

first time the teacher stopped the class. He acknowledged that this task could be difficult for students but also that he expected them to do it without his help.

After the 3-2-1 task for Level 2, the students were presented with a worksheet with game-related concepts (Figure 3). This task required students to use the mathematical concepts encountered in the game. Each student individually completed this worksheet. When the teacher distributed the worksheets, some students began to answer the questions, but the bell soon rang, ending the lesson.

DISCUSSION AND CONCLUSION

The purpose of this study was to investigate implementation of DGBL in mathematics lesson. In particular, the pedagogical activities performed by one mathematics teacher when trying to incorporate digital games into the classroom instruction. The case study presented in this paper illustrates 1) how a well thought lesson with DGBL can support the learning of probability and the pedagogical activities involved; and 2) two factors that have constrained students' learning of mathematics. Namely: lack of support in the elaboration phase in DGBL and time constraint component for the lesson.



Teachers need to take on a new set of responsibilities when facilitating gaming in the classroom. They must act as game administrators, lecturers, game tutors, subject matter anchors, and authority figures to keep students in an educational mode of play during a typical gaming session (Marklund & Alklind Taylor, 2018). In the case of subject matter, it is the responsibility of the teacher to ensure that all students meet the lesson learning outcomes. This indicates that the teacher must play an active role in the DGBL lesson and that digital games are intended to assist teachers in enhancing student participation and learning outcomes and not to replace teachers (Ke, 2016). In the case of Mr. T, four distinct DGBL pedagogical phases were identified: planning, orientation, tutoring during game play, and elaboration. However, the results show the difficulties experienced by the teacher and students namely in the elaboration and reflection phase of the DGBL process. Students had difficulty in 'extracting' the learning content out of the game. The teacher could not provide other support to the students than letting the students figure it out for themselves. Partially because he did not know what to do and partially because the lesson ended. Thus we may consider that the elaboration phase did not occur. Although Mr. T's activities in the elaboration phases were intended to reformulate and transform knowledge and the game played, we believe that the post-game discussions with the students would lead to a more grounded understanding of the probability concepts taught in the game. Mr. T performed his role as facilitator almost admirably up until the elaboration phase, which suggests he is unaware of the significance of group discussion. Bado (2022) and Kangas et al. (2017) highlighted the importance of this lesson phase in DGBL: game elaboration and reflection, or debriefing, is the transformation of gameplay into learning outcomes; teachers and students should discuss game content, gameplay, and curriculum to establish connections.

The final phase of the lesson appeared incomplete because time ran out and the teacher did not receive student knowledge feedback. This confirms the findings of various studies (e.g. An & Cao, 2016; Kaimara et al., 2021) in which the time constraint was identified as a barrier for DGBL. The formative assessment of students' knowledge must be incorporated into the DGBL lesson, and this must be considered when planning the lesson, as it is essential for the teacher's future work. Additionally, students should have the opportunity to test their understanding of the adopted concepts.

Mr. T's participation in the design of the classroom game can be viewed as gaining experience with game-based pedagogy (Nousiainen et al, 2018). Nonetheless, the lack of collaborative discussion during the elaboration phase demonstrates lack of adequate skills and knowledge in game-based pedagogy. Scholars argue that teacher education in game-based learning is crucial for the effective and meaningful use of games as an educational tool (e.g. Foster & Shah, 2020). According to the findings of Meletiou-Mavrotheris and Prodromou (2017), DGBL training should incorporate both experiential and theoretical learning. We advocate for DGBL professional development that includes knowledge on game elements (such as the MDA framework), learning process and the relation among them because otherwise it is not possible to connect the story, mechanics and dynamics in the game with the mathematical content, procedures and learning experience.

There is a dearth of research on the extent to which mathematics teachers use digital games, how they use them in the classroom, and what obstacles stand in the way of their use of these games (Denham et al., 2022). Our case study adds to the body of research in the area. To develop an appropriate pedagogy for the implementation of digital games in mathematics education, additional research on how teachers utilize DGBL in mathematics classrooms is required.

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EXPLORING THE ROLE OF 3D PRINTING TECHNOLOGY IN SUPPORTING UNDERGRADUATE STUDENTS' TOPOLOGICAL CONCEPTUAL KNOWLEDGE

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In this paper we explore the educational potential of using a 3D printer in a university mathematics setting. We investigate how 3D printing technology might help undergraduate mathematics students improve their conceptual knowledge and theoretical reasoning. Our approach intends to engage students in both the design of topological objects and their 3D printing manufacture as well as the study of their properties through the manipulation of physical examples. Extending the "example space" and moving from one semiotic representation to another are fundamental learning opportunities. The former is crucial in reifying a mathematical concept by including even a material example, and the latter is critical in activating cognitive processes to acquire conceptual understanding. Preliminary findings highlight that the 3D printing technology effectively integrated students' learning experiences.

Keywords: 3D-printing, conceptual understanding, example-space, topology, university mathematics

INTRODUCTION

In this age of digital transformation, 3D printing technology is being used successfully in a variety of industries, including manufacturing, medical, engineering, aerospace, and science. 3D printing is based on digital fabrication (DF), which is described as "the process of translating a digital design developed on a computer into a physical object" (Berry et al., 2010, p. 168). Using 3D printing in STEAM and STEM education improves students' creativity, cooperation, problem-solving skills, and higher-order thinking skills, as well as impacting their interests, engagements, beliefs, and careers, according to reviews (Ng, 2017; Cheng et al., 2021). In recent years, there has been a growing interest in employing 3D printing technology to improve mathematics education. The focus is on the impact that 3D printers can have on teaching and learning mathematics, as well as their potential for both knowledge construction and creativity development. According to a recent study (Stigberg, 2022), 3D printing is the most often used approach for creating manipulatives that reify mathematical concepts in geometry, algebra, functions, and fractions. The use of 3D printing helps students visualize concepts and proofs (e.g., geometry, calculus, volume) and enables them to develop mathematical, abstract, and spatial thinking (Dilling & Witzke, 2020; Huleihil, 2017). It not only scaffolds students' mathematical understandings, it is also a powerful tool to stimulate students' creativity and spatial and design thinking (Ng, 2017; Ng & Ferrara, 2020;). Ng and Ferrara (2020) report how primary students used 3D printing pens to create their own prisms and pyramids to learn the geometric properties and cross-sections. A physical 3D-printed mathematical object is simply one example of a mathematical concept. Research argues that engaging students in the production of examples is a necessary step to understanding definitions and overcoming difficulties in facing advanced mathematics problems (Watson & Mason, 2005; Moore, 1994). Encouraging students to analyse conventional examples, or produce examples their own, helps to deepen concept understanding and facilitates the discovery of proof. Watson and Mason (2005, p.51) say that "examples can be perceived or experienced as members of structured spaces" and introduced the term "example space" to describe such space.

Difficulties appear to depend on the delicate transition between seemingly conflicting approaches, operational-structural (Sfard, 1991), to grasp a concept and apply it to learn others and solve problems. According to Sfard an abstract notion can be conceived in two fundamentally different ways: *structurally*, as objects, and *operationally*, as processes, and these two approaches, although incompatible, are complementary. The transition from computational operations to abstract objects is a long and inherently difficult process, and consists of three steps: *interiorization*, *condensation*, and *reification*. In the first stage a learner gets acquainted with a concept and performs operations or processes on mathematical objects, then he/she has an increasing capability to alternate between different representations (Duval, 2017) of a concept, and finally can conceive of the mathematical concept as a complete, “fully-fledged” object. The paper addresses a topical issue of the use of digital technologies in mathematics education: the combined use of digital interacting tools, 3D printing technology (DF), and Embodied Learning (EL). The manipulation of 3D-printed objects is the last step of a process that involves the passage, through mathematical software, from one semiotic representation of the object to be produced to another. Indeed, students pass from an analytical representation to the corresponding graphic representation and from the latter to its physical one. The entire process is worthy of study by referring to Duval's theory of semiotic representation (Duval, 2006). In the study, the embodied learning approach also acquires great relevance. It manifests itself in a marked way in the final phase, when the student handles the object, and is highly influenced by the previous phase of digital manipulation of the concept, in an effective coupling between embodiment and instrumentation. EL technologies are employed in educational research in mathematics as an emerging technology with the potential to affect brain function to drive learning, particularly by integrating the physical body into the learning process (Abrahamson et al., 2020). Instrumental and embodied approaches to the use of digital tools in learning mathematics coordinate and align with the notion of embodied instrumentation (Drijvers, 2019), which is seen as the genesis of a body-artifact functional system (Shvarts et al., 2021). Drijvers images "productive learning trajectories on fundamental mathematical concepts, in which the development of sensorimotor schemes may gradually go hand in hand with, or even be part of, instrumental genesis."

We conduct a preliminary study on the use of 3D printing technology as a final step in a process coordinating digital and manual approaches with the aim of reducing the students' gap between the cognitive construction of the concept associated with a mathematical object and the real construction of a physical object representing it. To be engaged in a concept digital manipulation, to project 3D-printer constructions, to manage them, help students visualise concepts and proofs and discover new ones. Students' exploration of examples by themselves as they pass from a digital environment to a conventional environment contributes to achieving the familiarity useful to an aware learning process.

THEORETICAL FRAMEWORK AND RESEARCH QUESTIONS

The project foresees students' engagement in problem-solving activities stimulating cognitive and metacognitive processes: thinking, designing, producing, and then analysing, making argumentation, defining, conjecturing, proving, and reflecting. In a first phase, in a digital environment, students create and explore by themselves the objects to be studied by fingers in a second phase. The problem-solving activity aims to foster the acquisition of concepts and promote the generation of new mathematical knowledge by emphasising conceptual rather than procedural learning in specific digital learning contexts. Tall and Vinner (1981) relate the understanding of a concept to the distinct notions of “concept image” and “concept definition”. The former describes the “total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes” (Tall and Vinner, 1981, p. 152). The latter is “a form of words used to specify the concept” (p. 152) and refers to a formal definition. Moore (1994)

integrates the *concept image/ concept definition* construct with the notion of *concept usage*, “which refers to the ways one operates with the concept in doing proofs” (Moore, 1994, p. 252). Concept definition, concept image and concept usage define a *concept-understanding* scheme (Moore, 1994). In our context, this scheme also refers to another aspect: the digital manipulation and production of an object. The need to integrate the scheme, considering its digital counterpart, which we could define as “concept digital manipulation”, takes shape. *Concept digital manipulation* refers to the total digital structure and digital experiences associated with the concept, including all the digitally created graphs and physical objects. It could be seen as a kind of digital mediator between the ‘concept image’ and ‘concept usage’ as it refers to digital experiences such as the digital production and manipulation of a graphical representation and the digital fabrication of a material object. The notion fits well into the concept understanding in light of recent research on embodied learning, which foresees the interactive use of body and digital tools converging in a construct called *embodied instrumentation* (Drijvers, 2020). The educational potential of the use of different representations (Duval, 2017) and of the example space extension (Watson and Mason, 2005) is undisputed, as are the roles of the body and its interactions with digital instrumentation. We base the designing of our activities on these principles. The topological objects are examples enriching the example space: one graphic to visualize and one physical to handle. In particular, a 3D-printed example of a topological object is a concrete example that stimulates the abstract production of a structural example. Duval’s theory helps us to investigate the gap between the mathematical objects that we intend our students to build cognitively and those that, in reality, the student constructs. Transformations of semiotic representations are placed at the centre of the mathematical activity. According to Duval, transformations are at the heart of the mathematical way of working and teaching them is crucial to developing students’ cognitive skills to match different representations. To view the same object in different representations and to create correspondence between objects or between representations is crucial for developing cognitive processes. In the conversion process, we start with the analytical-algebraic representation and pass, through the software, to the geometric-graphic one and, through the 3D printer, to the geometric-physical one.

Research questions

We investigate students’ learning outcomes and how 3D printing technology affects the construction of mathematical knowledge through the lens of “concept digital manipulation”. Research questions: *What is the impact of 3D-printing technology on the construction of mathematical knowledge? Specifically, what is its role in supporting cognitive processes for conceptual learning in topology?*

METHODOLOGY

The context

The experience, to be planned for fifty students attending a third-year course of the bachelor’s degree Course in Mathematics, will be realised within an introductory course of algebraic topology, in which we give definitions of algebraic objects that are topological invariants, and we open the discovery of the classification of topological surfaces. In this context, the idea of engaging students in activities aimed at extending example spaces through 3D printing would add another dimension to the learning experience and increase motivation and curiosity. Students, as designers, print and build up an artifact, and the final product is a physical object they explore and classify by hand. We conducted a preliminary pilot study to test the potential of the idea. Three graduating university students in Mathematics were involved in a problem-solving preparatory activity dealing with the fabrication and classification of topological objects. The problem developed in the first phase as a combination of thinking, design, and digital production, and in the second phase as a manipulation of the 3D-printed objects. In line with the design, the students, divided into groups (only a group in our preliminary study), were assigned to print some spaces to explore, going from reconstructing

their analytic representations to exploring them, through sliders, and then, through fingers. In the conversion process, we started with the analytical representation and passed, through the software, to the visual-geometric-graphic one and, through the 3D printer, to the visual-physical one. Each representation was an occasion for reflection in which the treatment transformation already gave an answer, even partial, to the mathematical problem. The design foresees that a student is both a designer of the objects and a solver. Students design, explore, print, manipulate, classify, first in a digital environment made by the software and a 3D printer, and then in a conventional environment.

A designing and classification task

The project aims to involve the student in problem-solving activities focused both on the student's design and exploration of the digital object by sliders and on the student's printing and exploration of the physical object by fingers. As an example, let's take a one-sheet hyperboloid to shrink it until we get a cone. The spaces are no longer homomorphic because one has a trivial fundamental group and the other does not. Students collect printed objects and open the discussion on their properties in order to classify them topologically, even by calculating the Euler-Poincaré characteristic. Particular attention is paid to the classification of surfaces. In the second task question, through a set of printed objects, they pass from graphic exploration to a tactile exploration by fingers. The actual touching of physical objects can lead to “new gestural forms of thinking”. There are two problem-solving levels related to the actions to design and to solve. The task foresees students' engagement in the design and manipulation, in a digital environment, and in the 3D printing and handling:

Task

1. Design and print a set of mathematical objects (hyperbolas, cone, Möbius strip).
2. Study, compare and classify the objects through the topological properties discovered manipulating them during the process

Data collection and Data analyses

Our collected data, specifically students' written notes and recorded discourses, were qualitatively analyzed through a systematic and objective identification of some characteristics of meaningful learning, separately looking at the two phases, digital manipulation before 3D printing and handling manipulation after 3D printing, through the lens of concept digital manipulation, and then looking at their interactions to understand if they were harbingers of conceptual knowledge. Then we looked at which factors triggered them. In this paper, we confine our analysis to significant excerpts from the students' discussions in the different phases of the learning path. We investigate the factors contributing to conceptual learning and to what extent and why this has been supported by and supports 3D printing. To shed light on the impact that both the digital manipulation and the physical handling of 3D-printed objects can have on the learning of topological concepts, we focus on what emerged before and after the 3D-printing phase. We highlight how students' mathematics knowledge has improved by looking at their ability to distinguish, classify, and conjecture, as well as the factors influencing this learning improvement. In particular, we examine the function the “concept digital manipulation” has in determining a conscious and productive use of the 3D-printed objects.

PRELIMINARY FINDINGS

The task concerns designing some topological spaces and then interacting with them through moving sliders and then through moving the 3D-produced objects to learn their properties and the relationships between them. In the problem-solving activity, we ask students to digitally design and 3D-produce material objects as results of semiotic transformations (Table 1).

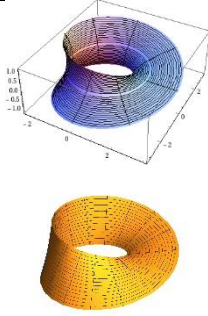
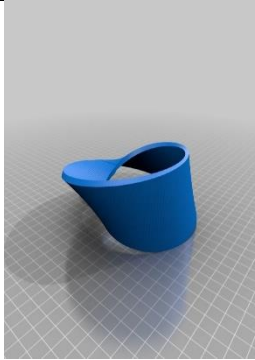
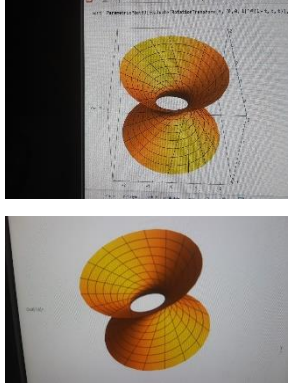
Topological space	Representation 1 Parametric Mathematica	Representation 2 Visual-Digital-Graphic	Representaion 3 Visual-Physical-3DP
<i>The Möbius strip</i>	$x[u, v] := (1 + (v/2) \cos[u/2]) \cos[u]$ $y[u, v] := (1 + (v/2) \cos[u/2]) \sin[u]$ $z[u, v] := (v/2) \sin[u/2]$ plot = ParametricPlot3D[{x[u, v], y[u, v], z[u, v]}, {u, 0, 2 Pi}, {v, -1, 1}, Boxed -> False, Axes -> False] Double rotation		
<i>The one-sheet hyperboloid</i> <i>Two models</i>	$x[u, v] := a \cosh[u] \cos[v]$ $y[u, v] := b \cosh[u] \sin[v]$ $z[u, v] := c \sinh[u]$ plot = ParametricPlot3D[{x[u, v], y[u, v], z[u, v]}, {u, 0, 2 Pi}, {v, 0, 2 Pi}, }, Boxed -> False, Axes -> False]		

Table 1. From analytic to physical 3D-printed representation of a topological space

Then we ask to create correspondences between different objects (topological classifications, as an example) or between other representations (Table 2), exploring them in the space with their fingers.

<i>A set of 3D-printed Topological spaces</i>	Classification 1 Metric type	Classification 2 Topological type	Classification 3 Homotopic type
	Five equivalence classes Why?	Three homeomorphism classes Why?	Three homotopy classes What if we took off the base of the tetrahedron?

Table 2. Classifying a set of a topological spaces through concepts by manipulating them

Every phase generates a reflection moment, helping students to reduce the gap between their concept understanding and their personal idea, while they are involved in an interactive instrumental (digital or not) action that goes from the use of the digital tool, up to the exploration of the 3D-printed material objects. We find that the factors having a positive influence on productive reflections and learning are visualization, both of the graph on the screen and of the material object, and digital and manual manipulation. Visual and physical approaches combine to realise an effective use of the 3D printing technology. In other words, concept digital manipulation positively affects concept physical manipulation of 3D models. Chosen a topological space, students propose the formulae (see Table 1) and generate the translation in the software language, a plot they have only first approached:

Student 1: Mmh, we could choose the spherical surface, which is easy to represent..and *plot* $x=r\cos u$ $y=r\sin u$ $z=..$ perhaps is too familiar and simple.....where the angles u and v run from 0 to 2π and we write $\{u, 0, 2\pi\}$, $\{v, 0, 2\pi\}$.

Student 1 highlights the meaning of the parameter range within a representation and reflects that the chosen sphere is an object that they have already had the opportunity to handle several times. The practise also has immense power to increase motivation and satisfaction, with a highly probable increase in ability to solve real problems, as seems to emerge from the answer of Student 2:

Student 2: ..ehm....It's an interesting space...but we can easily imagine what happens if we change the ray values ...So..Why don't we choose the Möbius strip? If we change the segment length in its double rotation, what we ...is it really getting closer to be a circle? This is a piece of every non orientable surface. A way to distinguish.

With the help of a computer technician, slider controls are added to allow interactive manipulation of the parametric values involved. Immediately, Student 3 recalls the notion of deformation and simulates it through the sliders, coordinating definition and manipulation, and makes previsions:

Student 3: I agree, but what is its representation?, ...to simplify I would add a quadric surface, a one sheeted hyperboloid for example, and study its deformations...[moving her fingers]...Let me change a !! It's a new more tight hyperboloid... Rather than running through a fixed graph, Manipulate lets you move back and forth at will.

Student 1 reflects about the variables and how to fix the range over which to vary it:

Student 1: [writing] The command `Manipulate[Plot[{x[u, v], y[u, v], z[u, v]}, {u, 0, 2 Pi}, {v, 0, 2 Pi}], {a, 1, 20}]` says that a is a value ranging between 1 and 20. You can click and drag the slider to see the displayed value change in real time... it changes while you are dragging the slider, but a ranges between 1 and 20. ..Mh and if a was 0?

Student 2: ...It's not possible!! It degenerates..into a conic, it seems so to me. I'll try...if you change the minimum value...

Visualizing helps students' reasoning. Students learn that the software lets you create a range through sliders, which you can use to vary the value of one or more parameters. It is not just a static result. You can interact with the graphic-visual object by deciding to manipulate the graphic through the sliders by hand, and this process aligns with the idea that instrumental embodiment strengthens concept understanding (Drijvers, 2019). This process also allows students to make some predictions about the objects. In the next discussion, students will handle 3D-printed models of two different one-sheet hyperboloids (see Table 1) and a cone. They explain that they feel the objects and deduce their properties and correspondences. The topological properties are intuitively pre-formally described, and further justifications are written. The discourse describes students' working with the 3D models.

Student 1: [moving the hyperboloids models around a finger] They both are rotational symmetric to my finger, the axis!!

Student 2 Discovers properties of objects by handling it.

Student 2: And my hand a plane with this slope [putting her hand in the middle of both] intersect one in a circle and the other in a small circle!! They are not isometric. But have them other symmetries?

Student 1: How did you do that? We want to know if they are homomorphic!

The reflection of Student 3 concerns the sensual experience of the characteristics of hyperboloid in relation to the possible actions on their graphs. To explain she recall the previous digital manipulation:

Student 3: Yes, they are!! You can imagine to restrict one to obtain the other. But.. Mhh... Look at this! [Places the finger around the object in the middle] When you put your finger on it like this, you obtain a cone. Imagine exploring by Mathematica sliders... Is it again homomorphic?

Student 2: Oh no!! The homeomorphism we have described identifying the one sheet hyperbolas is no longer good! This is one point! [Places the finger around the the middle] When you restrict here, you obtain a point. Is there another one?

After a teacher's hint to discover if some topological property distinguishes the surfaces:

Student 3: All are compact, connected. The Euler-Poincaré characteristic, it's the same,..mhh

Student 1: No, they aren't!! The cone is contractible. You can put continuously it in the vertex. While the cylinder.. have the same fundamental group of the circle.

The sensory handling of the models represented an important component of the students' reflections. Students are asked to classify a set of spaces (Table 2) based on feeling the shape and the holes of the 3D-printed objects. The explorations, after all, reveal that students developed a competent discourse about topological properties and relations, and the technology integrated their learning experience. 3D printing technology stimulates visual and spatial thinking processes and well intertwines with digital resources: digital manipulation positively affects physical manipulation of 3D models, and conversely. The passage from one learning environment to another, specifically from algebraic to variable visual graphics up to 3D-printed objects, allows students to strengthen their learning. We find that digital and material representations interact and merge to construct conceptual knowledge.

CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

Recently, 3D printing technology has occupied a place of honour in mathematics education as it provides an innovative way to handle mathematics concepts that enables students to develop theoretical and design thinking. Our research project foresees engaging students both in the design of topological objects and in their 3D printing to study their properties and relations. But what is the advantage of going from a digital representation to a 3D-printed one? Why don't we stop at digital representation or limit printing objects? What do we add to learning? When students handle spaces to classify them, they *imagine* retracting them into other ones, so they predict how a space should be added to a class and make other conjectures. Then they should verify their prediction by returning to the digital resource. So, there is a constructive back and forth from one register to another to strengthen the conceptual understanding, during which the body is immersed in a multimodal learning experience. 3D printing technology integrates students' learning by encouraging their imagination and enhancing the development of cognitive processes through a combination of digital tools, mind, and body. In conclusion, we may say that the use of the 3D printer is effective when integrated with the use of digital tools within body-mind interactions. The research opens up new perspectives for investigating the inclusion and autonomy of visually impaired and blind students. The perception of the real world depends on the senses. And a student, through touching, is put in a position to be able to identify the geometric properties of an object like everyone else. A visually impaired graduate math student interviewed says that "the use of 3D printing would certainly have been useful, especially in the representation of homeomorphisms, [...] much easier with the support of a tactile example."

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LEARNING GEOMETRY USING COMPUTATIONAL THINKING, SCRATCH AND PYTHON TURTLE

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Algorithmic thinking and computer programming are mandatory student skills to be implemented in all types of education. Over the past decades Curricula have been updated, but many mathematics teachers still lack competence and experience in computer programming, so they have to face new technologies and learn elements of computer science. The contribution of this paper is twofold: firstly, we present an educational program aiming to introduce basic programming concepts, such as loops and logical operations, to improve middle school students coding skills through their preexisting knowledge of mathematics and geometry. Secondly, we report a pilot study carried out by two teachers in their respective classrooms over a period of two months in a Italian lower middle school and in the first year of High School. Students were guided to develop their problem-solving skills using algorithmic strategies with Scratch and to increase their spatial awareness by tackling Python Turtle programming to draw plane geometric figures.

Keywords: Block programming, Computational Thinking, Scratch, Python Turtle, Geometry.

INTRODUCTION

This paper investigates the use of Computational Thinking and programming to teach geometry concepts using Scratch and Python Turtle. Computational Thinking is an educational practice that focuses on developing students' skills in problem-solving, abstract thinking, pattern recognition and logical reasoning through programming and algorithmic patterns. This area of learning is being supported by various initiatives such as "CS for All" and ISTE's Standards for Students in Computational Thinking, as well as the use of tools such as robotics, 3D printing, microprocessors and intuitive programming languages, as in Angeli (2020). Computational thinking is often related to problem solving in an algorithmic manner, i.e., defining problems and breaking them up into smaller solvable stepwise parts, much like the structure and purpose of a computer code (Barr (2011)). Several studies argue that programming can motivate students to study mathematics and increase their problem-solving skills (Barak (2016); Sinclair (2018)).

It is important for students, since primary levels, to understand and mastering the concepts of geometry, including angle, line, shape, translation and transformation. However, traditional methods of teaching geometry can be difficult for current students to understand. By using computational thinking and programming, students can gain a deeper awareness of geometry concepts. Scratch and Python Turtle are two programming languages that are often used to teach computational thinking and programming, Iskrenovic-Momcilovic (2020); Rahim (1997). Scratch is a block-based programming language designed for beginners. It allows students to easily create interactive stories, animations and games since their first years. Python Turtle is a text-based programming language that is more advanced than Scratch. It allows students to create complex programs and graphics.

The experimental part of this work is referred to a pilot study of an educational program carried out by two teachers in their respective classrooms over a period of two months in a Italian lower middle school and in the first year of High School. 45 students were challenged, individually and in small

groups, to solve geometric problems exploiting algorithmic strategies with the main functions of Scratch and Python Turtle programming. This paper finally discusses the benefits and challenges of geometry teaching by computational thinking and programming to students between 12 to 15 years, comparing student's results before and after teaching activities with the results of a control group.

THEORETICAL FRAMEWORK

Seymour Papert is an outstanding figure in the field of educational technology, as in Goldberg (1991). His seminal works (among others Papert (1972); Papert (1980); Papert, (1993)) have been widely cited and applied to various aspects of education. Papert's theories of computational thinking have been particularly influential in the development of educational applications that seek to promote problem solving and critical thinking skills. This paper is based on the theoretical framework of Papert's theories of computational thinking in education.

First, it is important to understand what Papert meant by computational thinking. Papert, 1980 defined computational thinking as "the ability to use abstractions, algorithms, and models to solve problems and create solutions." He proposed that, in order to effectively apply computational thinking to any problem or situation, there are four core components: abstraction, algorithms, models and problem solving. Abstraction involves breaking down complex tasks into smaller, more manageable components. Algorithms are the step-by-step instructions that can be used to solve a problem. Models are the representations of data or concepts that can be used to explain a concept or analyze a problem. Problem solving requires the ability to use all of the previous components with the aim of developing a solution to a problem.

In order to apply Papert's theories of computational thinking to education, it is important to plan activities and tasks that allow students to practice each of the four core components. Abstraction activities should involve breaking down a complex problem into smaller, easier to understand pieces. Algorithmic activities should teach students to develop step-by-step instructions to solve a problem. Modeling activities should inspire to use representations of data or concepts to explain or analyze a problem. Lastly, problem-solving activities should require applying all of the previously mentioned components in order to develop a solution to a problem.

In addition to activities that allow students to practice computational thinking, it is important for teachers to use instructional strategies that promote the development of computational thinking skills. For example, teachers can use inquiry-based learning to encourage students to explore complex problems. This type of learning requires students to identify and analyze problems, develop solutions, and evaluate their solutions. Teachers can also use project-based learning, which requires students to complete projects involving real-world problems. This type of learning encourages students to use their computational thinking skills to develop and evaluate solutions.

It is essential for educators to create a learning environment that is conducive to the development of computational thinking skills. This environment should encourage students to take risks and experiment with new ideas and solutions. It should also provide students with opportunities to collaborate and engage in meaningful dialogue regarding their solutions.

In conclusion, Papert's theories of computational thinking provide an important framework for the application of educational technology in the classroom. Through the use of activities and instructional strategies that promote the development of computational thinking skills, as well as a learning environment that encourages experimentation and collaboration, educators can effectively apply Papert's theories to create an engaging and meaningful learning experience for their students.

In 2011 Jeannette M. Wing proposed a new definition of this concept: "Computational thinking is the thought processes involved in formulating problems and their solutions so that the solutions are represented in a form that can be effectively carried out by an information-processing agent." Wing

(2011). A more recent and complete revision about Computational Thinking can be found in Grover (2018).

METHOD

Seymour Papert introduced a new way of working with geometry in the 1980s, the programming language Logo, which was called Turtle Geometry (Kynigos (2016); Papert (1980)). Logo is adapted to the use of children and young students to explore advanced geometrical figures and patterns, through the control of a physical turtle robot, which could be programmed to move around on a piece of paper to create drawings. Students could relate their body motion to the motion of the robot and transfer this knowledge into the work of learning formal geometry, including the conceptualization of angles.

In this paper, we propose a teaching strategy for computational thinking linked to geometric understanding and drawing of geometric figures via block-based programming. The developed education program is inspired by Scratch (Brennan (2012)) and Turtle geometry.

In other words, we proposed both the possibilities of programming to fully understand how geometric figures are drawn from a computational thinking point of view; conversely, we will try to propose the use of simple geometric figures to understand the principles of programming and computational thinking. Rather than using programming as a mean to explore geometry, we propose to use familiar geometric figures to learn the concepts using an algorithmic procedure. Teachers may apply the education program as an entry level to basic level programming concepts through geometrical understanding already owned by the students, before introducing formal, block-based programming. The programming activities were performed by Scratch and Python Turtle.

This article reports on a pilot study in which the two researchers who authored this article carried out teaching activities in the last grades of middle school and the first year of high school (12-15 years old). The classes were conducted over a period of about 4 weeks with two and a half lecturing hours per week. The activities carried out in each week were evaluated at the end of the same week, so that the activities proposed in the following week's lessons could be improved based on the students' reactions. Students' foreknowledge of geometry and mathematics was fairly homogeneous and in general none of them had ever used a programming language in a systematic way.

ACTIVITIES

After brainstorming in the groups and equipping the students with all the initial information about the learning path, the first part of lessons was devoted to the presentation of the working environments. Scratch and Python Turtle programming were presented at the initial part of the course.

The research questions this study attempt to answer, are:

- RQ1: How can learning geometry be fruitful to learn basic programming concepts?
- RQ2: Can computational thinking and engagement in programming simplify learning to solve geometric problems and make teaching geometry to 12-15 years old more effective?
- RQ3: How were the basic concepts of computational thinking grasped by the students after they have attended an introductory course in programming?

Related to RQ3, we use the definition of computational thinking proposed by Selby (2013), namely that it is a focused approach to problem solving, incorporating thought processes that utilize decomposition, abstraction, evaluation, algorithmic design, and generalizations.

Scratch

Brennan and Resnick's study focuses on CT in the Scratch programming environment (Brennan (2012)). The framework is organized along three dimensions: concepts, practices, and

perspectives. Their seven concepts resonate specifically with programming environments. Their computational practices section is strongly focused on students learning to work actively with code. The computational perspectives category focuses on students expressing themselves creatively through computation, enriching computational experiences through interaction with others, and questioning the complicated nature of technology.

This study successfully contextualized CT to math and geometry contexts. Thereafter, teachers explained the Turtle library, and that this library contains commands to move a robot object (an arrow by default) around the screen, and that a set of consecutive commands would eventually constitute a computer code, which could draw a desired geometrical figure on a virtual canvas when executed.

This laboratory activity is designed to help K-8 and K-9 students to understand the basics of geometry and black-based programming by an easy-to-use software. In Scratch environment an action can be considered and executed by assembling blocks of different colors based on their features and functions as in figures 1 and 2. The activity started by introducing Scratch and how to use the blocks. Students then began drawing simple geometrical elements such as points and lines. Once they have drawn an initial example of lines moving randomly, students were challenged to create simple geometric figures. Students first obtained a kind of rainbow of free lines, and then from the blocks used with Scratch, they tried to make first a triangle and then figures with a larger number of sides.

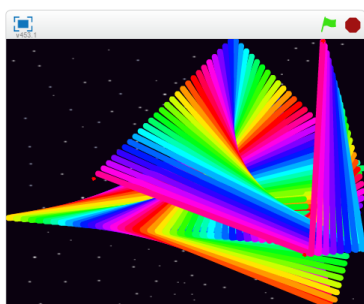


Fig.1

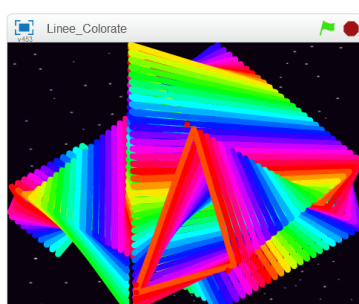


Fig.2



Fig.3

A fragment of the code used with Scratch is given in fig. 3: each point represents a vertex of a geometric figure, and using the reach Point block, the line joining two vertices is drawn, thus obtaining a geometric figure with n-sides.

The 'Reach Point' instruction allows students to draw a line from one vertex to another in the figure. Students drew the 3 sides of triangles using code similar to the one of fig. 3. By Iterating this process they can obtain more complex n-sided figures. Practising geometric concepts, students also learned to understand the principles of Scratch operation and computational thinking.

Python Turtle

Python Turtle (<https://docs.python.org/3/library/turtle.html>) is a library that is part of the Python programming language. It allows users to create graphical applications using a simple and small set of commands. Using Turtle, users can draw shapes and patterns, create animations, and even play games. Turtle is a great way for beginners to learn the basics of computer programming. Turtle is

based on the Logo programming language, which was developed by Wally Feurzeig, Seymour Papert and Cynthia Solomon in 1967. Logo was designed to help teaching children the basics of programming. The Turtle library was created to give Python users access to this same set of tools. Users can exploit Turtle to create shapes, patterns, and even characters (. The library provides a set of commands that allow users to control the turtle's movements. This includes commands such as forward, back, left, and right. In addition, Turtle provides commands for drawing lines, circles, rectangles, and other shapes. Turtle also provides a set of commands for creating animations. For example, users can make the turtle move around the screen in a specific pattern or make it move in a particular direction. They can also create a loop to make the turtle repeat the same action multiple times. In addition to shapes and animations. Turtle includes a set of commands for creating simple games. These commands allow users to control the turtle's movements to create the game. Using Turtle is easy for beginners. The library has extensive documentation and tutorials that provide step-by-step instructions for creating shapes, patterns, and games. Furthermore, the library is open source, so users can access the source code and customize it to meet their needs.

Overall, Python Turtle is a great way for beginners to learn the basics of computer programming, it is easy to use and provides users with a powerful set of tools for geometry learning (Laura-Ochoa (2022) Svistkov (2021); Nanavati (2020); Svistkov (2021)).

This workshop is designed to help K-8 and K-9 students learning the basics of geometry using the Python Turtle library. Students can discover how to draw simple shapes with the Turtle and use its commands to create designs of increasing complexity. The workshop begins by introducing students to the Turtle graphics library and the basic commands used to draw shapes. After the basics are covered, students used Turtle commands to draw a square and a circle. The lab will then move on to more complex shapes such as trapezoid and ellipse and then also a polygon with a variable number of sides and more complex concepts such as drawing patterns, creating complex designs, and drawing 3D shapes. At the end of the lab, students were able to use the Turtle commands to draw any shape they can imagine and create complex designs. They gained an understanding of the basic principles of geometry and were able to apply geometrical concepts just learned to real-life problems (Fig. 4a, 4b, 4c)

```
import turtle

wn = turtle.Screen()

square = turtle.Turtle()

#draw the square
square.forward(100)
square.right(90)
square.forward(100)
square.right(90)
square.forward(100)
square.right(90)
square.forward(100)

#draw the circle
circle = turtle.Turtle()
circle.circle(50)

wn.exitonclick()
```

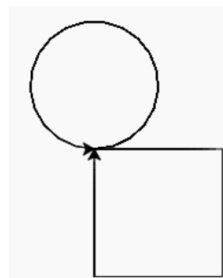


Fig.4 a

```
import turtle

t = turtle.Turtle()

t.penup()
t.goto(0, -50)
t.pendown()

# method to draw trapezoid
t.forward(100)
t.right(60)
t.forward(60)
t.right(120)
t.forward(160)
t.right(120)
t.forward(60)
t.right(90)

# method to draw ellipse
def draw(rad):
    # rad --> radius of arc
    for i in range(2):
        # two arcs
        turtle.circle(rad,90)
        turtle.circle(rad//2,90)

# Main section
# tilt the shape to be horizontal
turtle.seth(315)

# calling draw method
draw(50)
```

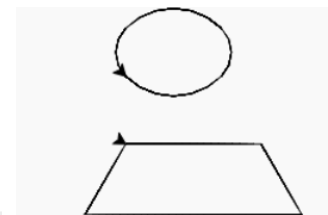


Fig.4 b

```
import turtle

sides = int(input("Enter the number of sides: "))

wn = turtle.Screen()
t = turtle.Turtle()

for i in range(sides):
    t.forward(50)
    t.left(360/sides)

wn.mainloop()
```

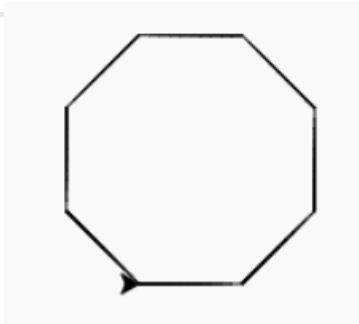


Fig.4 c

RESULTS

At the end of the activities, a questionnaire was administered to students with a twofold purpose. On the one hand, it was intended to assess the improvement of students' skills in plane Euclidean geometry; on the other hand, their reactions and enjoyment of this educational proposal were collected. The following are the most significant comments from some students that describe the general sentiment very well:

Student A (K-8): "It was really interesting to learn through geometry how to be able to use Scratch and Python Turtle Graphics and their fundamental instructions."

Student B (K-8): "We did not imagine that we were able to learn so quickly to use the fundamental functions of programs like Scratch and Turtle Library and to be able to write programs. So geometry seems easier!"

Student C (K-9): "I found that it is super-fast to draw geometric figures, with very few lines of code," Student D (K-8): "At first I had problems, I always got wrong figures, but from the mistakes I learned a lot."

Student E (K-9): "I would like to keep learning this way, it's more fun and I see the results right away." Student F (K-9): "I don't like geometry, and I didn't know that coding could be fun for me. I would like to do homework in class using coding."

The group of 45 students (20 K-8 and 25 K-9), at the beginning of the activities (pre-test), had, in geometry, equal score on average, to the control group. Comparing student's results after teaching activities (post-test), we report the most important result of our study: K-8 group reported an increase on average of 18%, and K-9 an increase on average of 21%.

At the end, students tried to make different geometric figures from the triangle, and then figures with a larger number of sides. A twofold effect was observed, which is a noteworthy result of this work. The use of Scratch helped students becoming familiar with the construction of geometric figures with an increasing number of sides, and the use of geometry helped students learning the basic concepts of programming and understanding the main elements of computational thinking.

CONCLUSIONS AND FUTURE WORK

The results of the pilot study show that the program was successful in introducing to K-8 and K-9 level students some basic programming elements in Scratch and that they were able to apply them to solve problems, besides to achieving a deeper awareness in geometric concepts. The students also developed their spatial awareness and problem-solving skills by using Python Turtle programming. Teachers reported that they found the program to be effective in teaching the students and that it was easy to adapt and apply it to their classrooms. Overall, this paper provides a valuable

resource for teachers in all types of classrooms, as it presents a simple but effective education program that can easily be adapted and applied to introduce basic programming concepts to students. The results of the pilot study demonstrate the effectiveness of the program in teaching by a multidisciplinary point of view and in increasing student's problem-solving skills and their motivation towards geometry with respect a control group. This study is a starting point to research about computational thinking and the interaction between computer science and maths in education from the youngest to college level pupils.

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ACCESS TO DIGITAL DEVICES, SELF-EFFICACY, PROBLEM-SOLVING, AND MATHEMATICAL ACHIEVEMENT: EVIDENCE FROM ETIMSS 2019

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This study contributes to a growing body of research on the access to digital devices in mathematics education. A large number of research studies highlighted the potential of digital technologies in mathematics education. However, there are research studies that found mixed results and the estimated impacts were often small and insignificant. In this paper, drawing data from eTIMSS, we reflect on the impact of three factors on students' mathematical achievement: students' access to digital devices during mathematics instruction, students' self-efficacy for using digital devices, and students' opportunities to work individually on problem-solving. Results from this study indicated the contribution of access to digital devices and self-efficacy for the use of digital devices, and individual work on mathematical problems on mathematical achievement.

Keywords: computer use, mathematics achievement, eTIMSS, individual work on mathematical problems, self-efficacy for the use of digital devices

INTRODUCTION

Digital devices, such as computers or tablets, and mathematical practices play an important role on mathematical achievement (Major et al., 2017; OECD, 2015). TIMSS was first conducted in 1995, and since then is repeated every four years. It seeks to evaluate factors that contribute to shaping learning outcomes, providing data on student performance and factors likely to be related to 4th and 8th grade students' performance in mathematics and science. TIMSS also provides the opportunity to study factors associated with mathematical achievement, in the classroom, school, and home context of students (Knezek & Christensen, 2008; Mullis & Martin, 2017). In mathematics, students are assessed on specific curriculum context (numbers, geometry, data) and cognitive domains (reasoning, applying, knowing) (Mullis & Martin, 2017). Several studies conducted on TIMSS data, found that digital device use has a significant impact on student learning (e.g., Jung & Li, 2021; Namome & Moodley, 2021; Skryabin et al., 2015). TIMSS assesses students' access to digital devices through the teachers' questionnaire with a question about the availability or not of digital devices during mathematical lessons (Mullis & Martin, 2017). Additionally, the individual work on problem-solving tasks refers to the frequency that students claimed that they do mathematics tasks and solve mathematical problems on their own (Mullis & Martin, 2017). Regarding students' self-efficacy for the use of digital devices, TIMSS provides data about the degree of confidence that students declare in the students' questionnaire for digital device use (Mullis & Martin, 2017). The recent computer-based assessment of eTIMSS in mathematics allows the investigation of the use of computers and tablets in mathematical problems.

Therefore, in this study we aim to investigate the effect of 4th grade students' mathematical achievement with the following three factors (1) access to digital devices in math lessons, (2) self-efficacy for the use of digital devices, and (3) opportunity for individual problem-solving work in mathematics. To fulfill this aim we used the data from eTIMSS 2019.

LITERATURE REVIEW

The literature review of the present study is organised in three parts. First, research on the contribution of digital device use to students' outcomes is reviewed, while in the second part, the literature review focuses on the effect of self-efficacy on student achievement in mathematics. The third part addresses the effect of problem-solving on students' mathematical achievement. This review leads to the purpose and research question.

Digital devices use and student achievement

Previous research evidence about the role of digital devices in mathematics learning is inconsistent. Research evidence indicates that access to digital devices at school has a positive relation to students' achievement (Carrasco & Torrecilla, 2012; Simões et al., 2022). This is consistent with research by Namome and Moodley (2021) who used TIMSS 2015 data. Their findings showed that access to digital devices during mathematics lessons had a positive effect on student achievement in mathematics. The positive effect of digital device use during mathematics lessons on student achievement, through TIMSS 2015 data, was also confirmed by Saal et al. (2019). Through TIMSS 2019 data, Ackay (2023) investigated the effect of digital device use in mathematics, with 4th grade students. His findings showed that digital device use is a significant predictor of high achievement in mathematics. Skryabin et al. (2015) using TIMSS 2011 data, showed that digital device use at school was a positive predictor for mathematics achievement of 4th grade students but a negative predictor for mathematical achievement of 8th grade students. In the same vein, Meggiolaro (2017), found a positive effect of digital devices on mathematical achievement when they are used for some, but not all, mathematical tasks. In particular, they indicated a positive effect of digital device use on knowledge, creation of content, and problem-solving tasks. In contrast, the time spent on digital device use for collaboration and communication tasks had a negative effect on the overall students' mathematical achievement. A negative effect of digital device use on mathematical achievement was also found by Eickelmann et al. (2017), who analysed PISA 2012 data for The Netherlands and Germany.

Self-efficacy and student achievement

Regarding self-efficacy, the findings by Simões et al. (2022) revealed that students with better self-efficacy for digital devices have higher achievement. Moreover, Zakariya et al. (2022) investigated the relationship between students' self-efficacy and approaches to learning in mathematics. Their findings revealed that students who have high self-efficacy are more likely to approach math instruction deeply. On the other hand, students who have low self-efficacy were more likely to approach math instruction more superficially. In the context of eTIMSS 2019, Jung and Li (2021) showed that students' self-efficacy is an important factor for their positive learning experience in a computer-enhanced learning environment.

Problem-solving and student achievement

Research findings showed a significant contribution of problem-solving on student achievement in mathematics. Merritt et al. (2017), in a review of experimental studies on the effectiveness of learning, concluded that an emphasis on problem-solving tasks in mathematics is an effective teaching method that contributes to students' knowledge, attitudes, and conceptual development. Similarly to Merritt et al. (2017), Brown et al. (2013) showed that an emphasis on problem-solving has a positive effect on student learning outcomes. In a computer-based problem-solving context, Ku et al. (2007) investigated the effect of the individual work on mathematical word problems on mathematical achievement of 6th to 8th grade students. Their findings revealed that individual computer-based learning focused on mathematical problems, can be used to enhance lower-ability students' mathematical achievement. This is consistent with Hwang's et al. (2012) findings which revealed a positive effect of the emphasis on problem-solving tasks using technology, on student achievement.

Research question

More research is needed to investigate the effect on the mathematical achievement of access to digital devices (Namome & Moodley, 2021), student self-efficacy for digital device use (Jung & Li, 2021), and individual work on mathematical problems (Liu et al., 2019). The research question is: Are there differences in 4th graders' mathematical achievement who participated in eTIMSS based on their (a) access to digital devices during mathematical lessons, (b) self-efficacy for the use of digital devices, and (c) opportunity for individual work on problem-solving in mathematics?

METHOD

Participants

For this study, eTIMSS 2019 data were used. The participants were 4th Grade students, their teachers, headteachers, and parents, from all participant countries. The sample for our study consisted of 20161 students and 2196 teachers from all 30 participating countries.

Data source and variables used

This study used data from the student's assessment booklets, from the student's questionnaire, and from the teacher's questionnaire. During eTIMSS 2019 all students completed the assessment booklet on a computer or tablet. The booklet consisted of mathematical problem-solving tasks. In addition to this, all students completed a questionnaire. In this questionnaire, students responded to questions about their school environment, their attitudes toward learning, and their familiarity with digital devices. Similarly, the teachers of these participating students responded to a teacher's questionnaire. Teachers were asked about their educational background, teaching experience, and professional development. In addition, they responded about challenges they face while teaching, level of preparation they need for their teaching, curriculum subjects they cover, assessment practices, and accessibility to digital devices for instruction.

The mathematical achievement refers to the overall score of each student in the mathematical tasks provided by TIMSS. To calculate students' mathematical achievement, the mean score of the five plausible values was computed (Marchant, 2015). The minimum mathematics achievement score was 215.43 and the maximum score was 785.34. The mathematical achievement score was used as the dependent variable.

For independent variables, we used the relevant responses from the students' and teachers' questionnaires. Specifically, independent variables were (1) the frequency with which students work on mathematical problems on their own during math lessons (student questionnaire), (2) the frequency of access to digital devices during math lessons (teacher questionnaire), and (3) the students' level of self-efficacy for the use of digital devices (student questionnaire). In particular, the individual work on mathematical problems was measured based on how often they worked on problems on their own (1="every or almost every lesson", 2="about half the lessons", 3="some lessons", and 4="never"). The access to digital devices was from the teachers' questionnaire and was measured with a questions about the availability or not of digital devices from students during mathematics lessons (1="Yes", 2= "No"). Finally, students' self-efficacy was measured as a categorical variable: 1= "high self-efficacy", 2= "medium self-efficacy", 3= "low self-efficacy".

Data analysis

The data were analysed using IBM SPSS Statistics (Version 27). To answer the research question a three-way ANOVA was used with dependent variable the mathematical achievement and independent variables the access to digital devices, self-efficacy for digital devices, and individual work on mathematical problems.

RESULTS

The results of the analysis of variance (ANOVA) are shown in Table 1. The access to digital devices, students' self-efficacy for digital devices, and the opportunity for individual work on problem-solving tasks in mathematics had a statistically significant effect on mathematics achievement ($p < .05$).

Predictor	Sum of Squares	df	Mean Squares	F	p- value
Access to digital devices use (A)	104975.49	1	104975.49	15.54*	<.001
Self-efficacy for digital devices (B)	132545.63	2	66272.81	10.34*	<.001

Opportunity for mathematical problem-solving (C)	2264505.67	3	754835.22	111.72*	<.001
A*B	2037.32	2	1018.66	.15	.860
A*C	84192.25	3	28064.08	4.15*	
B*C	135256.67	6	22542.78	3.34*	.003
A*B*C	50337.61	6	8389.60	1.24	.28
Error	136058024.63	20137	6756.618		
Total	5656777140.31	20161			

*p<.05

Table 1: Analysis of variance of access to digital devices, self-efficacy for digital devices use, and the opportunity for mathematical problem-solving

However, the effects of these three variables might have been masked by the interactions between (a) access to digital device use and opportunity for individual work on mathematical problems, and (b) the self-efficacy for digital devices and opportunity for individual work on mathematical problems on mathematical achievement, which were also significant. The interpretation of the results depends on the interaction effects of these three variables. We found that there were statistically significant interactions between access to digital devices and individual work on mathematical problems ($F_{(3, 20143)}=3.58$, $p<.05$) and self-efficacy for digital devices and individual work on mathematical problems ($F_{(6, 20143)}=3.03$, $p<.01$). On the other hand, we found no statistically significant three-way interaction ($F_{(6, 20143)}=1.24$, $p>.01$).

The interaction effect between access to digital devices and individual work on mathematical problems on mathematical Achievement

As shown in Table 2 and Figure 1, when students do not have access to a digital device, the frequency of individual math problem-solving has a greater effect on their mathematical achievement than when students have access to a digital device. However, students who have access to digital devices and work individually on mathematical problems in half of the lessons ($\bar{x}=518.03$) or every lesson ($\bar{x}=542.50$) have higher achievement than students who have not access to digital devices and work on mathematical problems in half of the lessons ($\bar{x}=507.15$) or in every lesson ($\bar{x}=536.50$). Moreover, when students do not work frequently on mathematical problems but have access to a digital device ($\bar{x}=465.59$ and $\bar{x}=505.62$ respectively), access to digital device has a greater effect on their mathematical achievement than when students have not access to a digital device (never $\bar{x}=455.96$, some lessons $\bar{x}=490.08$).

Access to digital device during math lesson	Opportunity for mathematical problem-solving	$\bar{X}$	SD
yes	never	465.59	92.12
	some lessons	505.62	79.95
	about half lessons	518.03	79.39
	every or almost every lesson	542.50	82.03
no	never	455.96	85.79
	some lessons	490.08	79.44
	about half lessons	507.15	82.74
	every or almost every lesson	536.50	85.72

Table 2: Means ($\bar{X}$) and Standard Deviations (SD) of students according to access to digital devices during math lessons and the opportunity for mathematical problem-solving

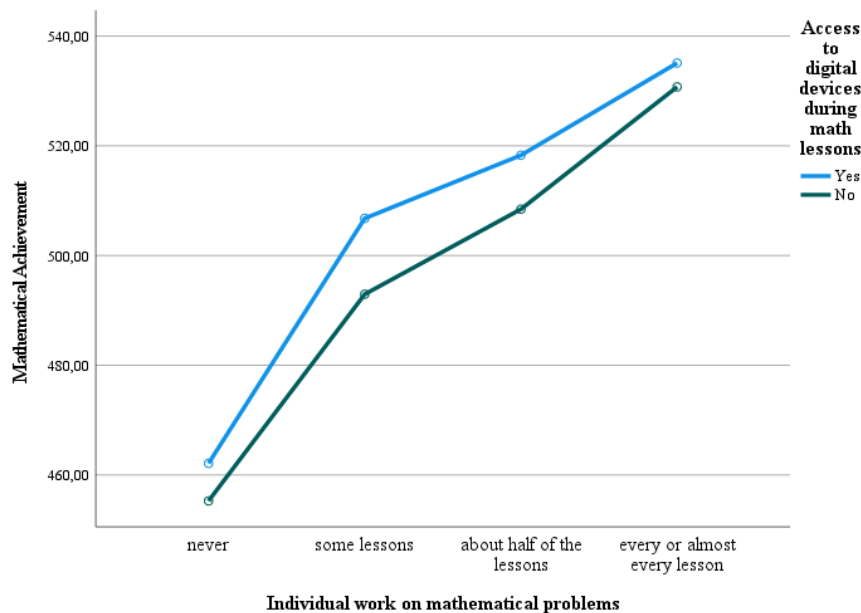


Figure 1: The interaction of opportunity for mathematical problem-solving and access to digital devices during math lessons

The interaction effect between self-efficacy and individual work on mathematical problems on mathematical achievement

As shown in Table 3 and Figure 2, students who have the opportunity to work individually on problem-solving in every or almost every lesson have higher achievement in mathematics. Among these students, those with higher self-efficacy always have higher achievement than those with lower self-efficacy. Among students who do not work individually on mathematical problems, students with higher self-efficacy ($\bar{x}=469.59$) have higher achievement than those with lower self-efficacy ($\bar{x}=444.56$). Furthermore, when students work on mathematical problems more often, their self-efficacy for digital devices has a greater effect on their mathematical achievement. In, particular, students with medium ($\bar{x}=534.83$) or high self-efficacy ($\bar{x}=543.72$) have higher achievement when working individually on mathematical problems every or almost every lesson, than students who work on mathematical problems half of the lessons ($\bar{x}=510.73$ with medium and $\bar{x}=514.28$ with high self-efficacy).

Self-efficacy	Opportunity for mathematical problem-solving	$\bar{X}$	SD
high self-efficacy for digital devices use	never	469.59	85.25
	some lessons	499.89	75.76
	about half the lessons	514.28	78.48
	every or almost every lesson	543.72	80.31
medium self-efficacy for digital devices use	never	459.15	91.82
	some lessons	495.81	82.56
	about half the lessons	510.73	82.82
	every or almost every lesson	534.83	87.68
low self-efficacy for digital devices use	never	444.56	89.08
	some lessons	497.3	85.08
	about half the lessons	510.41	96.33
	every or almost every lesson	515.76	97.53

Table 3: Means ($\bar{X}$) and Standard Deviations (SD) of students according to their self-efficacy for the use of digital devices and the opportunity for mathematical problem-solving

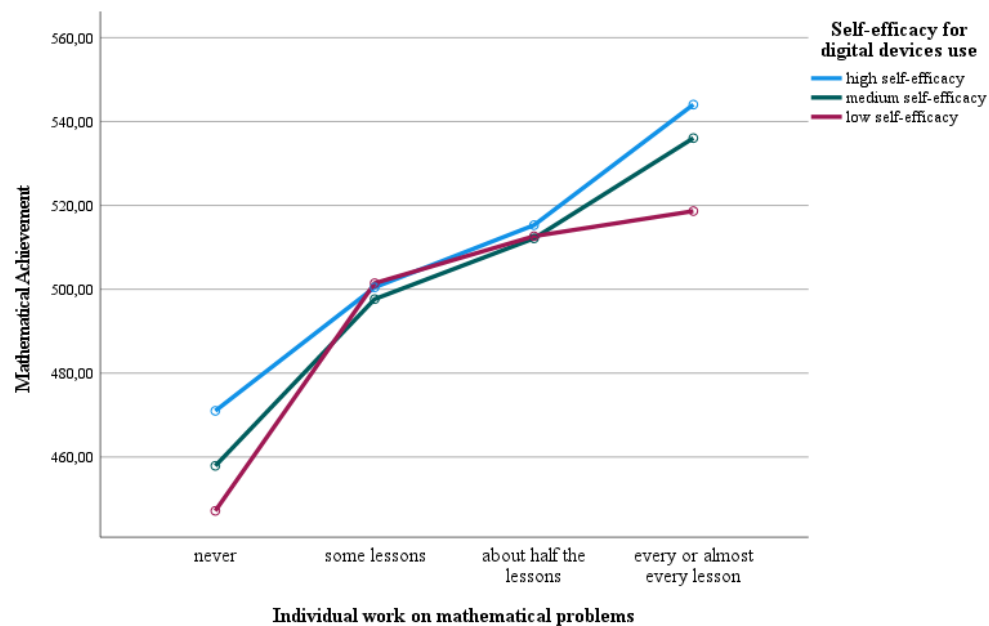


Figure 2: The interaction of opportunity for mathematical problem-solving and self-efficacy for digital devices use on mathematical achievement

DISCUSSION AND CONCLUSIONS

The purpose of this study was to explore the effects of access to digital devices during math lessons, students' self-efficacy for digital device use, and opportunity for individual work on mathematical problems on students' mathematical achievement through eTIMSS 2019 data. The findings emphasise suggest that access to digital device during mathematics lessons, self-efficacy for digital devices, and the opportunity to individually work on mathematical problems affect students' mathematical achievement.

In line with previous studies, our findings confirmed that access to digital devices (Ackay, 2023; Namome and Moodley, 2021; Saal et al., 2019; Simões et al., 2022) has an effect on mathematical achievement. The results also confirmed that individual work on problem-solving tasks is significant for student achievement (Brown et al., 2013; Merritt et al., 2017). Furthermore, the results revealed that students who had access to digital devices during math lessons had higher achievement in mathematics when they worked on mathematical problems frequently. This occurs even if students who do not have access to digital devices. Opportunity to work individually on problem solving positively affected their mathematical achievement. However the level of mathematical achievement of those with access to digital devices always surpassed that of students who did not have access to digital devices. This finding highlights the significant positive effect on students' achievement when they work individually on mathematical problems and also have access to digital devices.

According to the results on students' self-efficacy for digital devices, the findings are consistent with previous studies (Jung & Li, 2021; Simões et al., 2022; Zakariya et al., 2022). The results revealed a significant effect of students' self-efficacy for digital devices on mathematical achievement. The findings also showed that students with high self-efficacy for digital devices have higher achievement in mathematics. Moreover, it appeared that students with medium or high self-efficacy have higher achievement when they work on mathematical problems individually more frequently. Whereas the achievement of students with low efficacy on digital device use is not affected if they work sometimes or frequently on mathematical problems. This result suggests that

higher self-efficacy on digital use can prove more beneficial for students' mathematical achievement especially if they are frequently given the opportunity to work individually on problem-solving.

A limitation of this study is that, although TIMSS secondary analysis gives the opportunity to study multiple factors on a large scale and from several countries at the same time, it does not provide rich information about the type and context of digital devices used in the mathematics lesson. Another limitation refers to the variables used for the analysis. The three independent variables were taken from two different questionnaires, the student's questionnaire and the teacher's questionnaire. Future studies can shed more light on investigating the context in which digital devices are used in mathematics classrooms and identify the most productive and appropriate uses for the development of mathematical knowledge and competencies.

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REPRESENTING DISTANCE-TIME SCENARIOS IN A DIGITAL EMBODIED LEARNING ENVIRONMENT

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The purpose of this study was to explore how students use the feedback provided by a digital embodied learning environment to graphically represent qualitative and quantitative aspects of distance-time scenarios and overcome typical graphing mistakes. Fifty-nine 11-year-old-students were asked to construct graphs for two scenarios and got feedback in form of an animation of the corresponding time-distance scenarios. The results indicate four different categories of how students used this feedback to effectively revise their work. Further, the analysis shows that students' interaction with the learning environment was beneficial since several students exhibited a covariation approach to distance-time scenarios when interpreting their graphs and grasped that the two quantities are varying simultaneously and every value of one quantity determines exactly one value of the other.

Keywords: covariation; functional thinking; embodied digital tools; graphical reasoning

INTRODUCTION

Researchers still struggle to find effective ways of integrating technology into mathematics teaching and learning (Polly, 2014). This also holds true for the area of functional thinking and graphical reasoning. It is a timely issue to examine how portable and handheld digital technologies offer opportunities for enacting embodied learning in functional thinking situations that require representing functional relations (Abrahamson & Bakker, 2016). Indeed, contemporary digital tools provide learning environments for understanding graphs of change and motion that incorporate students' own motion experiences. The present study focuses on students' competency of graph construction and to what extent motion experiences help students to evaluate the correctness of their graphs. The goal of this study is to empirically investigate how students use the feedback of a digital and embodied learning environment to represent distance-time scenarios and overcome typical mistakes. To address this question, we explore how 11-year-old students revise their graphs that represent distance-time scenarios after watching the animation provided by the learning environment.

THEORETICAL BACKGROUND

Covariation and Graphical Reasoning

Distance-time scenarios mainly entail a covariation approach to functional thinking. The covariation approach is mostly studied by contextualized dynamic functional relationships (Thomson & Carlson, 2017). It is viewed as a way to make sense of relationships of dependence, causation, and correlation between quantities, and thinking in terms of rates of change. Creating the graph of a dynamic functional relationship requires reasoning about changes of dependent and independent variables, and the direction of these changes. Moreover, translations between natural language and graphical representations to model dynamic functional situations and imaging and coordinating the simultaneous changes of the involved values are crucial (Ellis, 2011).

The covariational reasoning framework, proposed by Thompson and Carlson (2017), suggests that students should be able to think about how two quantities vary and to understand that the two quantities vary simultaneously. Indeed, students' ability to reason with quantities and relationships fosters their functional thinking (Moore et al., 2013; Pittalis et al., 2020). Covariational reasoning

involves the mental coordination of the values of two quantities. Covariational reasoning can contribute towards developing graphical reasoning, since it entails assimilating a graph as a trace in progress (Moore et al., 2019). However, the interpretation of graphical representations that describe dynamic scenarios can be challenging for students (Friel et al., 2001). Discerning among discrete and continuous representations of change and differentiating between the shape of a graph and the characteristics of the scenario pose various difficulties.

Graphical reasoning encompasses both graph interpretation and construction. Graph interpretation consists of three essential processes: recognizing visual features of a graph, interpreting relationships represented by these features and connecting the identified relationships with what the graph represents (Shah & Hoeffner, 2002). For distance-time scenarios, students should be given the opportunity to connect the physical situation (i.e., own motion experiences) with visual elements of the graphical representation and vice versa. To do so, two instructional approaches have been suggested. The first one emphasizes on quantitative or local aspects of graphing, while the second one underlies the importance of grasping qualitative or global aspects. The latter one highlights the graph's general shape as it facilitates visualizing the relationship between the quantities before constructing a graph, as shapes of trends are mapped onto the graphs' axes. It relates to Castillo-Garsow et al.'s (2013) description of thinking about the relationship between two variables as continuously changing. This type of thinking models the notion of motion and facilitates mapping young students' everyday experiences with motion to the abstract concept of continuous change. Research showed that students encounter difficulties when graphically represent changes over time, e.g., when interpreting a graph as a representation of a real event or when plotting points of a graph without considering the values in between and consequently, without understanding that the graph represents a relationship between continuous covarying quantities (Thompson & Carlson, 2017).

Embodied Learning Environments for Graphing Motion

A systematic review by Duijzer et al., (2019) shows that embodied learning environments, which immediately link students' own motion with the mathematical representation provided by a digital tool, are effective in supporting students' understanding of graphs. To become familiar with the graphical representation of scenarios involving covarying quantities, it would also be possible to use dynamic simulations. Therein, students can either focus on single points to connect different types of representations or manipulate variables and observe the consequences. In the latter possibility, students can observe the effect of a systematic variation in all representations simultaneously, which makes the covariation of quantities perceptible. A digital environment that links a graph with an animated motion in real-time provides a valuable entry-point into reasoning about continuous change represented in the distance-time graph, because the motion observed corresponds to a time value at every point in time. For the case of graphing motion, digital learning environments can be categorized in respect to bodily involvement and immediacy (Duijzer, et al. 2019). Bodily involvement can be distinguished between own motion and observing others/objects' motion. Immediacy is defined in terms of immediate and non-immediate. An immediate task provides a simultaneous interaction with the physical environment, while the graph is plotted, whereas in the second case this interaction is based on an embodied simulation. Research showed that embodied learning environments, which make use of students' own motion immediately linked to its graphical representation were more effective (Duijzer, et al. 2019). In the present study, we turn attention on students' understanding of graphing motion, as they observed and influenced an objects' motion in which the bodily involvement (of the animated object) took place in the absence of direct environmental stimuli.

THE PRESENT STUDY

The purpose of this study was to investigate how students use the feedback provided by a digital embodied learning environment to represent qualitative and quantitative aspects of co-varying quantities (distance-time scenarios) and overcome typical graphing mistakes. The research questions are: (a) How do students use the feedback provided by the digital learning environment to revise their graphs and (b) to what extent does students’ interaction with the digital environment facilitate the emergence of a covariation approach to distance-time scenarios?

Participants, Intervention, Evaluation and Procedure

The participants of this study were fifty-nine 11-year-old-students, 28 girls and 31 boys from three Grade 5 classrooms. Consent to participate in this study was given by the students and their parents. They exhibited a broad spectrum of academic achievement levels. Students had used tablets in mathematics several times before, but they had not previously been taught functional relationships involving measures that covariate simultaneously.

As intervention context, the research team designed a module with an emphasis on the covariation aspect of functional thinking consisting of four 40-minute lessons. The lessons were delivered by one of the members of the team in a two-week period. They were designed based on four principles: situatedness, inquiry-based learning, embodiment, and utilization of digital tools. The module included embodied activities that required conceiving co-varying quantities, representing graphically distance-time scenarios (with paper and pencil and digitally), and animating with bodily movement distance-time graphs that were developed in the framework of the FunThink project. While working on the module, all students were given an activity worksheet and tablets with the online applet *Turtle Crossing* from the Desmos platform (desmos.com). The applet presents a turtle that walks away from the sea and students had to make connections between several turtle-crossing scenarios and the respective distance-time graphs. In the present study, we used the functionality of the applet to draw a distance-time graph and then watch an animation of the turtle’s corresponding journey (see Figure 1 right). The user can pause the animation, scroll the play-slider back and forward, and observe in the form of a vertical line the time-correspondence of the animation with the graph. When the user draws the graph, a sign displaying the position of the turtle in the animation-setting appears.

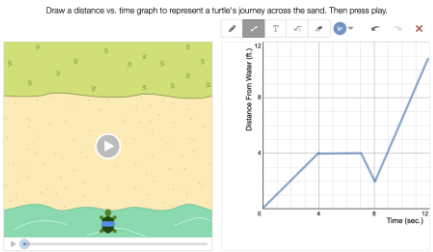
Turtle-crossing scenarios	Desmos environment
Task 1 focuses on global aspects of the graph: The turtle moves away from the sea. Suddenly, it stops for a while. Then, it continues moving away from the sea. Task 2 focuses on local aspects of the graph: The turtle moves 8 ft out of the sea in 4 seconds. It pauses for 2 seconds. It then returns to the sea in just 2 s.	

Table 1: Description of Tasks in the digital Learning Environment

Two weeks after the completion of the intervention program, each student was interviewed in a session that lasted approximately 20 minutes to evaluate students’ learning. Students were asked to construct four graphs for given scenarios in the *Turtle Crossing* applet. They were informed that they could revise their work based on the feedback provided by the animation. Two of the scenarios required conceptualizing global aspects of the graph, while the other two involved grasping local aspects. In the present study, we analyze students’ work in two of these tasks (see Table 1 left).

Data Collection and Analysis

This study used qualitative methods for data collection. We videotaped students' work on the tablets to capture their actions and oral explanations while working. A qualitative interpretive framework was used in the analysis of the data (Miles & Huberman, 1994). We used the constant comparative method to compare students' work and formulate categories of students' ways to use the provided feedback and explanations. The categorization was developed within an iterative back and forth process, that was both literature-based as well as data driven.

RESULTS

We addressed the first research question of the study by examining how students revise their graphs based on the feedback provided by the digital learning environment. We examined in depth the sequence of students' actions that did not construct a correct graph based on the given scenario from their first attempt. We present here the categorization of students' ways of utilizing the provided feedback based on our analysis: (a) Delete the whole graph and start over after watching the animated turtle trip (without identifying which are the correct and the wrong parts of the graph), (b) delete and revise the whole graph or parts of the graph to correct the wrong parts of the graph, by focusing mainly on the animated turtle trip (keeping or reconstructing the right parts of the graph), (c) delete only the perceived wrong part of the graph and revise it by interpreting the effect of this part of the graph on the animated turtle trip, and (d) validate the graph section to section and revise selected parts by controlling the enactment of the route through the use of the play/pause button of the animation. Below, we illustrate the four categories with students' answers.

In Task 1, 32 out of the 59 students revised their work (the remaining students provided a correct graph in their first attempt). Twelve of them correctly revised their graph after one or two corrections, fifteen after multiple attempts and the rest of them did not provide a correct answer. The students that provided a correct graph after multiple attempts or did not manage to do so could mainly be assigned to the first two categories, while their revising procedure took a long time. This may have resulted from the fact that students revised their work based solely on evaluating whether the animated trip of the turtle corresponds to the given scenario, without making direct links between each part of the graph with the animated trip and the scenario. Figure 1 presents Helen's attempts to construct the graph in Task 1. Helen's work is indicative of students that used the first two categories in an attempt to revise their graphs. Their work suggests an iconic perspective of graphs or difficulty to coordinate the different directions of change of the two quantities. First, Helen constructed the graph shown in Figure 1i(a). Then, she selected the play button to see the animated turtle journey. When she was asked to comment on the animation, she responded that the turtle disappears instead of staying still. She explained that the disappearance of the turtle was due to the fact there was a blank space between the two parts of the graph. Afterwards, she selected the "delete all" button of the app and started over to create a new graph. She constructed the graph presented in Figure 1i(b). While watching the animated turtle movement, she did not figure out the new trip immediately, so she decided to watch the animated video two more times. Then, she commented that the turtle moves forward faster than staying still, without trying to match the different parts of the graph to the turtle's move. She started over again and constructed a graph representing the stop interval as a vertical line, explaining that leaving blank space does not work, as the turtle disappears or moves fast (Figure 1i(c)). Again, she watched three times the animated video, without noticing the difference of this video to the previous one and commented that the turtle flashes out and then runs fast again. This time, she explained that the graph should stop and start again and constructed the graph presented in Figure 1i(d). After watching the new animation twice, she concluded that she should have represented the blank space in the graph in a different way because her construction made the stop invisible. Then, she made a different graph, by representing the stop interval with a dotted line parallel to the horizontal axis. When she validated

the new graph, she explained that it matches the scenario as the turtle stays still for a while. When the interviewer prompted her to rethink why the turtle was flashing on and off during the stop, she answered that flashing intended to emphasize its stop.

Figure 1(ii) presents the graphs made by Stella in Task 1. Stella's work is indicative of students that deleted only the perceived wrong part of the graph and revised it by interpreting the effect of the wrong part of the graph on the animated turtle trip (third category). First, Stella constructed two diagonal segments to represent the scenario "the turtle moves forward" and left a blank space for the turtles' stop (see Figure 1ii(a)). After watching the animated video twice, she explained that the turtle disappeared instead of making a stop for some seconds. When asked to explain the gap in her graph, she answered that the turtle should stay in the same position for a while, thus, she decided to "continue the line after a while from the same position" (indicating the corresponding y-value). Then, she used the segment tool and constructed a dotted horizontal line to represent the turtle's stop (Figure 1ii(b)). Watching the animated video helped her conceptualize that the dotted line resulted in the appearance and disappearance of the turtle. Then, she used the "eraser" tool to delete the dotted line and sketched a horizontal segment instead, explaining that "I used previously a dotted line for the stop, but it did not work, when the line of the graph stops the turtle disappears, I must make a segment like this (indicating a horizontal direction) to connect the two parts that the turtle moves forward".

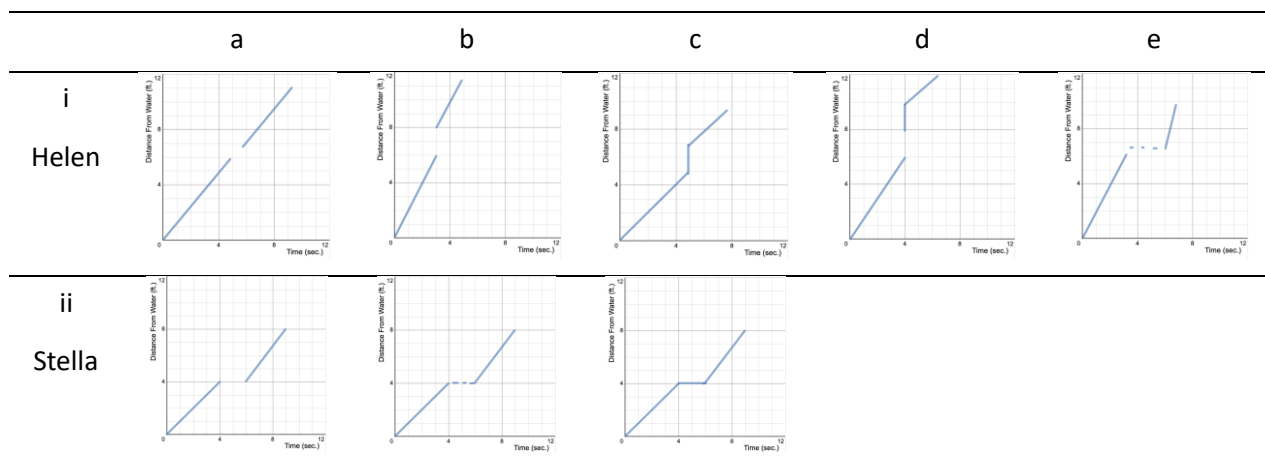


Figure 1: Helen and Stella's graphs in Task 1

In Task 2, 35 out of the 59 students revised their work. Twelve of them correctly revised their graph after one or two corrections, seventeen after multiple attempts and the rest of them did not manage to provide a correct answer. Figure 2 presents Andrew's graphs in Task 2. Andrew's work is indicative of the second category, as he revised only the part of the graph that did not correspond to the scenario. First, Andrew constructed the graph presented in Figure 2i(a), as he represented the scenario "returns to the sea" by extending the line graph to the origin. After the first attempt, he watched the animated video three times because he was surprised by the fact that the animation included two turtles and thought that there was a bug in the app. He explained that he could not figure out the problem and the best option was to delete a part of the graph. Then, he deleted a segment of the graph and watched the animation of the new graph (Figure 2i(b)). He explained that he realized what was going wrong and that one part of the graph (Figure 2i(b)) should be kept, as it represents the first two parts of the scenario and add a third segment to show the return of the turtle to the sea. However, he did not interpret the effect of the segment he deleted to the route of the turtle and the reason of the appearance of two turtles in the first graph. Consequently, he used a trial-and-error procedure to add a third segment to the graph (see Figures 2i(c)-(e)). After watching the animated video of the last graph, he explained that his graph was correct, besides the fact that the duration of the return of the turtle was six instead of two seconds. It seems that as the graph with a

vertical segment did not work, he opted to construct a diagonal segment till the last value of the axis, emphasizing the direction of the segment and ignoring the numerical constraints (global view of the graph).

Zoi's work is indicative of the fourth category, as she efficiently used the play/pause button of the animation to revise her graph. Zoi's first attempt showed that she conceptualized the statement "the turtle returns to the sea in just 2 s" as the turtle reaches the sea in the 2 seconds (see Figure 2ii(a)-(b)). Zoi watched the animated video many times and paused the video, when a second turtle appeared. When asked to explain why two turtles appear from the second to the sixth second, she scrolled the slider of the play button back and forward and observed the vertical line in the graph that shows the time-correspondence of the video and the graph. She observed that during this time-interval (the appearance of two turtles) the vertical line crosses the graph twice. Then, she deleted the wrong segment, played the video, and paused at the 6th second. Afterwards, she started constructing a diagonal segment to represent the return of the turtle and watched at the same time the dotted line in the left screen that displays the position of the turtle along the time (see Figure 2ii(c)-(d)). Based on the feedback, she completed the graph. She played again the video and used the pause button at the fourth, sixth and eighth seconds to validate her graph in terms of time-accuracy.

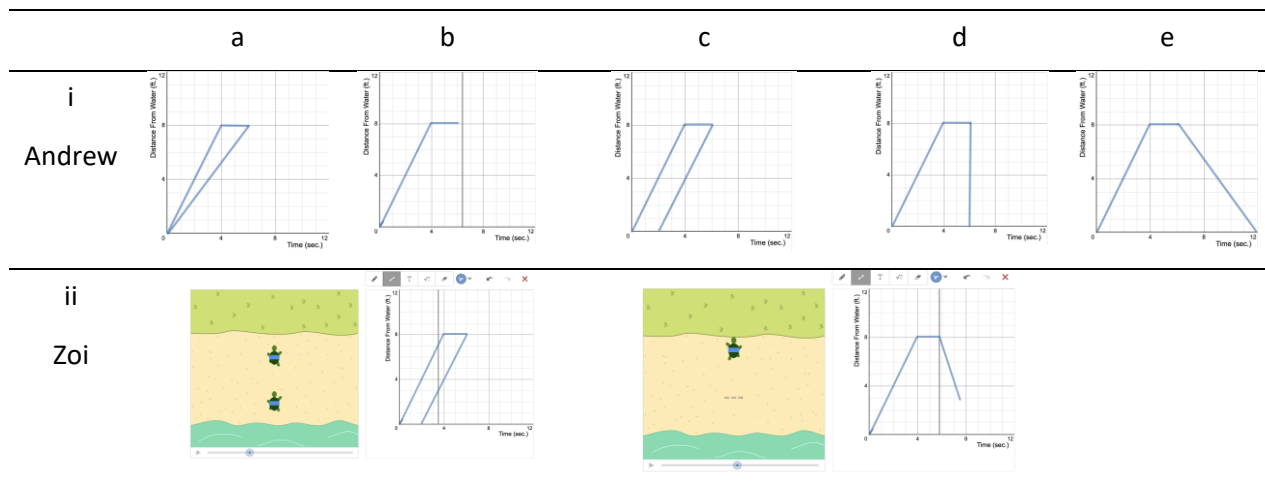


Figure 2: Andrew and Zoi's graphs in Task 2

The second research question examined to what extent does students' interaction with the digital environment facilitate the emergence of a covariation approach to distance-time scenarios. The analysis showed that specific features of the app facilitated conceptualizing that (a) the two quantities (distance and time) vary simultaneously, (b) every value of one quantity determines exactly one value of the other, and (c) the quantity of time is constantly increasing, while the quantity of distance may increase, decrease, or stay constant. Below, we present examples of such instances. In Task 1, several students represented the situation "the turtle stops for a while" by leaving a blank space or making a dotted line. While watching the animated trip of the turtle, students observed that leaving a blank space or a dotted line results in the disappearance of the turtle from the screen. Students commented that "I thought that if there is a space between them [meaning the two other parts of the graph], the turtle would stop, but it disappeared, that's not possible, we cannot disappear the turtle, we are not magicians, the turtle should appear from the beginning to the end", "I killed the turtle, poor turtle, the turtle should keep going or just stand watching all the time", and "what happened to turtle?... I left a blank space for two seconds to show that it rests for two seconds, but it did not work, I wanted to show that the distance is the same for two seconds". We conclude that these three examples indicate an intuitive understanding of the fact that distance and time should vary simultaneously in this scenario, as students mention that the turtle should appear

throughout the video. In Task 2, the most common mistake was to represent the return of the turtle by constructing a segment from right to left (see graphs in Figure 2ii). In that case, two turtles appear in the animated video. Students shared the following interpretations: “Oh no... two turtles, when did it give birth? ... it seems that the app gets two positions of the turtle in the same second and gets confused”, “I played the video back and forward, I made this [showing the back segment] to show that the turtle returns, but two turtles appeared from the beginning, the second turtle shows the return...wrong timing”, “it is wrong, I managed to return time back! Look [displaying a vertical line for the third second], in the third second the turtle had to be in two places simultaneously, so it copied itself”. These interpretations indicate an emergent understanding of the fact that every value of one quantity determines exactly one value of the other quantity. In terms of the specific scenario, in each time-point, the turtle can appear in only one distance point away from the sea. A variety of explanations emphasized that time runs, while distance increases when the turtle moves forward and decreases when it returns to the sea. These explanations provided evidence of students’ interpretations using their own wording that the quantity of time is constantly increasing, while the quantity of distance may increase, decrease, or stay constant.

DISCUSSION

The contribution of this study lies on the empirical examination of the way students use the feedback provided by a digital embodied learning environment to represent qualitative and quantitative aspects of distance-time scenarios. The analysis showed four different categories in respect to the extent to which students used the provided feedback, in the form of an animated video of the constructed graph, to effectively revise and interpret their work. Revising based on the first way included deleting the whole graph and starting over after watching the animated trip. The difference of the second way compared to the first one was editing only the wrong parts of their graph. The third way was different in a qualitative way, as students revised the wrong part of the graph by interpreting the effect of this on the animated trip. Finally, the fourth way took full advantage of the environment functionalities, such as utilizing the play/pause button to validate the graph section by section. Further, the analysis showed that students’ interaction with the learning environment was beneficial since several students exhibited a covariation approach to distance-time scenarios. They connected the animated video with the constructed graph. Their interpretation showed an understanding that (a) the two quantities (distance and time) are varying simultaneously, (b) every value of one quantity determines exactly one value of the other and (c) the quantity of time is constantly increasing, while the quantity of distance may increase, decrease, or stay constant (Thompson & Carlson, 2017).

These results could be valuable for educators, curriculum, and software developers as the study showed the potential of embodied digital learning environments to connect graphs with a corresponding real-life animation. It seems that the animated video contributes to an emergent intuitive understanding of difficult mathematical concepts, such as the covariation approach to functional relationships. For instance, observing that a single turtle cannot be in two different positions at the same time helped visualizing that every value of one quantity determines exactly one value of the other. Further, the study highlights the need of adopting appropriate didactical approaches and the role of the teacher to maximize the learning benefit for students (Polly, 2014). Several students did not use all the functionalities of the learning environment, validated superficially their work based on the animated video, and did not attempt to develop an in-depth understanding of the effect that each part of the graph has on the turtle’s trip. Targeted guidance and appropriate questions would prompt students to link the available representations and reason about the effect of the form and the direction of each part of a graph in a real-life scenario and grasp the corresponding limitations in terms of the involved varying-quantities. This teaching approach

could exemplify the importance that learning environments hold as means to concretize important mathematical concepts.

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THE IMPACT OF TEACHING SCRATCH ON GROWTH OF COMPUTATIONAL THINKING OF SEVENTH GRADE STUDENTS

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In recent years many educational systems around the world start to integrate some sort of computational thinking (e.g. programming and coding) in their school curriculum. In Iran educational system, can be identified vein and roots of computational thinking in both school, curriculum and after school program. In current study we investigate the potential benefits and possibility of using Scratch as blocky programming in enhancing middle school students' capabilities in computational thinking. So, the main purpose of the study was to measure the effect of programming with Scratch on the level of computational thinking of 7th grade students. In this study 50 seventh grade students from two lower secondary school) were participated. Results of analysing students' scores upon Dr. Scratch show that the teaching blocky programming could improve seventh grade students' capability in CT.

Keywords: Scratch, Blocky Programming, Computational Thinking, Middle School Students, School Curriculum,

INTRODUCTION

One of the latest educational perspectives is to introduce computer programming in classrooms from K-12 for developing students' computational thinking which defined by Wing (2006). So that today, governments in many countries around the world have made decision to put programming in their national curriculum. Indeed, human beings in society have become more and more technology-based in the 21st century and must have appropriated knowledge to perform their work in a more efficient manner, in which such a society is heavily integrated with technology (Bocconi et al., 2016). One of the important aspects that everyone has to know is Computational Thinking (CT) as Wing (2006) suggested, "to reading, writing, and arithmetic, we should add CT to every child's analytical ability" (p. 33). Shodiev (2014) defines CT as a way of thinking algorithmically using design trees from computer science as a guiding structural, and sometimes metaphorical, framework. Hoyles and Noss (2015) consider CT as abstraction, algorithmic thinking, decomposition, and pattern recognition. CT involves concepts (e.g. loops, conditions) and practices (e.g., abstraction and debugging) (Lye & Koh, 2014; Kafai & Burke, 2013). However, CT is not simply programming and rote skill, but it is a conceptualizing and fundamental skill; a way that humans think (Wing, 2006).

CT changes the nature of some contemporary research in the mathematics domain. For example, we can see the computer-based proof in mathematics (e.g. four-color theorems). For another example, a new domain of research related to mathematics and computation such as Bioinformatics emerged recently. In this regard, the European Mathematical Society (2011) recognized an emerging way of engaging in mathematical research: "Together with theory and experimentation, the third pillar of scientific inquiry of complex systems has emerged in the form of a combination of modelling, simulation, optimization, and visualization" (p. 2). Weintrop et al. (2016) try to address CT as a more sophisticated concept upon literature review and interview experts who use CT in their carrier. They develop a taxonomy of CT skills which has close relation with the third

pillar of scientific inquiry. This taxonomy contains four main categories: data practices, modelling and simulation practices, computational problem-solving practices, and systems thinking practices.

Educational system in many countries start to introduce computational thinking in their students at K-12. For example, coding and programming integrated into the school curriculum at Japan, Canada, Cyprus, Greece, Italy, Luxembourg, Hungary, UK(England), Slovakia, Portugal, Ireland, France (Balanskat and Engelhardt, 2015). Investigation of the Iranian school curriculum show that computational thinking gradually inters to some grade textbooks. For example, there is a chapter about building a computer game using Scratch software in the grade nine “work and technology” textbook (Esmaili et al., 2020). Recently, from 2022-2023 school year, we can see some sort of programming and coding in the textbooks at several grade levels. However, there were several after school programs about computational thinking from several years ago, especially in math houses.

In the current study we focused on Scratch as blocky programming and use it at a lower secondary school to examine this research question. What is the effect of Scratch (as a tool for blocky programming) on fostering computational thinking of seventh grade students? This is a part of bigger study which investigate the impact of Scratch (as a tool for blocky programming) to enhance probabilistic thinking of seventh grade students. Although in current study, we focused on fostering computational thinking of seventh grade students and don't refer to mathematics in particular, but we have to keep in mind that there are many common aspects between computational thinking and mathematics. For example, Shute, Sun, and Clarke (2017) mentioned full set of shared concepts of computational and mathematical thinking as problem solving, modeling, data analysis and interpretation, and statistics and probability.

LITERATURE REVIEW

One of the pioneers in using programming in education was Symour Papert who created the famous and well-known programming language named Logo. In Papert's (1991) constructionism perspective, learning means building relationships between old and new knowledge in interactions with others while building collective artifacts. Papert mentions in his writings that one day he was passing through an art class at the university, and he saw how the students were trying to make different shapes and forms by cutting of a bar of soap and from it. From that moment, Papert started to think about how we can use such experiments in math classroom. Indeed, from that time the idea of Logo software development began. Papert and Harrell (1991) have described the concept of constructionism, that is, learning by making. Papert (1980) briefly described the early evolution of the idea of laboratory microworld such as turtle geometry, which was a mathematical land where certain kinds of thinking occurred on that. Mathematics could develop with particular ease in these "small lab worlds," each with its own set of assumptions and constraints. In doing so, students learn to transfer habits of discovery from their personal lives to the formal domain of scientific theory building.

Recently the ideas of Papert to use programming as microworld alive again in a new soul which is blocky programming such as Scratch. In this type of programming, students in all age and grade level could engage in programming project and construct their own thinking and learn mathematics and other subjects which are important for 21st citizen. Martin's (2016) case study, specifically investigating the effectiveness of the Scratch programming language for exploring computational thinking concepts and methods in sixth grade students, provided promising evidence that a well-designed student task or assignment, It can provide all students with the opportunity to develop the ability to use computational thinking concepts, and students who use Scratch are easily able to quickly acquire computational thinking concepts and methods. In Martin (2016) study, many students who worked on a programming project with minimal help from their teacher, gained

experiences in the field of computational thinking concepts and methods only by spending four hours training. In another study, Maloney et al. (2008) examined Scratch as a tool for students in the computer Clubhouse organization, by examining 536 Scratch projects and analyse student interviews. They concluded that students' coding abilities and understanding of computational thinking concepts were developed through the using of Scratch.

In current study we use Brennan & Resnick (2012) theoretical framework for studying and assessing computational thinking which used by several studies in literature (e.g. Xue and Liu, 2021). This framework summarized at table 1.

Key components of CT	Concept	Definition
Computational thinking concepts that were used in Scratch	Sequences	Sequences is a series of individual steps or instructions that can be executed by the computer.
	Loops	A sequence can be repeated in a loop for a certain number of times or an unlimited.
	Events	What happens on a computer can cause something else to happen. Events are an essential component of interactive media.
	Parallelism	sequences of instructions could happening at the same time.
	Conditionals	Conditionals – the <i>if</i> block – means the ability to make decisions based on certain conditions, which will be caused multiple outcomes.
	Operators	Operators enabling the programmer to perform numeric and string manipulations.
	Data	Data involves storing, retrieving, and updating values. Two types of data in Scratch are as below. • variables (which can maintain a single number or string) • lists (which can maintain a collection of numbers or strings)
Computational thinking practices that were used in Scratch	Incremental and Iterative	Coding is not a clean, look like final program that work and well-design. Instead, it is an adaptive process, in which changed gradually through feedback and new ideas.
	Testing and Debugging	A developed program usually does not work immediately as written, but finding logic errors and debugging is an essential part of coding.
	Reusing and Remixing	Coding upon available programs and developing the work of others projects.
	Abstracting and Modularizing	Abstracting and modularizing means to build large program with using several small programs.

Table1. Framework for Assessing of CT (Brennan & Resnick, 2012)

METHOD

The current study had adapted through a quasi-experimental quantitative method (one group rpe-test post-test). In this regard, 50 seventh grade students from two lower secondary school in city of

Rafsanjan in Kerman province (at south-east of Iran) was selected randomly for participating at this study. Data collection occurred in 2020-2021 school year. In the time of data collection, the pandemic of covid-19 virus exists in all over the world and schools in many countries provide lessons in virtual mood for students. In Iran, also like other part of the world, students participated at virtual class. They use their special platform namely SHAD which designed at national level for supporting teaching and learning process of Iranian students. In this special platform (SHAD) all teachers and students have their own account. Teachers organised online teaching through this special platform (SHAD) and set assignment for the students and then students follow the instruction and complete their assignment. All types of assessment and test also designed in the SHAD and students follow their teacher instruction to pass the exam and course.

For data collection in this study, we also used SHAD as our platform. All of students announced that they didn't have any sort of knowledge or experience in programming and coding. Almost all of them have the same level in their education progress (according to their scores in math and science). All fifty students at experimental group participate in four 45-minute online sessions which became familiar with basic information about Scratch blocky programming and its capability for designing different program and animation. All of the class time video recorded and uploaded in www.apparat.com which is a platform for producing and sharing videos in Iran. So, students have access to the content of the online class as offline lesson in any time through www.apparat.com. We also share videos in educational network of SHAD for further accessibility of these materials.

After each training session, the students make a program about their creative ideas in their projects and sent them to the researchers. The researchers saved their projects and analysed them with the help of Dr. Scratch program, which examines the students' projects and assigns a computational thinking score. Authors saved and recorded the score of students in an excel file. In the results section, brief results of the students will be reported.

In literature, there are several tools for assessing computational thinking, for example in Román-González, Moreno-León and Robles (2019) writers try to review evaluation framework in domain of computational thinking and then introduce a comprehensive model intends to assess CT. However, in current study we use Dr. Scratch as a assessment tools of CT which designed upon component of Brennan & Resnick (2012) framework. Dr. Scratch (Moreno-León et al., 2015) is a free and open-source web application that analyses, in an automated way, projects programmed with Scratch language. The score that Dr. Scratch assigns to a project is based on the degree of development of seven dimensions of CT competence which are abstraction and problem decomposition, logical thinking, synchronization, parallelism, algorithmic notions of flow control, user interactivity, and data representation. These dimensions are evaluated through analysing project codes and given points from 0 to 3 and results will be reported in a total at ranges from 0 to 21. Upon final results, Dr. Scratch consider three level of progress which are basic, developing and master level. In the basic level, students get 0-7 points at total. In the developing level, students get 7-11 points at all seven dimensions of CT competence. In the master level, students get 14-21 points at total. Furthermore, Dr. Scratch also generates some feedback for students to improve their abilities in programming.

RESULTS

Descriptive analysis of student score upon Dr. Scratch tools shows that most of the students' capabilities in programming has grown well (see table 2). As shown in table 2, 86% of the students who participate at this study scored by Dr. Scratch at developing and master levels of computational thinking, which show that using blocky programming (in this case Scratch) has a positive effect on the growth of the level computational thinking of seventh grade students.

	Level of CT	Number of Students	Percentage of Students
	Basic	7	14 %
	Developing	24	48 %
	Master	19	38 %
Sum		50	100 %

Table 2. Level of students CT capability scored by Dr. Scratch

In order to get a general summary of the number of changes in the computational thinking capabilities of all students, the computational thinking score of all projects that students submitted at each stage was calculated with the help of the Dr. Scratch program. Then, the average computational thinking scores were calculated in each stage, and in total, to show the progress of the students, The broken line graph of the average computational thinking scores in each stage was drawn (See diagram 1).

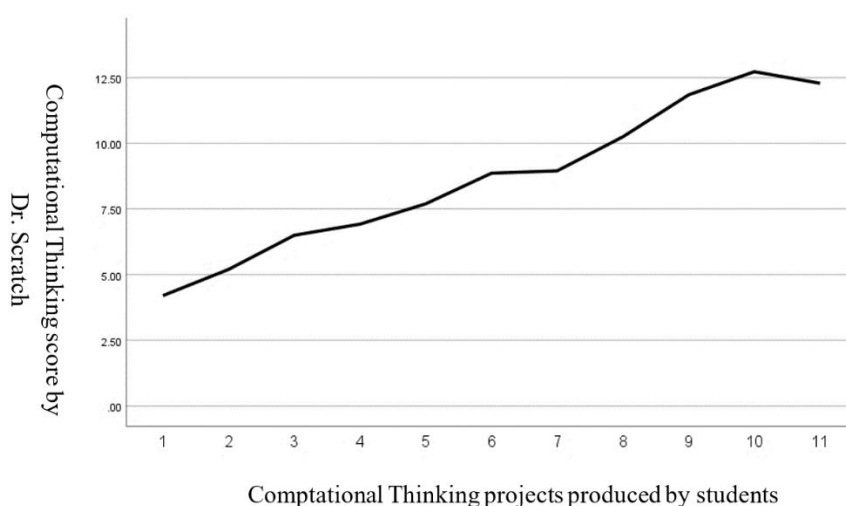


Diagram 1. The broken line diagram of the average changes in the computational thinking score of all students scored by Dr. Scratch

The ascending graph in diagram 1 generally shows the progress of computational thinking of the participating students. Therefore, according to this evidence we can claim that the teaching blocky programming could improve seventh grade students' capability in CT. Figure 1 shows one of the projects that students design and implement it by Scratch. In this project, students design a game which with the help of the racket we have to prevent the ball from hitting the red line, and every time we stop the ball, we get a point. After passing time, the game becomes harder and harder, and the ball moves faster. In this programming, students use loops and conditionals with appropriate operations to create harder stages for this game. In such complicated programming, students integrate computational and mathematical thinking for their creation.

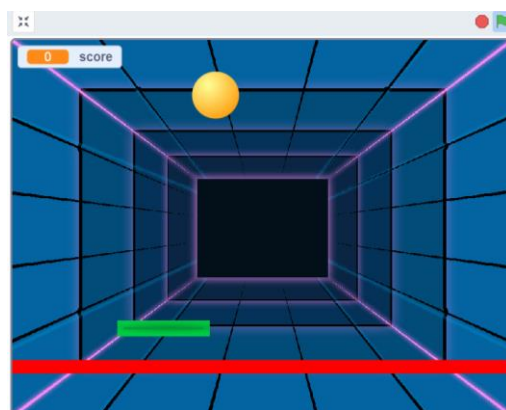


Figure 1. Game of racket and ball

DISCUSSION AND CONCLUSION

One of the important things that everyone citizen in the 21st century should be known is computational thinking. In this regard, after reading, writing and arithmetic, we need to add computational thinking to every child's analytical ability. CT is not only related to computer science, but it is also located in the heart of many other sciences such as mathematics, science, engineering, etc. A suitable platform for developing students' computational thinking is blocky programming (e.g. Scratch). In the current study, with a constructionism point of view, we attempt to develop a course for teaching Scratch programming to seventh grade students and examine gradually develop of students' programming skills and computational thinking abilities. Upon table 2 and diagram 1, we can see that the score of students in their CT related project (which provide by Dr. Scratch) dramatically improved after only four 45-minute session training. The results of this research are similar to the other research from literature review (e.g. Maloney et. al., 2008, Burke, 2012, Martin, 2016, and Wolz et. al. 2011). In more details Maloney et. al. (2008) pointed that participants They never realized that they were programming, but they actually do it. This situation repeated in current stusy. Wolz et. al. (2011) say that almost all of participants found Scratch as an interesting things which is reported by current study participants.

Furthermore, students who participate in this study become an autonomous learner and start to create new programming projects. Sometimes this progress of students surprised researchers because most of the students have limited access to technology tools, but they motivated to make more advanced programming project with Scratch and share their ideas with others. Many educational system defined autonomy as one of their final goals and try to obtain such qualified output which is motivated for self-learning to prepare for lifelong learning.

Finding of this research comes with some limitations. First of them was related to virtual education which spread during the pandemic of Covid-19 virus. In this time, researchers have limited access to the students because all students follow their lessons from home and the teachers had less influence in the process of learning of students. Second limitation was small number of training sessions which were four 45 minuts sessions and this time is quite short. Third one returned to online assignments and students may have taken help from their elders or others and this lack of direct observation of the students by teachers' effects on the results of this study.

According to the research paths that was taken in Iran and other part of the world, it can be concluded that the research on Scratch programming education is generally growth more and more and provides many benefits such as improving programming learning, enhancing problem solving skills and create more positive motivation to enhance students learning.

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TECHNOLOGY INTEGRATION BY PRIMARY-SCHOOL TEACHERS IN RURAL MEXICO AND THE DESIGN OF A PROFESSIONAL DEVELOPMENT PROGRAMME

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In this paper we describe parts of a (pre-pandemic) research where we investigated the access and use of DT for mathematics by primary school teachers in a rural region of Mexico, in order to design a professional development (PD) course for such teachers. The research included: First, a diagnostic phase to investigate teachers' previous training in the use of DT and their access to these resources. Then, a design-based study where we designed, implemented and refined a PD course for integrating DT in the teaching of mathematics. Concurrently, we examined some of the participating teachers' uses of DT. We describe here some of the results from that research, which show the difficulties in access that such teachers face, how they orchestrated the DT available for mathematics lessons, and the need for teacher training that takes into account their context, reality and possibilities.

Keywords: Technology-use in developing countries, primary-school education, instrumental orchestration, teacher training

INTRODUCTION AND THEORETICAL FRAMEWORK

In developing countries, such as Mexico, there is a digital gap, that needs to be addressed (Sunkel et al., 2014), and schools are ideal spaces for this purpose (Peres & Hilbert, 2009). In fact, the integration of digital (DT) resources to the teaching of mathematics is a social and pedagogical necessity (Sunkel et al., 2014). However, deficiencies have been observed in that regard in sectors of the Mexican primary education system. In Mexico, and more so in rural areas, the use of digital technologies (DT) in schools, particularly in state primary-schools, is scarce. One problem is the lack of hardware and software. In fact, in 2021, it was found that fewer than 45% of Mexican homes had a computer (INEGI, 2022), with most people relying on their mobile phones for connectivity (e.g., WhatsApp, which became how most teachers and students communicated during the COVID pandemic), and only around 50% of people used the Internet in rural areas (INEGI/SCT/IFT, 2021).

Another problem is that teachers lack professional development (PD) for the integration of DT to their teaching practice, and particularly in terms of its didactical use: in fact, as could be seen in the syllabus of the official teacher training programs (SEP, 2018), DTs were given very little importance in primary-school teacher training.

Moreover, in the past fifteen years, there have been few successful initiatives for DT integration in Mexican state schools. The last significant programmes by the Ministry of Education, were in the early to mid-2000s –the Enciclomedia programme for primary education, where schools were equipped with computers and smartboards, and interactive apps accompanied the official textbooks; and the EMAT (Teaching Mathematics with Technology) programme that introduced, among other tools, Spreadsheets, Dynamic Geometry and Logo programming for mathematics in middle-school education (see Sacristán & Rojano, 2009; and Trouche et al., 2012). But after 2006, DT in state schools, and the training of teachers on their pedagogical use (such as for the teaching of mathematics), became very limited or non-existent.

In this paper we describe parts of a (pre-pandemic) research where we investigated the access and use of DT for mathematics by primary school teachers in a rural region of Oaxaca, Mexico, in order to design a professional development (PD) course for such teachers. The research included: First, a

diagnostic phase to investigate teachers' previous training in the use of DT and their access to these resources. Then, a design-based study where we designed, implemented and refined a PD course for integrating DT in the teaching of mathematics. Concurrently, we examined some of the participating teachers' uses of DT. For that analysis, several theoretical approaches were used: that of Instrumental Orchestration (IO) (Drijvers et al., 2010); Hughes's (2005) categories on technology use; and, for examining the changes in teachers' participating in the PD courses, the Documentational Approach to Didactics (DAD) (Gueudet & Trouche, 2012).

The IO was used to analyse the way in which teachers organized and used digital resources in their mathematics classes. In particular, we examined teachers' IOs by analysing their didactic configuration of the existing artifacts and their arrangement in the teaching environment; the expected method of exploitation of said artifacts; and the didactic performance of the teacher in the class using the planned technology. For this, we relied on the six ways of orchestrating technology identified by Drijvers et al. (2010) –technical-demonstration, explain-the-screen, link-screen-blackboard, discuss-the-screen, detect-and-show, and sherpa-at-work– and also identified other orchestrations.

Hughes' approach (2005) identifies three types of technology use: (i) as a replacement (without changes in the way of teaching, promoting the same learning processes and goals) ; (ii) as an amplifier (class tasks are carried out more efficiently and effectively, even though they are the same); and (iii) as transformative (changes are made in teaching practices, in student learning routines, and in the roles of teacher and students).

The DAD studies teachers' practice and their professional development by looking at their interaction with the resources they use (select, adapt, review, reorganize) for teaching mathematics, which is called a documentation work (Gueudet & Trouche, 2012). This work produces documents, which are composed of recombined resources and the usage schemes associated with them; a teacher's set of documents is their documentation system (DS). Throughout a teacher's professional career, their DS evolves as they work and experiment with old and new resources. Thus, the teacher's documentation work reveals their professional development, that is, the evolution of their practice, knowledge and beliefs (Gueudet & Trouche, 2012).

THE FIRST PHASE: IDENTIFYING ACCESS AND USE OF DT

In the diagnostic phase of our study, in 2017, we explored the access and use of digital technologies (DT) for the teaching of mathematics by primary school teachers in the region of our research. In 10 schools of that region, we interviewed each school principal, and 67 teachers answered a survey, in order to inquire on the availability and use of DT resources, as well as teachers' perceptions of their potential for mathematics learning. We then observed some of the lessons of ten teachers who had stated in the survey that they did use DT to teach mathematics and interviewed those teachers.

We classified the results in terms of three categories: (i) Access to DT (both in schools and in teachers' homes); (ii) Perceptions of teachers and principals on the use of DT for the teaching of mathematics; (iii) DT use for mathematics teaching. This information was considered, in the later intervention phase, for the design of the PD course aimed at promoting the integration of DT for mathematics teaching.

In terms of access, most schools lacked hardware; there was old, mostly abandoned, Enciclomedia hardware from 2005-06 that was present in most schools. The ratio of students per computer ranged from 11.5/1 at one extreme, to 170/1 at the other. Internet access was limited (of the five schools where Internet access existed of them it was of very poor quality and two of the other schools lacked the equipment to take advantage of it. Added to that, school policies limited the use of what little is available. Thus, DT access was limited (see Fig. 1).

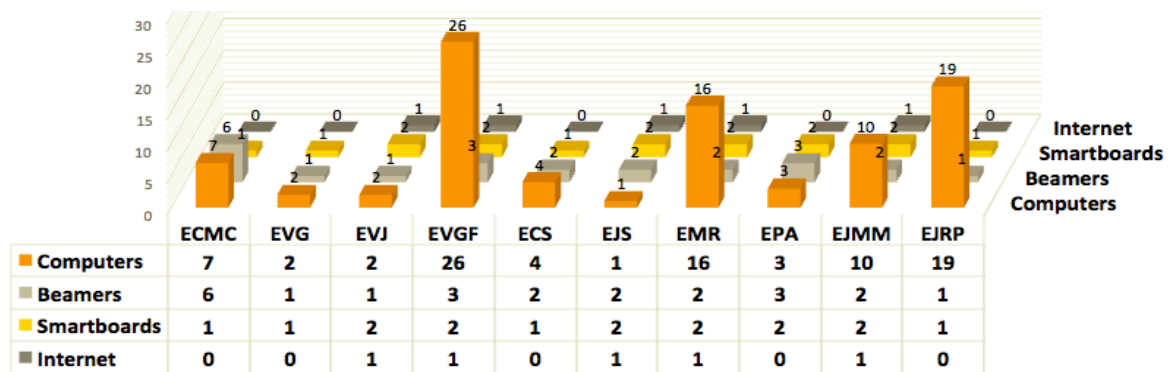


Figure 1. Available DT hardware in the observed schools.

In terms of teachers' perceptions, DTs were seen as something positive that they would use "if they had them" and "had the knowledge to use them". Most participating teachers were unaware of the potential of DTs for mathematics. In fact, the majority of the teachers, had little previous training in the use of DT for teaching mathematics: only 25% had taken courses related to the use of technology, although many were mostly technical and did not focus on its didactic use.

Thus, their use of DT in their practice was scarce, and inconsistent. However, some teachers tried to do their best with what was available. Through the survey we found that, for mathematics, 21% (14) said they never used it, 24% (16) rarely, 44% (29) occasionally and 11% (7) almost always. When asked in the survey about the type of resources they had used for mathematics and the activities carried out with them, 61% (41) of the teachers answered. Of these, 32% (13) indicated that they used videos to support their explanations of a topic (e.g., to explain procedures); 34% (14) used beamers to project images, problems or exercises; another 34% (14) used interactive apps (e.g., apps from Enciclomedia or others, such as for addition and subtraction); 24% (10) printed materials for the class downloaded from the Internet (e.g., riddles, problems, geometric drawings); and two teachers (5%) mentioned having used the calculator.

Class observations allowed us to examine further the ways in which these teachers used and orchestrated the digital resources (videos, projected information, interactive apps, printed material and calculators) in their mathematics classes. We present information from nine of the ten observed teachers, whom we called (using pseudonyms): Armando (Ar), Alicia (Al), Alfredo (Alf), Paola (P), Carmelo (Ca), Cecilia (Ce), Rosi (R), Alexa (Alx), and Hortensia (H). Table 1 summarizes the type of orchestrations that we identified for each of the resources, as well as the types of uses based on Hughes's (2005) categories. A detailed analysis of the orchestrations (Drijvers et al, 2010) of two of the teachers is presented in Enríquez and Sacristán (2019).

Resource	Teachers	Observed IOs	Types of uses
Video	Ar, Al, Ce, H	Explain-the-topic; explain-the-screen; repeat-the-screen	Replacement & Amplifier
Projected information	R, Alx, H, Ca	Copy-the-screen; discuss-the-screen	Replacement & Transformative
Interactive app	Alf, Ca, H	Technical-demonstration; explain-the-screen; Sherpa-at-work	Amplifier
Calculator	Ca	Detect-and-show	Amplifier/ Transformative

Printed material	P	-	-

Table 1. Orchestration and types of use of the digital resources used by teachers.

Five teachers (Ar, Al, Alf, Ca, Ce) used a laptop and a beamer; three (R, Alx, H) took their students to a computer room (with one or two children per computer); and one (Ca) provided calculators to each student. We observed the use of videos in four cases: three of them (Ar, Ce and H) orchestrated it in the Explain-the-topic mode, using it to provide an explanation of the topic, either to remind students of a concept already seen (e.g., angles –H’s case); present a new one (e.g., mean, median and mode –Ar’s case); to identify procedures (obtaining mean, median and mode –Ar’s case; conversions from fraction to percentages –Ce’s case). Other ways of orchestrating the video were Explain-the-screen and Repeat-the-screen: for example, teacher Al would stop the video to explain a part of it again, and then carry out with the children an activity suggested in the video (make the largest or smallest amount possible given four figures on cards). These orchestrations show a use of technology as a replacement and amplifier, since it was used to replace and/or expand the teacher's explanation, with illustrations and animations to optimize the activity.

In the survey, 34% of teachers used beamers to project information (e.g., a document or a web page). Four of observed teachers used this in class: in three cases (R, Alx and Ar), the orchestration was Copy-the-screen, carried out in a computer lab: the children copied the information shown on the computer screen (e.g., representations of fractions, angles) into their notebooks –we considered this use of DT as a replacement. Four other teachers used the projection to pose a problem (with response options); in the case of Ca, children, with the help of the calculator, found and discussed the solutions. We considered this mode of orchestration as Discuss-the-screen: using DT to validate, compare and experiment, activities that could be considered as transformative (Sandoval, 2017).

The use of interactive apps was pointed out by 34% of the teachers. In two of the observed cases (Alf & Ca) these resources were orchestrated as: Technical-demonstration, where students were given guidance on the use of the equipment; as Explain-the-screen where how the app worked was shown; and in Sherpa-at-work mode, where some kids led their teams and used the apps. In Alf and Ca’s orchestrations, there was also an amplifying use of technology: in both cases, mental calculation exercises were encouraged together with the interactive apps use in order to promote the attention and participation of the children.

The calculator was another resource pointed to, in the survey, by 5% of the teachers. It was used by one of the observed teachers (Ca), where the orchestration was Detect-and-show, since he asked some children to present their solutions to multiplication tasks and contrast them with those of others (i.e., promoting activities of comparing, experimenting, and validating), although without discussing the reasoning involved. This DT use approached a transformative use of DT.

Finally, another resource was printed material downloaded from the Internet. We observed P doing this for the topic of addition and subtraction. For 24% of teachers this was a use of DT, but strictly speaking, DT is not used in the class.

In summary, the way that the participant teachers used DT was mainly as a replacement and amplifier, rather than as transformative; this has been noted by others (Santiago, et al., 2013; Trigueros et al., 2014). But we consider that DT resources that teachers used have limited possibilities for enhancing mathematics learning. The videos and the projected information were used, in general, to make the students listen, watch and transcribe explanations and information of some topic, without providing possibilities to experiment to do mathematics (a replacement type use). On the other hand, the interactive apps had an amplifying use, since they were used to practice skills (mental calculation), but they did not promote the exploration and understanding of concepts. Regarding the use of the calculator, since it only allows operations with quantities, its didactic use

depends on the design of the activities that the teacher does (in Ca's case, it approached a more transformative use). In any case, it was necessary to promote teacher training and access to DT.

THE PROFESSIONAL DEVELOPMENT PROGRAMME

After the diagnostic phase, we designed and implemented a PD course seeking to address some of the challenges identified in that study for DT integration. As design-based research, the PD course was refined through two cycles; that is, we implemented two versions of the PD course with different cohorts over a two-year period. The 67 teachers who participated in the first study were invited to participate in that PD course and associated study; 15 of them accepted for the first cycle. Each PD course lasted 5 months (6 hours per week).

For the design of our PD programme, we took into account the schools' and teachers' context in this rural area of Mexico. We designed a course that would have official validity, in a venue close to the teachers' schools and/or homes. We chose a few offline (i.e., not requiring an Internet connection) digital resources, that could run on the old hardware available in the region's schools. These had to be easy for teachers to use, allowing for a significant and constructionist implementation in class, for "doing mathematics" rather than learning about mathematics (Papert, 1971).

The PD course consisted of three modules, each focused on studying a certain DT resource – respectively, diverse interactive apps for specific topics (including the Enciclomedia apps); dynamic geometry (GeoGebra); and Logo. The interactive applets in the first module deal with specific curricular contents and are easy to manipulate. GeoGebra helps address many geometry activities of primary-school. Finally, Logo also helps address the compulsory curricular contents of primary-school mathematics, through programming –allowing children to express themselves (construct), be creative, and develop computational thinking (an important skill in today's world). As explained in Sacristán & Enríquez (2020), we chose Logo over the nowadays popular Scratch, because it has less technical requirements, allows a more straightforward access to Turtle Geometry (allowing students to engage more directly in math-related tasks), and could be run offline on the available computers.

We didn't just introduce teachers to digital resources; rather, we aimed to promote a use that was transformative (Hughes, 2005), changing teachers' pedagogical practices and students' learning-processes and roles in the classroom. Thus, the work with teachers during the PD programme attempted to follow constructionism principles (Papert, 1991; Kynigos, 2015): e.g., by having tasks that would promote active, collaborative and autonomous exploration and expressivity of the learners. The tasks of each module were presented in four stages: (i) The study of the resource; (ii) the design of mathematics lesson plans integrating the digital resources studied in the course; (iii) the implementation of the designed classes; and, (iv) group reflections on the implementation experiences. Thus, teachers and trainers engaged in discussions of strategies for using the DT resources, taking into account the material limitations (e.g. of hardware), in terms of group organization, didactical interventions, etc.

For data collection, we used: (i) initial interviews of the teachers, and subsequent ones after each implementation of their lessons using DT; (ii) the lesson plans designed by the teachers during the course; (iii) in-class observations of the implementations of the designed lessons; (iv) the teachers' presentations of their class experiences, which they shared, during the PD course, with their colleagues. We then used the DAD approach (Gueudet & Trouche, 2012) to study the changes in the participants' documentation systems (DS). Details of this are beyond the scope of this paper, but a comparison of one teacher's DS (see Table 2), before and after the PD course, is presented in Enríquez and Sacristán (2020).

Stage	Activity	Aim	Resources	Teacher's actions
Initial DS (without DT)				
1	Solve problems in teams	Assess and review previous knowledge	Official activities and guidelines; Printed materials. Internet. The children's settings. Stories.	Following the resolution processes
2	Share solutions (whole group)	Deal with weaknesses and reinforce knowledge	Children's solutions Videos and concrete materials Exercises and problems	Guides Sets exercises Uses resources Decides problems for M3
3	Solve problems individually	Promote an expected learning	Official activities and guidelines; Printed materials. Internet. The children's settings. Stories.	Guides Deals with difficulties Sets problems
4	Solve textbook tasks	Assess the learning	Textbook	Guides. Assesses. Closes theme.
DS1: "Capacity measures [of liquid containers]" interactive app				
1	Pose questions on liquid volumes (liter, milliliter)	Assess and review previous knowledge	Questions	Pose questions & assess answers
2	Solve in teams problems on capacity of liquid containers	Promote an expected learning	Bottles, pails. Worksheet.	Guides
3	Solve individually problems on capacity of liquid containers	Promote an expected learning	"Capacity measures" App. Official activities. Worksheet.	Guides Deals with difficulties
4	Solve textbook tasks	Assess the learning	Textbook	Guides. Assesses. Closes theme.
DS2: Heights of triangles with GeoGebra				
1	Build triangles in teams (to describe characteristics)	Assess and review previous knowledge	Tangram Geometry set Colored pens and paper	Assess students' descriptions
2	Recognize the height of a triangle (individually)	Assess and review previous knowledge	Geometry set Pen and paper	Guides Reinforces knowledge
3	Solve individually the GeoGebra activity on heights of triangles	Promote an expected learning	GeoGebra Official activities. Worksheet	Guides Deals with difficulties
4	Solve textbook tasks	Assess the learning	Textbook	Guides. Assesses. Closes theme.
DS3: Constructing polygons with Logo				
1	Team game (characteristics of regular and irregular polygons)	Assess and review previous knowledge	Polygon bingo game Polygon comparison table	Designed and showed a comparison table of polygon characteristics
2	Draw regular polygons (individually)	Promote an expected learning	Geometry set Pen and paper	Guides
3	Construction of polygons with Logo (in pairs & individually)	Promote an expected learning	Logo Worksheet	Guides Deals with difficulties

Table 2. Changes in the documentation systems (DS) of one teacher (Enríquez & Sacristán, 2020)

As explained in Enríquez & Sacristán (2020), we observed that the integration of digital resources, as well as the training (the PD course), generated modifications to teachers' DS, with resources being substituted, or used in combination with previous ones. Each digital resource presented new possibilities (of implementation, knowledge and even motivation), as well as some limitations: The interactive app was easy to use and was adapted to the curricular content, although the learning tasks were restricted; GeoGebra was difficult to use, but allowed the teachers to design materials; and Logo brought about more drastic changes, because it modified how the curricular content was approached, which was also difficult for teachers (see also Sacristán & Enríquez, 2020).

FINAL REMARKS

Our study showed, first, some of the difficulties in DT access and training that many teachers in rural areas of developing countries face, as well as their use and orchestrations (Drijvers et al., 2010) of the available DT in their classes. The design-based study of a PD course allowed us to analyse how teachers integrated into their DS, the digital resources (e.g. GeoGebra, Logo) proposed during the course, finding that reaching a transformative use (Hughes, 2005) is complex but necessary. In order to have meaningful DT integration, it requires the collaboration of teachers, trainers, researchers and authorities, as well as access to hardware, with a development of teachers' pedagogical technology knowledge (PTK) (Thomas & Palmer, 2014).

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TINKERING WITH ANIMATED MODELS OF LETTERS: INSIGHTS IN THE USE OF VARIABLE

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In this paper we discuss the generation of 9th grade-students' meanings of the concept of variable as a generalised number and their competence in using mathematical properties while tinkering with animated models of letters of the Greek Alphabet with the programming tool MaLT2. Using a combination of the 'Teaching for Robust Understanding' Framework and the '5Es' Framework in designing innovative activities, the study explores students' mathematical experience by focusing on how students use the concept of the variable in the mathematical properties embedded in the dynamic models of letters, and provides some insights into their misconceptions about the role of the variable as a generalised number.

Keywords: Mathematics, variable, programming, debugging, animated models.

INTRODUCTION

In this study the author investigates the meanings that students generate about the concept of the variable while tinkering with animated models of letters of the Greek alphabet and negotiating the use of mathematical properties embedded in the letter models in the programming environment of the digital tool MaLT2. The research doesn't aim to study how students learn a new mathematical concept or property, but how they put into use a concept or property in a context based on mathematical activity with generalised models through programming and engineering (Kynigos & Diamantidis, 2022). The variable plays a fundamental role in mathematics, but when it comes to its use as a generalised number, research shows a number of difficulties or misunderstandings for students in both algebra and geometry. The problems stem from the lack of the essential basis of the definition and functional role of the variable, leading to students' inability either to abstract and generalise or to deal with variables in other contexts of mathematics beyond algebra (Dede, 2004). At the same time, in geometry, students are reported to struggle with the Pythagorean Theorem or trigonometric numbers, when the use of variables is required (Rudi et al., 2020; Orhun, 2004). Some episodes are presented specifically about the use of the variable, which can have multiple meanings in mathematics, while both the tool and the designed activity are based on the use of the variable as a generalised number in geometry. The design of the activity, which involves the construction of dynamic models of letters using the programming tool MaLT2, was based on the approach that refers to mathematics as "ideas" rather than fixed "products", as usually found in school curriculum (Li & Schoenfeld, 2019) and considers knowledge as an active construction by the learner through the exploration and the expression that the digital medium offers. The capital letters of the alphabet and the programming environment have also been used in the past to explore students' meaning making around the concept of ratio and proportion (Noss

& Hoyles, 1996; Psycharis & Kynigos, 2009). However, in this study, the researcher paid attention to different mathematical properties that could be embedded in the selected letters from the secondary school (variable as generalised number, Pythagorean theorem, trigonometry), without separating the content knowledge into different chapters and classes. Beyond this aim, the author made an attempt to deploy the Teaching for Robust Understanding (TRU) framework (Schoenfeld & the TRU Project, 2016) with the use of digital technologies, by using it in combination with the pedagogical framework of '5Es' (Benton et al., 2016; 2017) in designing innovative learning activities based on programming, and analysing students' mathematical experience during their interaction with the programming tool from an extended perspective.

THEORETICAL FRAMING

The Teaching for Robust Understanding (TRU) Framework

The theoretical framework of Teaching for Robust Understanding (TRU) provides a way of identifying powerful learning environments by shifting attention from the lesson content to the creation of experiences for students to be developed into powerful thinkers and problem solvers in class (Schoenfeld & the TRU Project, 2016). According to this, the quality of a learning environment depends on the extent to which it provides opportunities for students in the following five dimensions: *Dim.1: The Content (The Mathematics)* refers to the quality of students' mathematical experiences and the opportunities provided for them to make connections with previous concepts or procedures and to develop strong mathematical thinking, *Dim.2: Cognitive Demand* consists of the challenges ('productive struggle') for students to be actively engaged in mathematics and to make sense of important disciplinary ideas and their use, *Dim.3: Access* involves the meaningful engagement and participation of all students in the learning process, *Dim.4: Agency, Ownership*

- *Identity* concerns students' willingness to get involved, ownership (feeling of control) of content and acquisition of a mathematical identity as people who can do mathematics through opportunities to generate and share ideas or arguments and to build on others' ideas and vice versa, *Dim.5: Formative Assessment* relates to the responsiveness of the environment to students' thinking. It is clear that the five dimensions are interrelated, rather than isolated, and according to the TRU framework, all dimensions must be met for a learning environment to be considered powerful.

The '5Es' Framework

The pedagogical framework of '5Es' (Benton et al., 2016; 2017) offers a way of thinking about and designing activities with programming media, by incorporating a constructionist approach to learning. It was developed in ScratchMaths project and has a clear focus on children's engagement with mathematics and programming. The '5Es' model consists of the following unordered phases that provide opportunities for students to: 1) *Explore* ideas by taking control of their own learning, debugging errors and looking for reasons behind different outcomes, 2) *Explain* and discuss their own ideas and being encouraged to use the programming language as a 'tool to think with', 3) *Envisage* by predicting outcomes before creating or running the code and then reflecting on and comparing the actual outcome with their prediction, 4) *Exchange* and defend their own ideas and debug or build on others' perspectives, 5) *bridgE* by making links between the programming environment and the mathematics through a reconstruction of ideas.

Based on the fundamental principle of constructionism, where students generate meanings as they construct or tinker with digital artefacts and discuss their mathematical ideas in relation to these artefacts and their behaviour (Kynigos, 2015; Papert, 1980) and considering that the TRU framework does not propose a specific way of teaching or learning with the use of digital technologies, in this research, the author tried to investigate the TRU framework with the use of a programming medium combined with the pedagogical framework of '5Es' in order to provide students with opportunities to generate meanings through a different way of interacting with the mathematical properties, compared to the siloed approach of the textbooks. Of course, there is not a strict one-to-one correspondence between the TRU dimensions and the '5Es' phases, as the TRU dimensions overlap and the '5Es' phases are combined at many points in the learning process. On the one hand, the dimensions of the TRU framework provide valuable aspects for teaching mathematics, as they are designed to be easily understood and open to the teacher or researcher. However, they wouldn't show a deep understanding of students' mathematical experiences or details of students' ideas during their interaction with the programming tool. On the other hand, the '5Es' phases provide this deeper insight into the mathematical experience through opportunities for students to engage with and construct meanings with and through the tool, but they remain focused

on these elements and don't refer to the overall value of the learning process or the role of the teacher. Therefore, the researcher discusses the use of a combination of these two theoretical frameworks as a better perspective for the creation of a strong learning environment based on programming for the development of children's mathematical thinking and competence to use mathematics in different contexts and, thus, she addresses the following research question: How do students use mathematical properties and, in particular the concept of the variable, while they are tinkering with animated models in the programming environment of MaLT2?

DESIGN OF THE RESEARCH

The activity in the programming tool MaLT2

MaLT2 is an online tool for symbolic expression in mathematical activities through programming for the creation of 2D and 3D graphical models, recognized by the Greek Ministry of Education. It integrates the UCBLLogo language with the affordance of dynamic manipulation of procedures with variables (Kynigos & Grizioti, 2018) and provides three interconnected representations: (i) symbolic, (ii) figural, (iii) dynamic manipulation of generalised variable values through sliders (Fig.2c). Its affordances allow the construction of animated models, where students have to apply mathematics in such a way that the models maintain their properties as the values of their variables change. The designer of the activity decided to take advantage of the mathematical properties that can be embedded in the construction of the Greek capital letters N, Z, Σ, M,

& in a programming environment because, firstly, they are not typical mathematical shapes and therefore do not immediately reveal their mathematical nature to the students, and secondly, the construction or debugging of the letter models could be quite flexible, providing many entry points for the students to engage in the activity and opportunities to develop their mathematical thinking. Thus, the design of the activity was mainly based on the didactic design of "half-baked" microworlds (Kynigos, 2007), where buggy models 'hide' their mathematical properties behind their engineering way of construction and aim to engage students in tinkering and debugging them. The phases of the activity are shown in Table 1, each coloured differently and linked to the '5Es' framework during their design stage. However, the author only refers to those in bold, as it was during these activities that the episodes presented took place.

Activity	'5Es' phases
(A) Prediction & debugging of half-baked generalised model N	Explore – Envisage – Explain
(B) Find the code of a given animated model of N (in a square)	Explore - Explain
Free construction of an animated model of letter Z	Explore – Explain - bridgeE
Half-baked words ΝΗΣΙ & ΑΜΜΟΣ (mean island & sand)	Envisage – Exchange – bridgeE
Free construction of a dynamic poster with all letters	5Es

Table 1. A brief description of the activities and their design according to the '5Es' framework

Methodology of the Research and Data Collection

The research was conducted with four 9-grade students of secondary school who worked together in two groups in an after-school class. They participated voluntarily in the intervention, which lasted 8 teaching hours over 3 weeks. The students had already been introduced to the basic components of MaLT2. Each group had a laptop, a worksheet for each activity and a notebook. The researcher played the role of the facilitator during the intervention, encouraging students to express and justify their ideas, while data was collected from computer screen and voice recording, students' notes and MaLT2 codes from the letter models constructed by the groups. This study was a pilot study of a larger design-based research (Cobb et al., 2003). The data were analysed through the lens of the

TRU dimensions and the ‘5Es’ phases based on episodes (Noss & Hoyles, 1996) regarding the generation of students’ meanings or the debugging of students’ misconceptions about the use of the variable during their interaction with the models of the letters in the programming environment and their classmates or the researcher.

RESULTS

The researcher identified instances of students’ discussion and actions concerning the use of the variable in the animated letter models in MaLT2. Table 2 presents students’ actions related to the five dimensions of the TRU framework identified through the ‘5Es’ phases. The following episodes took place during phase 1 of the activity and therefore (A) refers to the 1st activity presented and (B) to the 2nd one (see Table 1, in bold).

Dim. 1 The Mathematics Students made connections to previous concepts and procedures and developed a strong mathematical thinking, when:	- the role of the variable as a generalised number emerged after debugging each other’s ideas (<i>Exchange</i>), through the challenge of the activity to give the code, but not the execution values (A) - they thought and decided to use the concept of sine in their model (<i>Envisage - body sintonicity</i>) by thinking with the tool (B) - they realized the link (<i>bridgE</i>) between mathematics (i.e., the concept of the sine) and the programming environment (i.e., the specific procedure needed (B) to its use in the code)
Dim. 2 Cognitive Demand Students met challenges and engaged in a ‘productive struggle’ through:	- Their predictions (<i>Envisage</i>) and discussions (<i>Explain & Exchange</i>) that led them to take control of their own actions and learning by <i>Exploring</i> the model of a triangle with the sliders without being asked (A), by working on the misconception that x always plays the role of the unknown (B) and, gradually, by building on the perspective that x is used as a generalized number in the models (A & B) - The iterative tests with the digital tool during the <i>Exploration with the sliders, the 3D camera and the scene’s grid</i> that led the students to realise that the given model of N fits into a square, to make sense of the concept of sine or the Pythagorean theorem in the construction of this animated model and, above all, to put these ideas in use in the code, which requires an engineering approach to mathematical properties (B)
Dim. 3 Access to Mathematics Active and meaningful engagement of all students through:	- The prediction (<i>Envisage</i>) of both the model (A) and the code (B) - The many entry points of the activities (<i>Explore</i>), such as the non-unique answer (A - e.g., letter N or triangle) or the variety of mathematical properties that could be embedded in the letter models (B - e.g., Pythagorean theorem, trigometric numbers, ratio)
Dim. 4 Agency, Ownership, Identity Students developed their willingness to engage in the activity and the control of their creations by:	<i>Explaining</i> their thinking and building on each other’s ideas about the role of the variable: unknown vs. generalized number (A & B) <i>Exploring with the sliders</i>, making conjectures about their models and the given variables (A & B) and gradually replacing stable numbers by variables (side dependence) & vice versa (A) Students developed a positive mathematical identity as doers by generating and sharing ideas about the use of properties in the generalised models and by correcting their misconceptions about the use of the variable (A & B) throughout all the ‘5Es’ phases
Dim. 5 Formative Assessment The responsiveness of the environment to	- The digital tool, which promoted the comparison between the students’ predictions and the real outcomes (<i>Envisage</i>) and led to the active exploration of the limits of their current perception though the <i>Exploration</i> of their artefacts (A & B) - The researcher’s question, which tried to address the students’ emerging

students thinking appeared through:

misunderstanding about the use of the same variable in all sides (B) of the triangle, without giving the answer, but by trying to engage children in a “productive struggle”

Table 2. An overall view of the presented activities’ results through TRU and “5Es” Frameworks

The following episode took place during Activity A (Fig. 1a), where students were given a “mystery” code and asked to predict what it could construct.

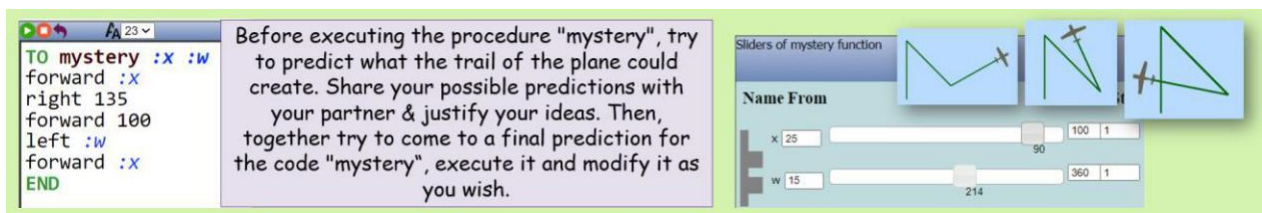


Figure 1. a. The given code for students’ prediction, b. Screenshots of students’ dynamic manipulation of the model through the sliders after the code execution

Student A: So, I think that it gives the letter N.

Student B: Yes, I agree, but wait. It goes forward x. The right command is fine. Then forward 100. It doesn’t go forward that much.

Student A: What do you mean? It doesn’t say how many steps forward the plane goes. It goes less.

Student B: How do you know it’s going less than 100, since it doesn’t say so?

Student A: Well, we only predict. But, yes... We don’t know if it is more or less than 100.

Student B: Exactly. Forward x means the plane goes forward as many steps as we want. And then left w. Wait! Could it also be a triangle? ... If the left command goes over 180 degrees, then this side (shows in notebook) turns a lot to the left and closes, so a triangle is constructed.

The crucial point in this episode is the children's discussion about the role of the variable in their predictions. At first, both students implicitly assumed that x was a certain number, so they seemed to use the letter x as an unknown. In fact, Student A stated that the letter x was a number less than 100, while Student B assumed a number greater than 100. It is interesting to note that even though both students had a certain number or range of numbers in mind for the variable x, they were at the same time aware of the uncertainty of this number, as they questioned each other. Through the expression of their thoughts, students made connections to previous knowledge about the role of the variable as a generalised number (*Dim.1 The Content*), which emerged after debugging each other’s ideas (*Exchange & Dim.4: Agency & Identity*) and both built on the perspective that x is not used as an unknown (*Dim.2: Cognitive Demand*) in the model (“as many steps as we want”). The key feature of this activity that triggered the students’ discussion was the concealment of the execution values of N at the end of the code, a fact that seemed to encourage the students’ willingness to engage in the prediction of the model and to be involved in a discussion about the role of the variable x. If the designer had given the execution values along with the half-baked model (e.g., x=50 w=120), the students probably wouldn't have made these different assumptions. Thus, the *Exchange* phase from the ‘5Es’ model was prominent here, even though the researcher hadn’t included it during the design of the activity. Furthermore, the opportunity to predict the model before executing the procedure seemed to challenge students to look deeply into the code, intuitively explore the possible outcomes and discuss the command that has a variable in the left turn that could actually construct a triangle (Fig. 1b). After their predictions, the students executed the code, compared their predictions (*Dim.5: Form. Assessment from the tool*) with the real outcome and explored the triangle model for some time using the sliders (*Explore*) before deciding to finally modify it to construct a generalised model of the letter N, using the concept of ratio.

In contrast to Activity A, in Activity B, students were given an animated model of N and asked to create a procedure (code) that could construct this model. The key feature of Activity B was that

this letter model fitted into a square. After *exploring* the model N using the sliders, the 3D camera and the scenes' grid, the students identified the square and one of them thought about using the concept of sine in the code (Fig. 2a).

Teacher: And how did you think of using the sine in the code?

Student D: When I saw that two triangles were created in the square, in this triangle, I went and stood on the top, here (shows on the screen), and from there the plane was looking across. So, when I saw the opposite side, I also thought about the hypotenuse and that's how the sine came into my mind.

Even though the students had the figural representation and were looking for the code that constructed this model of N, the *Envisage* phase had also worked the other way round. Student D mentally put himself in the place of the airplane (Fig. 2a) and imagined the path and the movements that this avatar had made, by means of their body ('*body syntonicity*' - Papert, 1980), using two different verbs to describe his thought ("I stood – The plane was looking"). This possibility, offered by the programming tool, allowed the student to generate the idea of the sine (*Dim.4: Agency*) by making connections to previous knowledge and procedures (*Dim. 1: The Mathematics*). However, the students' actions and the discussion that followed, showed that they had difficulty using it in the code because of a misunderstanding concerning the concept of the variable.

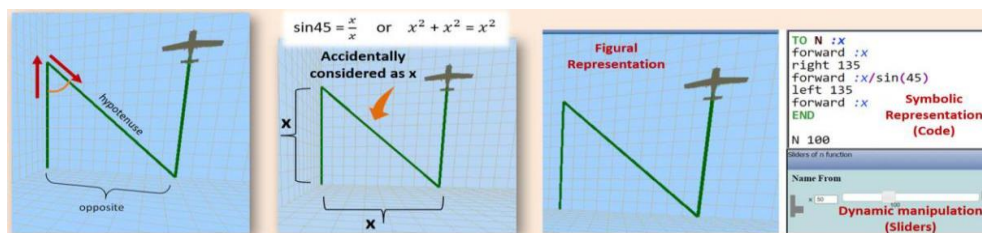


Figure 2. a. Student's thought about the sine through body syntonicity, b. Students' misunderstanding and tests, c. Final code & model construction using correctly the concept of sine in the command 'forward'

During the children's repeated testing on the notebook and the tool, and before they reached the final code shown in Figure 2c, the following episode occurred.

Student D: Something is wrong... It doesn't make sense, and it doesn't work either with the sine that I thought of firstly or with the Pythagorean Theorem that we tried afterwards.

Teacher: Which side is x?

Student C: The hypotenuse, that is what we are looking for.

Student D: (after 1 minute...) Wait! What did we set as x?

Student C: The hypotenuse.

Student D: Oooh that's it! We put x on the hypotenuse, but the other 2 sides are x!

Student C: I don't understand!

Student D: In our model the two vertical sides are x, right? Because of the square outside the N. You see? Forward x! The hypotenuse we are looking for is not x.

Student C: Yes, that's right. It's more than x, because it's a lateral side.

Student D: Right. In our notebook, we also put it as x, because at school we use x for whatever we are looking for.

Student C: That was like an explosion in my head! Let's try it again!

In this episode, the responsiveness of both the digital tool through the instant feedback and the teacher's reflective question "Which side is x?" (*Dim. 5: Formative Assessment*) who had immediately understood the students' misconception about the variable, acted as a facilitator in the learning process of the use of the variable x. Even though the students initially seemed to be able to justify the choice of the sine in their code (Fig. 2a), they appeared rather confused about how to use it in the code command, where they set all sides as x and made also the same mistake when they tried the Pythagorean theorem, instead of the sine (Fig. 2b). Students engaged in a 'productive

struggle' (*Dim.2: Cognitive Demand*) and managed to make sense of the different role of the variable through repeated tests from the paper to the tool, indicating a rather serious problem of the school curriculum, which follows a siloed approach to many concepts and has led students to the misconception that an unknown concept is always represented by x ("...at school we use x for whatever we are looking for."). Thus, based on this episode, the researcher could claim that the students reconstructed their ideas between the way that mathematical concepts and properties are used in the programming environment of MaLT2 and the way they express these ideas in 'official' mathematics (*bridgE*), as the use of the variable is different in the command 'forward : x ' (generalized number) and in the hypotenuse of the triangle (unknown), as the students did. In addition, the fact that the students remained positive in testing their ideas and checking their model over and over again, allows the author to assume that they built up a positive identity as people who can do mathematics. (*Dim.4: Ownership & Identity*).

DISCUSSION

The results presented showed that the students' mathematical identity was fostered as they became designers, used mathematical properties and discussed the concept of the variable and its use in a programming environment where mathematics took on more of an engineering substance. The groups used the concepts of ratio, sine and the Pythagorean theorem to construct or debug their letter models, in an engineering way rather than the usual school way, which gave them the opportunity to express their thoughts about the variable in the animated models. The episodes showed that the children approached the variable mainly as an unknown number that they will determine at some point. On the one hand, they tried to think of the variable as a generalized number, but it seemed that in their minds they actually had x as a specific number during Activity A. However, through the prediction of the model, the students debugged and at the same time built on each other's arguments, reshaping their knowledge on the variable as a generalized number. On the other hand, during Activity B, the students directly expressed the procedural process they have been following in school where x was always what they were looking for, without even thinking about it. However, the digital tool gave them the opportunity to develop a critical attitude towards the use of mathematics in a different context. The affordance of dynamic manipulation of the models by changing the variable values and the instant feedback offered by MaLT2 were crucial to the activity, as they encouraged students' repetitive cycles of exploring, assuming and testing. This encouraged them to make connections between the different uses of the variable, and ultimately to feel competent in using mathematics as a tool for thinking, constructing and reasoning. More than that, the use of the TRU framework, enriched with digital technologies and combined with the '5Es', enlightened the analysis of the students' meanings and ideas about the use of the variable as a generalized number and the study of their mathematical experience as a whole in a programming environment. Besides, the appearance of all the TRU dimensions illustrated a powerful learning environment based on the designed activities using the '5Es' phases and the MaLT2 digital tool. Thus, the author could suggest that these theoretical perspectives may provide an insightful way of designing activities for students, aiming not only to teach 'new' mathematical concepts, but also to develop students' competence and agency to use the mathematics that they already know through a different approach and in different contexts. Clearly, there are limitations and the results of this research cannot be generalised. However, it seems worthwhile to further explore the combination of these theoretical approaches as a better perspective for creating strong learning environments based on programming for the development of students' mathematical thinking and competence, also exploiting 3D dynamic models.

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CONCEPTUALIZING NEW CONSIDERATIONS FOR TECHNOLOGY: TEACHING ELEMENTARY MATHEMATICS IN A VUCA WORLD

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The COVID-19 pandemic led to global disruptions in education that extend beyond a specific moment in time. Lessons learned about teaching mathematics during the pandemic can inform future practice in a VUCA (volatile, uncertain, complex, ambiguous) world. Rapid, mandatory shifts to technology-enabled distance education challenged teachers at all grade levels, but were particularly novel for teachers in elementary schools where distance learning had been rare. This paper draws upon existing literature and interview data from twelve elementary teachers in U.S. rural communities to conceptualize new considerations for technology that emerged from teaching mathematics remotely during the pandemic. Underlying issues surrounding technology in mathematics teaching suggest avenues for future research and practice in a VUCA world where education continues to be impacted by extreme circumstances such as climate change, conflict, and catastrophe.

Keywords: VUCA, elementary mathematics education, educational technology

INTRODUCTION

Rarely does an occurrence disrupt and change educational experiences as the COVID-19 pandemic did on a global scale. The United Nations Educational, Scientific and Cultural Organization (UNESCO) estimated that 94 percent of the world's children were impacted by school closures in the early months of the pandemic (UN, 2020). Educators and educational systems worldwide made rapid shifts to minimize the impact of this disruption, overwhelmingly involving the use of technology for educational purposes. The worldwide incorporation of technology for teaching during the pandemic also presented an opportunity to reconceptualize the role of technology in education. Although the COVID-19 pandemic was unprecedented in its global impact on education, the lessons learned from this experience can inform and improve educational experiences in other extreme circumstances, both current and in the future. The purpose of this paper is to conceptualize new considerations for technology in the teaching of school mathematics in elementary grades, particularly within a volatile, uncertain, complex, and ambiguous (VUCA) world. This paper draws upon existing research literature regarding technology in mathematics education and interview data from twelve teachers who shared their experiences and perceptions about teaching mathematics and STEM during the COVID-19 pandemic to address the research question: *How did rural elementary teachers' use of technology for teaching mathematics change in the face of an evolving educational landscape?*

The acronym VUCA owes its origin to the U.S. military, rising to prominence during one of the more uncertain times in the nation's history, the Cold War (Reeves & Reeves, 2015). The notion of attempting to describe unpredictable and tumultuous events was subsequently adopted by the business world in an effort to make sense of a series of disruptions to the norm, including, but not limited to: social and political states of unrest, global economic crises, and world-wide health dilemmas, such as COVID-19 (Stein, 2021). These disturbances carry with them a trickle-down effect, impacting even the young among us with little say in decisions being made on their behalf. Because of the far-reaching element of these incidents, educational researchers have begun to see the merit in using this acronym as a lens for viewing the fluctuating needs of classroom teachers and students alike in an attempt to better prepare individuals for the varied, sometimes erratic, nature

of the world we live in (Hadar et al., 2020). In order to better develop modern solutions to the modern problems in a transforming society, existing academic systems must adapt to consider issues that may have previously been overlooked. Sudden introduction of new technology demands thoughtful support for its use; a landscape that extolls the virtue of being open about social-emotional strife calls for teachers to be trained in better attending to such needs (Luthans & Broad, 2022); and as we continue to accept that actions having consequences that can be measured on a global scale, we are obligated to think of ourselves in a role of global citizenship (Waller et al., 2019). In a volatile, uncertain, complex, and ambiguous world, the best path forward is illuminated by a strong educational framework that is poised to make sense of these ongoing transformations. While VUCA does not function as a theoretical or analytical framework in this paper, it does offer a useful context for making sense of long-term educational implications from the COVID-19 pandemic.

Framing the Study

As technologies for teaching mathematics have advanced, so too have theoretical perspectives for teaching mathematics with technology. Drijvers et al. (2011) outlined the development of theoretical frameworks relevant to integrating technology in mathematics education from the 1960s to 1990s, and how maturing theories in didactics for mathematics can apply to research about technology integration. Emerging theoretical perspectives in mathematics education continue to build upon how researchers frame the integration of technology in mathematics teaching (see, for example, Trouche et al., 2019). Kaput et al. (2020) suggested a shift toward technology as infrastructure in mathematics, a perspective Yeo (2020) echoed regarding integrating digital technology in elementary mathematics. While numerous theoretical perspectives exist for understanding technology integration in teaching mathematics, making sense of how teachers worldwide responded when suddenly required to facilitate remote mathematics learning for students of all ages posed new, unexpected challenges and opportunities for researchers interested in this phenomenon.

Drijvers (2018) described potential benefits of digital tools in mathematics but emphasized the need for teacher implementation to meet that potential. Thus, this paper draws upon research specific to teachers' attitudes toward and implementation of technology, with specific focus on rural, elementary contexts that align with participants in this study. Over a decade ago, Loong et al. (2011) found little difference between the use of information and communication technologies for teaching primary mathematics in rural and urban environments in Australia. When differences did exist, frequency of use in rural contexts almost always exceeded that of urban schools. Likewise, Howley et al. (2011) found that rural elementary teachers demonstrated more positive attitudes toward technology integration compared to their urban or suburban counterparts, though challenges with access and preparation for teaching with technology persisted. A study of rural middle and secondary math and science teachers concurred; while teachers tended to want to use technology in their teaching, access and familiarity with new technologies inhibited implementation (Kalonde, 2017). More recently, Hatch and Clark (2021) studied the instructional decisions of highly effective elementary teachers and found that rural teachers demonstrated less fidelity to curriculum, often adapting and using web-based resources. This aligns with Baniahmadi et al. (2021) who found an increase in the use of teacher-created math curriculum during COVID-19, often to make resources more accessible. Overall, research suggests that despite challenges of access and training, rural elementary teachers of mathematics may hold positive views toward integrating technology in teaching and may use or adapt web-based curriculum resources to support their students.

NEW CONSIDERATIONS FOR TECHNOLOGY

Teachers involved in this study taught primary and intermediate grades ranging from Kindergarten to Grade 6, all in rural schools in the Midwestern region of the United States. Participants included two men and ten women whose teaching experience ranged from 6 to 25 years. All participants earned a Master's degree in education and participated in a federally-funded grant program to develop rural teacher leadership for STEM. The data upon which this paper draws represent a subset of the larger overall funded project. Each teacher was interviewed twice via Zoom, which automatically generated transcripts. The interviews themselves were semi-structured in nature, with participants afforded the latitude of expounding on points where they saw fit, but the same set of questions were asked of each individual. Interview data were then openly coded using MAXQDA qualitative analysis software, wherein researchers went through a subset of the transcripts and each coded them based on initial conversations regarding the interview content. Researchers checked interrater reliability through MAXQDA and negotiated discrepancies before proceeding to independently code all interviews. After identifying commonalities, axial codes were agreed upon to match broader themes in the interviews, and the rest of the interviews were re-coded accordingly. Findings from this analysis are described below. In lieu of an extensive literature review at the beginning of this brief paper, each consideration begins with an overview of relevant research before describing results from this study.

Technology as an Obligation

Teaching during the COVID-19 pandemic pushed teachers of mathematics to use technology as an obligation rather than an optional possibility. Literature indicates that even when technology was available and teachers exhibited positive dispositions toward technology for teaching and learning mathematics, it often went largely unused, emphasized presentation technologies, or was used for low-level computational tasks (e.g., Padrón et al., 2012; Perienen, 2020; Polly, 2014). Similarly, Thurm (2018) conducted a quantitative analysis of German secondary teachers' beliefs and practices when teaching mathematics with technology and found a seemingly contradictory category, "positive beliefs—infrequent users." Thus, positive beliefs and access to technology do not necessarily imply regular use of technology as a tool for teaching and learning mathematics. Yet, the rapid shift to teaching mathematics remotely during COVID-19, necessitated many teachers' use of technologies.

Adapting pencil-and-paper work for remote learning
I downloaded Adobe Scan on [my] phone where you can take pictures and turn them into a PDF; we put together weekly packets, and we posted all those PDFs on Google classroom where the kids couldn't type on them. Parents were printing these. Some parents [asked] "hey could you try to make it editable?" so then I spent hours transferring them into Google slides so then they could type on them. But I thought that was kind of interesting like you could make it interactive.
I can make fillable PDFs for the kids to do and I have done that from the start because that made sense to me. So I guess I've used Adobe to make fillable PDFs.
I finally told my students that if you have a tablet, download the classroom App on that tablet take your pictures of your homework, using the tablet instead of your computer because one it's better quality and you it's easier for kids to upload those and take the picture from a tablet device or smartphone.
Adapting / expanding familiar technologies for remote learning
Everything was on Google Classroom because instead of having those face-to-face instructional times, I recorded myself on a video and they had to watch the video of the lesson and then do their work. And so it was all just on Google Classroom instead of only having part of it on Google Classroom.

I've had a Doc camera for a while, so the week my Smart Board was out I took a giant light poster paper covered my smart board so then at least the projector could use my Doc cam and projected for the kids and then I would just write on the paper underneath the Doc cam, so that did work. So I worked around that issue, so I knew what to do.
Adopting / adapting new technologies for interactive remote learning
We have become very tied to our devices, partly because we have those virtual learners and stuff and so kids are watching more simulations on the Chromebook and that kind of thing. Back to the hands on, I always do a math lesson with money, like trading in tens for hundreds and I wasn't sure about using the actual money and having them all touch it because you can't really disinfect it, so I found an interactive site that they could actually do it on their own. The same with fractions; we always did fraction strips which I did end up using but, at the beginning, we just used an interactive site for that.

Table 1. Teacher Interview Excerpts Relating to Technology as an Obligation

When technology use became an obligation rather than an option, elementary teachers adopted, adapted, or expanded their use of technology for teaching mathematics. In some cases, teachers adapted what would have been pencil-and-paper work for remote learners. Although this use of technology may not have been particularly novel or specific to mathematics content, the interview excerpts in Table 1 describe how teachers in this study used technology to address basic needs and functions in teaching elementary mathematics. In other instances, teachers used technologies familiar from face-to-face mathematics teaching, but adapted or expanded them to better serve students in remote settings. Excerpts in Table 1 highlight two such instances, both for presentation purposes. Furthermore, two teachers highlighted more interactive uses of technology when required to teach mathematics remotely, including simulations and virtual manipulatives.

Technology as a Connector

Though not always embraced for classroom learning, especially at the elementary level, technology proved an essential tool for facilitating student-student, student-teacher, and teacher-family connections during remote teaching. The essential role of communication and collaboration in mathematics teaching and learning has been widely studied (e.g., Morgan et al., 2014), but when face-to-face interactions were disrupted, the role of technology as a connector was elevated. The essential role of families in their children's mathematics education (Knapp et al., 2017) was also amplified as millions of children studied from home during school closures. Examining three cases of Latin American countries during the COVID-19 pandemic, Castro et al. (2021) called for redefining the relationship between school and family for the objectives of mathematics education and citizenship. Mathematics education researchers also viewed interaction, collaboration, and school-home connections as more important post-pandemic versus in 2019 (Bakker et al., 2021).

Facilitating student collaboration and communication
Whoever was [a remote learner's] partner would just go to the iPad and communicate with them during that group time I had. A couple of kids were out at one time, and so I just put them in a group and their own little zoom meetings so that they could talk about things, and then I would pull them back into our classroom zoom when it was time for us to go back to the whole group learning. I sent materials home for the different activities we were doing, while [remote students] were gone and they would do them on zoom with us. I really just tried to give them activities they could do that weren't just a worksheet or something like that. That way they were still collaborating with their classmates. So it was a lot of ideas on how I can keep them doing something active and also feeling like they're still part of the group as well. But then [students in the classroom] saw [remote students] come in with our zoom meetings, every day and they were able to see them and communicate with them so they knew they weren't sick at home.

Partnering with Families
So this year, we did not have any in person parent teacher conferences, they were all virtual and that's never happened before. I would say we have a lot more email interactions. But I will say technology, helped to communicate with parents faster and more often. We did adopt seesaw, which is a program that we use to send out homework and stuff in case we were just closed down again; it's like sending text messages, so if a parent wants to communicate during the day. Last year was one of my first years I did a Facebook group for my class, but that actually came in the handiest during quarantine because I could just open up my Facebook and message parents and that's something that pops up on their phones and I know they're going to see it versus an email-it might be two days later that they see it, so that was very beneficial for me. Yes, it's probably not the most professional way but it worked with our parents here.

Table 2. Teacher Interview Excerpts Relating to Technology as a Connector

Interviews with rural U.S. elementary teachers highlighted the importance of technology as a connector during the pandemic. Table 2 highlights the important role technology played in keeping students connected and engaged with one another, during both remote and hybrid mathematics teaching. Other teachers emphasized how technology helped them stay connected with families, often in new ways such as virtual parent-teacher conferencing and through social media groups.

Technology for All

When technology became an obligation, disparities in access were put in sharp focus. Globally, the poorest and most vulnerable students tended to disproportionately lack access, connectivity, and digital skills necessary for remote learning (UN, 2020). This finding reflected decades-old digital equity challenges in mathematics education (Forgasz et al., 2007). A post-COVID survey of mathematics education researchers found increased concern about the digital divide and making free, high-quality curriculum resources available for multiple technology formats (Bakker et al., 2021).

New adoption of 1-1 Devices
The one-on-one iPad; they did have it in the middle school and high school but not in elementary. I don't think we could have done it without it, because when [we] had to zoom or quarantine, we sent that iPad with them ... and that they were able to communicate. Having an adult putting that into the students' hands was essential, because not everybody obviously has a technology at home. I would have to say that just our students being able to have their own Chromebook, their individual Chromebooks, has been a blessing. We never had that prior to the pandemic. We did purchase more iPads with the CARES Act money, so that in the event that we would need to send them home with students, we'd be able to. We have enough that we could send at least an iPad home with each family.
Expanding Connectivity and Access
They can get a hotspot from the school to be able to get Internet, like last spring [when school was closed] or if they're a virtual learner, that kind of thing. But so that's the big thing, is the equity part of it. Trying to make sure it's the same for all kids. I could see like with natural disasters, if there's internet access and that kind of thing. We still have a lot of kids trying to recover from the flood two years ago. Our kids are one-to-one with Chromebooks in [Grades] 4-6. And so any kid that didn't have a computer at home got issued one from the school and then internet. Our internet company and clerks actually discounted parents that had [to have] internet for something every day.

At least we could get one [iPad] to each family now. How do we structure a day so that the kindergarten student and the fourth-grade student don't have to be on zoom at the same time using the same iPad or bogging down the Internet system here in town? How do we even do this and make it work? We had parents who were not doing zoom's with our school earlier because their parents needed the Internet to do work, and it was bogging them down and they weren't able to do the things that they needed to do to make money so they said forget it.

Table 3. Teacher Interview Excerpts Relating to Technology for All

Access and connectivity were important considerations for this sample of rural teachers, as well. Most teachers described prior or new 1-1 device availability that expanded student access. Others described initiatives to provide rural students with internet connectivity, also raising new questions about how to balance connectivity for parents' remote work with students' remote schooling. One teacher noted expanded connectivity possibilities for targeted remote instruction during future unplanned events such as weather-related closures and natural disasters. Overall, expanded access to devices and connectivity helped narrow the digital divide in these teachers' rural communities but that is not necessarily representative of communities worldwide.

IMPLICATIONS, LIMITATIONS, AND FUTURE RESEARCH

Because of their positive views and experiences with technology (Halley et al., 2011; Kalonde, 2017) and tendency for lower curricular fidelity (Hatch & Clark, 2021), rural teachers were uniquely positioned to adapt mathematics curriculum with technology during the pandemic and offer insight about these changes. Findings from rural U.S. elementary teachers in this study help illuminate technology-related changes for teaching mathematics during this tumultuous time. Although this study was situated during the COVID-19 pandemic, the implications extend beyond a relatively brief moment in time.

The considerations suggested in this paper undergird elementary teachers' ability to teach, and in turn, students' opportunity to learn mathematics with technology during distance and hybrid learning. In a VUCA world, educational systems must be agile, teachers must be well-prepared, and researchers must be mindful of the many dimensions relating to teaching and learning mathematics with technology. Ongoing study of theory and practice specific to technology integration in mathematics education is essential for understanding current and future teaching, learning, and classroom practice. But it is important for stakeholders at all levels to consider what technology can and cannot do when the world shifts in unpredictable ways that directly impact the teaching and learning of mathematics. Lessons learned about teaching mathematics remotely during COVID-19 are already relevant in more geographically-isolated VUCA events that have disrupted business as usual in schools (e.g., weather events, conflict). Prioritizing digital equity, facilitating connections among students, educators, and families, and supporting teachers' incorporation of technology in a variety of ways are essential considerations for supporting math learning in non-traditional contexts.

While the shift to remote teaching during the pandemic was somewhat universal, the ways in which remote teaching was accomplished were not and the perspectives of the teachers in this study do not capture the experiences of teachers across contexts and cultures. Further study of elementary teachers' use of technology for remotely teaching mathematics should draw from diverse populations of educators. Additional study is also needed to understand the extent to which the three considerations for technology proposed in this paper correspond with high quality mathematics teaching and learning experiences, as well as how these considerations relate to technology integration for teaching mathematics, as framed by a variety of existing theoretical perspectives.

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UNPACKING TEACHER BELIEFS ABOUT MATHEMATICAL DIGITAL COMPETENCY

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The recently proposed construct of mathematical digital competency (MDC) entails that student understanding of mathematical concepts may be almost inseparable from digital technology (DT) use, and that students might only be able to “do” mathematics with DT. We report on a case-study in which we investigated how two German pre-service-teachers (PSTs) think about the connected notion of MDC, and discuss their opposing views. We found that the belief that “mathematical understanding” is reflected by being able to explain and do mathematics without DT was a fundamental reason to oppose the notion of MDC for one of the two PSTs. In contrast, the other PST supported the notion of MDC by placing the DT in relation to other tools. We conclude that beliefs about MDC are likely a critical part of teachers’ own MDC.

Keywords: mathematical digital competency, digital technology, teacher beliefs, predicative knowledge, operational knowledge

INTRODUCTION

Our research work entails the investigation of a somewhat novel dimension of teacher beliefs in the context of teaching mathematics with digital technology (DT). The starting point for the research study was the work of Geraniou and Jankvist (2019), who argue that mathematical competencies and digital competencies are rarely seen as a connected whole, even though students will have to simultaneously activate and use these competencies. They thus conceptualize the construct of “mathematical digital competency” (MDC), describing an amalgam of mathematical and digital competencies. They show that such an amalgam entails that a student’s understanding of a mathematical concept may almost inseparably be connected to DT and the student’s instrumented techniques. In this paper, we investigate PSTs’ beliefs about such an amalgam, how they justify their beliefs, and how their beliefs are related to other beliefs about the potentials of DT. Throughout the paper the term “digital technology” (DT) refers to mathematics-specific DT such as function plotters, dynamic geometry systems, computer algebra systems and multi-representational tools.

THEORETICAL BACKGROUND

Mathematical competency can be defined as “someone’s insightful readiness to act appropriately in response to a specific sort of mathematical challenge in given situations” (Niss & Højgaard, 2019, p.14), while digital competency has been conceptualized as “the set of knowledge, skills and attitudes [...] that are required when using ICT and digital media to perform tasks; solve problems; communicate; manage information; collaborate; create and share content; and build knowledge” (Ferrari, 2012, p.43).

Geraniou and Jankvist (2019) linked both kinds of competencies by using the theory of instrumental genesis (TIG) and the theory of conceptual fields (TCF). TIG (e.g., Guin & Trouche, 1999) describes the process of transforming DT (an artefact) into a mathematical instrument, which is a psychological construct that combines (parts of) the artefact and cognitive schemes in which technical knowledge about the artefact and mathematical knowledge are intertwined. TCF (Vergnaud, 2009) highlights that a concept comprises a set of schemes, a set of situations and a set

of linguistic and symbolic tools of representation, and that different concepts and situations are interconnected forming conceptual fields. The set of situations gives meaning to the concept and acts as a point of reference. TCF also emphasizes the distinction between operational knowledge that makes it possible to do something and predicative knowledge that makes it possible to describe and give reasons.

Geraniou and Jankvist (2019) use TIG and TCF to investigate the simultaneous activation and development of mathematical and digital competency, which they call “mathematical digital competency” (MDC), and highlight, that the situations that make up students’ conceptual fields “[...] may be embedded so deeply in a techno-mathematical discourse that, potentially, also their understanding of the mathematical concepts involved is almost inseparable from the digital tools and the students’ instrumented techniques” (p. 42). This could for example mean that the set of situations that students use as points of reference to give meaning to a concept will largely comprise situations involving DT. Hence, a student’s predicative and operational form of knowledge will be inherently intertwined with DT, which means that a student might only be able to think about, describe and explain mathematics with reference to a DT (predicative knowledge) and do mathematics with DT (operational knowledge).

Continuing the work on the MDC framework entails to investigate what teachers believe about such a potentially close interwovenness as a first step in convincing teachers for the value of supporting students developing MDC. This very argument is aligned with Tabach’s plenary talk at PME44 (Tabach, 2021), in which she pointed out that besides MDC for students the research community must try to provide a parallel conceptualization of MDC for teachers. We propose that such a parallel conceptualization comprises teachers’ beliefs about MDC. Teacher beliefs – which can be defined as “psychologically held understandings, premises, or propositions about the world that are thought to be true” (Philipp, 2007, p. 259) – are particularly important, since they act as a bridge between knowledge and action and hence may have a profound impact on classroom practice (Thurm & Barzel, 2020; 2021). Clearly, a teacher can have different belief positions with respect to the relation between mathematical and digital competencies. On one extreme, one can fully embrace the “*connected position*” of MDC (e.g., that students’ understanding of the mathematical concepts involved may be almost inseparable from DT). On the other extreme, one could strongly favour an “*independent position*” believing that mathematical and digital competencies should be clearly separated (e.g., a student should be able to think about, explain and do mathematics without DT).

In a related quantitative study (Thurm et al., 2022), with n=198 PSTs from three German universities, the authors of this paper showed that many PSTs oppose a “*connected position*” as conceptualized by MDC and strongly favour an “*independent position*” instead. This can be problematic, if the goal is to develop students’ MDC. However, investigating teachers’ belief position (e.g., “*connected position*” vs. “*independent position*”) should be augmented with investigating teachers’ *belief argumentations* – i.e., how teachers reason for their position (Rott, 2021). Furthermore, it is important to consider teachers’ *belief system* (Green, 1971; Philipp, 2007) in which beliefs form clusters, since beliefs “come always in sets or groups, never in complete independence of one another” (Green, 1971, p. 41). Beliefs within a teachers’ belief system may be logically connected. In addition, some beliefs may be more central than others, while some beliefs may be inconsistent with one another. The results of Thurm et al. (2021) indicate that PSTs’ beliefs about the potentials of DT and beliefs about whether DT should only be used if mathematics is understood without DT, may be central in the context of beliefs about MDC.

RESEARCH QUESTIONS AND METHODOLOGY

The study aims to explore how PSTs justify their beliefs (*belief argumentation*) about MDC and to investigate beliefs about MDC in relation to PSTs’ *belief system*:

- RQ1: How do pre-service teachers justify their beliefs about the relation of mathematical and digital competencies? (*Belief argumentation*)
- RQ2: How are beliefs about the relation of mathematical and digital competencies related to beliefs about DT? (*Belief system*)

To answer these research questions, we use the case-study approach (Flyvbjerg, 2006) and investigate two PSTs' beliefs by means of semi-structured interviews. Case studies provide deep insights into phenomena and help to capture the complexity of a phenomenon. In particular, prototypical cases allow to exemplify important features and tensions among teachers' belief systems (e.g., Andrà et al., 2021).

To prepare PSTs for the case-study interview, we constructed an initial “information sheet” that outlined the “independent position” and the “connected position” and illustrated each of these with a concrete example. In a pilot study, 20 PSTs were asked to read an information sheet and subsequently elaborate on the reasons underlying their position. The pilot study indicated that some PSTs had difficulties understanding the difference between the two positions. Furthermore, the PSTs were often focusing on the operative form of knowledge, and several PSTs were quickly side-tracked to talk only about the potentials or risks of DT use. Based on these observations, we revised the information sheet as well as the interview questions. We focused the information sheet on predicative knowledge and extended the illustrating examples. The final information sheet is given in figure 1.

Independent position

A learner should be able to think about and explain mathematics independently with DT.

This means:

- A learner should be able to explain and think about a mathematical concept or relationship without making references to a DMT

Examples:

- A learner can explain the role of the parameter "a" in $f(x)=ax^2$ without verbally referring to a DT, e.g., the learner says: "If I draw a parabola with $a=2$, it will be wider than if I draw a parabola with $a=3$. Therefore, "a" determines the width of the parabola").
- A teacher asks a student what comes to his mind when he thinks about the derivative. The student can think about the concept independently from DT. The student answers: "I think of tangents to the graph of the function."
- A learner can explain linear functions without making references to DT: "A linear function is a function of the form $y=mx+b$, where m and b are numbers, for example $3x+5$ ".

Connected position

It is acceptable if a learner can only think about and explain mathematics in connection with DT.

This means:

- It is acceptable if a learner can explain and think about a mathematical concept or relationship only by making references to a DMT.

Examples:

- A learner can explain the role of the parameter "a" in $f(x)=ax^2$ only by verbally referring to a function plotter, e.g., the learner says, "If I take a slider that controls "a" then the width of the parabola would change. Therefore, "a" determines the width of the parabola."
- A teacher asks a student what comes to his mind when he thinks about the derivative. The student answers: "I think of a function in GeoGebra where I can move a tangent dynamically along the graph of the function."
- A learner can explain linear functions only by making references to GeoGebra. He says for example: "If I use GeoGebra to vary m and b in $y=mx+b$ then we would get many different examples of linear functions like $3x+5$ "

Figure 1. Information Sheet

Next, we set up four subsequent interview questions, each clearly addressing one distinct aspect:

- **[beliefs about MDC-predicative]:** "What are the reasons underlying your decision about whether a student should be able to think about, explain and give examples with or without DT?"
- **[beliefs about MDC-operational]:** "To what extent should a student be able to "do" mathematics without DT, e.g., solving equations or drawing graphs?"
- **[beliefs about time point of DT use]:** "How much do you agree with the following statement: "DT should only be used after the mathematics has been thoroughly understood without DT. Please explain the reasons underlying your position."
- **[beliefs about potentials of DT]:** "Do you believe that DT can help support the learning of mathematical concepts? If yes, why? If no, why not?"

After another pilot in which no further problems arose, the final information sheet (figure 1) was given to 12 mathematics PSTs studying in the 5th semester at a German university. The PSTs were asked to read the sheet carefully and position themselves on a 6-point-scale with 1 being the "independent position" and 6 the "connected position". Afterwards, two PSTs, one strongly favouring the "independent position" and the other "connected position", were asked to take part in an interview that addressed the four questions given above. In the following, we describe the two cases of Lara and Pete.

CASE 1: LARA (“INDEPENDENT POSITION”)

[beliefs about MDC-predicative]: Lara stressed that if a student can explain, think about, and give examples only with respect to a DT, this would not allow her to see whether the student understands the mathematics or whether (s)he only possesses technical competency:

Lara: I think it’s important to make sure that you teach the whole thing without DT, so that you can really make sure that the students understand it.

Moreover, Lara stated that being able to explain and think about mathematics without reference to DT is more “true” mathematics.

[beliefs about MDC-operational]: In the same sense, Lara argued that if a student cannot “do” mathematics without DT, then she cannot be sure that the student understands:

Lara: I see the same again, what I have just said. For example, if I’m supposed to draw something or maybe determine some things, I find that it’s not clear whether the students are aware of what they’re doing.

Furthermore, she stressed that doing mathematics by hand / pen & paper helps to foster understanding. For example, she argued that a student should always be able to create all geometrical constructions without use of DT.

[beliefs about time point of DT use]: Because of her previous points, Lara clearly believed that DT should only be used when the mathematics is already understood thoroughly (without DT) in order to make sure that the students understand the mathematics, and to be able to check the understanding of the students:

Lara: I think that is exactly what I just said. [...] because I think that they should first try it out themselves, or construct it themselves, so that one can see that the children have understood it and have the chance to learn to understand the connection between different things. And when the knowledge is available, you can say now I’m working with DT.

[beliefs about potentials of DT]: Lara was generally positive about DT and had even considered to write her Bachelor thesis about DT. She stressed that DT can be helpful, if knowledge and understanding have already been established. She highlighted that DT offers ways to dynamically modify parabolas, and to easily construct geometric objects. Furthermore, she stresses the potentials to discovery mathematical relations on your own:

Lara: One can experiment, for example. [...] we had drawn a triangle with GeoGebra and drawn the bisector, and then we had just pulled on the triangle, and we suddenly realized that we had an equilateral triangle, where the bisector, which we had drawn before, was suddenly the perpendicular bisector. And you can do something like that with the program and explore such things, which is not possible on paper.

CASE 2: PETE (“INDEPENDENT POSITION”)

[beliefs about MDC-predicative]: Pete stated that the world is constantly changing and progressing, and new and emerging DTs are part of this progress. He believed that it is totally fine, if students can only explain and think about mathematics with reference to DT. For him, the “*connected position*” and the “*independent position*” were actually quite the same – except that students may refer to different tools in their thinking or explanations:

Pete: For example, many students cannot describe angles without having a set square in their hands. And a set square is also a tool but not a digital one. And the digital

tools, are just somehow a further development and they can be used gladly and can also be used gladly in justifications.

[beliefs about MDC-operational]: Pete said that students are currently required to do mathematics without DT in German schools by law, and therefore it is currently important that students can do mathematics without DT. If this was not the case, he explained, it would be perfectly fine for him, if a student could only do mathematics with DT. He argued that everything a student can do by hand can also be done with DT – e.g., solving equations. He stated that with DT, one can even do certain mathematical procedures in different ways. For example, plotting a function can be done directly by entering the expression or by creating a table of values and plotting these values, which he argued is very close to drawing a function by hand:

Pete: For example, a graphic calculator can also be used to insert tables or similar things and then a function can be drawn without just simply entering the function and it is there. So, they can also draw functions on the graphic calculator, like they would do it by hand, so to speak.

[beliefs about time point of DT use]: Pete strongly opposed the view that DT should only be used after the mathematics has been understood thoroughly without use of DT. This is because DT provides opportunities to learn, he said:

Pete: I think that DT offers a possibility for many students to access mathematics in a different way. And that gives them the opportunity, for example, to discover graphs or similar things that I [they] don't have the opportunity to do by hand. Then the process is faster. The process is maybe even clearer. I think there are many possibilities that help the students."

[beliefs about potentials of DT]: Pete strongly believed that DT supports the learning of mathematical concepts, and he refers to the points he made when answering the previous question. He augmented his position by referring to a personal experience during his own schooling, where his teacher had used DT for introducing the parameters of functions:

Pete: I actually noticed for myself that this [it] was a good access with the slider, because I saw exactly what was happening with the function, and it worked quickly. I understood relatively quickly and well what the individual parameters of the function were and what they were good for. This was a quick, understandable access for me.

DISCUSSION

In this case-study we have taken a first step to investigate how PSTs justify their beliefs about the relation between mathematical and digital competencies, and how their beliefs relate to other beliefs in their belief system. Lara's case shows that for PSTs "to understand mathematics" can mean to be able to think about, explain and do mathematics without DT, i.e., that students' predicative knowledge is not intertwined with DT. Consequently, a derivative belief of Lara is that a student should be able to explain and think about mathematics without any reference to DT ("*independent position*"). However, Lara is positive about using DT because of its interactivity and the potentials for discovery learning. Lara's belief that DT should only be used if the mathematics is thoroughly understood without DT, can be explained as a way to manage the tension of using DT (acting in line with her positive beliefs about DT), while at the same time maintaining an independence between mathematical understanding and DT (acting in line with her "*independent position*"). However, there seems to be some salient contradiction in her belief system, because a question arises as to what should be discovered by using DT, if concepts and relationships have already been

taught without DT. Lara could also have considered the case of using DT to consolidate knowledge and deepen understanding of concepts following their introduction without DT.

In a preceding quantitative study, the authors of this paper found that many PSTs held a strong “*independent position*” (Thurm et al., 2022). This potentially becomes problematic, if the goal is that these PSTs support students to develop MDC. Lara’s case indicates that teacher educators will need to discuss with students, what “mathematical understanding” means and how it can be witnessed. This will entail to make clear that if DT is used in an epistemic way (i.e., to create understanding or support learning within the user’s cognitive system), the conceptual fields, and the set of situations that students use as points of reference to give meaning to a concept, would not be independent of DT. Also, PSTs need to understand that if students use DT extensively in epistemic ways (to understand mathematics) and pragmatic ways (to “do” mathematics), their operational knowledge will be inherently linked to DT.

We can conclude that it is not enough to focus on PSTs’ beliefs about the potentials of DT and convince them of the potentials of DT. Rather Lara’s case shows that PSTs’ beliefs about the potentials of DT may be less central than other beliefs – leading to PSTs strongly opposing the notion of MDC based on fundamental beliefs about the nature of mathematical understanding in relation to DT, despite their positive beliefs about DT.

Pete’s case illustrates that a “*connected position*” may be justified by placing DT in relation to other tools that have been used in mathematics education for a long time. For Pete “understanding mathematics” has nothing to do with which tool a student refers to in his or her explanations or when doing mathematics. Such a more “holistic” concept of mathematical understanding, i.e., one not bound to certain tools, could be beneficial for developing PSTs’ beliefs about MDC towards a “*connected position*”.

Tabach (2021) has pointed out that besides MDC for students the research community must try to provide a parallel conceptualization for teachers. Teachers’ beliefs about MDC and beliefs about “mathematical understanding” in relation to DT indeed seem to be a crucial aspect of such a parallel conceptualization. Nonetheless, the present study is only a first step in investigating beliefs about MDC. We are currently analysing more cases to contribute to a better understanding of teachers’ MDC related beliefs.

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USING GRAPHING CALCULATORS IN TEACHER EDUCATION

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This paper focuses on the use of graphing calculators in a mathematics laboratory where teachers become familiar with this kind of technology. In-service high-school teachers were involved in didactic activities on successions and limits of successions through the use of calculators. A qualitative analysis was developed, teachers were interviewed in focus groups and semi-structured questionnaires and data was collected from a qualitative point of view.

Keywords: *Graphing calculators, technologies in math education, costructivist approach, integrated laboratory*

INTRODUCTION

In 2017, the Italian Minister of Education amended the legislation to allow the use of GC during the final examination in scientific high schools (Ministerial Ordinance no. 257/2017, Article 18). This amendment generated increasing interest in this technology and prompted manufacturers to develop training courses. However, it has yet to significantly alter the way teachers approach mathematics and its models with such a tool. Previously, the GCs were little used in teaching activities, probably because they were not allowed in the final exams of the courses, therefore, until 2017, few works on the subject had been produced and in Italy there is no significant relevance on the effectiveness of its use in teaching. At an international level, on the other hand, GC is a teaching tool used by many school systems for almost 40 years and has been the subject of research in teaching.

This paragraph will provide an overview of experimental results, interpretations, and conclusions of educational activities developed with GC. We are able to review over four decades' worth of experiments and models conducted in various school levels and in relation to various mathematical topics. In 1996, Penglase and Arnold (Penglase and Arnold, 1996) conducted an extensive review of the published works on GC and presented an in-depth analysis of educational levels, activities, structures, and instructional outcomes, with a focus on their potential for mathematics teaching from both curricular and pedagogical perspectives (Tnan, 2011).

In the coming years, numerous studies have been conducted on the topic of GC, with specific focus on their impact on students' and teachers' behavior (Nzuki, 2016; Parrot, 2014 and 2018; Mc Culloch, 2009; Leng, 2011, Quesada, 1994 and 2008; Ross, 2017; Mitchelmore 2000; Asli Ozgun-Koca, 2009; Tan, 2015; Lee, 2005). Furthermore, a number of authors have examined the process of teaching, learning and the formation of mathematical understanding via these technologies (Burril, 1996 and 2002; Drijvers, 1996; Ellington, 2003; Trouche, 2010; Robutti, 2010), as well as teachers' attitudes towards GC (Bologna et al. 2019 and 2021; Tortoriello and Veronesi, 2022). Generally, these papers describe educational activities conducted on a small group of learners, typically a class, and typically centering on a single topic.

The Mathematics Education Research Group of the Department of Mathematics at the University of Salerno has been working for years on an educational experimentation project that adds to the students' extracurricular activities and sharpen their critical thinking skills through the introduction of interdisciplinary topics that enrich their knowledge of Mathematics and its role in connecting classical and scientific fields (Rogora and Tortoriello, 2021). This plan also seeks to enhance the students' skills by providing them with the latest technological tools to interpret a rapidly changing reality (Tortoriello and Veronesi, 2021 and 2021).

In this paper in particular we decided to present and discuss a case of teacher training, and specifically the activity of a lesson dedicated to successions, to see how teachers consider the use of new technologies appropriate for teaching mathematics.

There is a need for practical analytical research tools and frameworks that offer the potential to analyze teachers' technology integration in mathematics. A commonly used framework, derived from Shulman's (1986) pedagogical content knowledge, is Koehler and Mishra's (2009) Technology Pedagogical and Content Knowledge (TPACK), which focuses on the aspects of teacher knowledge that are needed for the effective use of technology in the classroom.

The aim of this study is to assess how teachers consider it appropriate/effective to integrate technologies into the pedagogical and methodological aspects of mathematics. Through the lens of the Mishra framework, a training on the use of GC technology has been designed to improve teaching so that the technologies are integrated into teaching and disciplinary skills at the intersection of TPACK's knowledge domains. The activities carried out have been structured with the purpose of acting as an example and stimulating teachers to reproduce the same teaching model.

In this study, a qualitative analysis was conducted to record and analyze the impressions of teachers through observation during activities, focus groups and semi-structured questionnaires administered to teachers at the end of the activities.

MEANS AND TOOLS FOR ANALYSIS

This paper addresses issues of integration of theory and practice in the maths education and describes a laboratory that, beginning with episodes from real-life problems, proposes a set of theoretical models and situates these models in examples. The educational project focuses on the aim of creating and implementing activities that are tailored to the teachers' needs of developing the students' curricula through the GC to be used to facilitate learning. The project involves researchers who are knowledgeable in the fields of education, curriculum, and instruction and provides guidance to teachers in the development of activities to support the learning process. The activities are designed to promote engagement and collaboration between teachers and focuses on developing skills and knowledge that are essential for success in the classroom. Through this process, teachers are able to gain valuable insight into how to effectively utilize new technology, learning strategies, and content materials in the classroom with the aim of benefiting students from engaging with the new content in a meaningful way.

The aim of the research was to pay attention to the experiential data emerging in the performance of the activities using the research techniques of observation of free and semi-structured interviews (in the form of questionnaires) and focus groups. For the qualitative analysis of data through the lens of TPACK, a positive attitude towards the GC tool seems to emerge, the use of GC designed with a teaching perspective, the acquisition of new technological skills.

METHODS

The modality that appears most suitable, as it is easily repeatable in the classroom and therefore effective for the acquisition of technological skills by teachers, is the one that sees the domains of knowledge integrated and exemplified through modeling by the researcher-trainer and put into practice in "authentic" contexts by the participating teachers.

Contents and participants

In order to provide participants with didactic tools immediately usable in the classroom, prior to the beginning of the course the themes to be developed were agreed with them and two topics of interest were identified: the approach to the analysis and the problems of choice. The activity presented in the paper concerns the first theme and focuses on the concept of succession.

The participants in the educational workshop are four teachers of scientific high schools who teach in the fourth class, penultimate year of the course of study.

The laboratory of graphing calculators

Given the importance of the changes made by the Minister of Education to the final exams of scientific schools and the positive educational impact that the use of GC can have in terms of skills acquired by students, the research group designed an experimental laboratory. In the activities, teachers worked together in group to gain a deeper understanding of mathematical objects by viewing them in a variety of ways through the GC menu with different approaches, numerical, analytical, graphical, tabular. This provides an effective learning experience and allows participants to gain a more comprehensive understanding of the strategies that can be used in teaching.

Data collection and data analysis

As specified in the introduction, the analysis of the research was qualitative and developed in three different areas: the administration of questionnaires, the organization of focus groups and the interview of the participating teachers.

The questionnaires prepared are semi-structured and are divided into two parts, one with multiple answers and the other with open questions. Multiple choice questions concern the organization of the course in its entirety. We investigated the level of difficulty, the commitment and the workload required to follow the course, the organization with reference to the contents and methods and asked for an evaluation of the didactic impact in their teaching. It was asked to express an evaluation on the choice of contents addressed, on the clarity and organization, on the achievement of the learning objectives. Another group of multiple choice questions investigates the behavior of the course participants and the interaction between trainer and teachers and between the teachers themselves, the climate established during the activities, the stimulated interest and the full use of the paths by everyone. In the open question part, the teachers were asked what were the most useful aspects of the course from their point of view, what were the strengths and weaknesses of the course and they were asked to propose improvements for this type of activity to be repeated in the future.

In the interviews the trainer met each of the four participating teachers and asked them to evaluate the skills acquired with respect to the use of GC and the design of a mathematics didactics implemented with technology.

In the focus group (Zammuner, 2003) the researcher acted as a facilitator, directing the conversation towards the GC topic to garner the teachers' perspectives through questioning that guided the discussion. Group interviews proved to be an effective method in gathering data efficiently due to the interactivity between participants.

The activity

The activity took place in four meetings held during extracurricular hours. Using a heuristic approach, the participants explored the functionality of GC by engaging in simple content exercises. The topics were already known to the participants, allowing for an easy transition into the exploration process. Through this method, the group was able to dive deeper into the functionality of GC and further their understanding. After becoming familiar with the various menus of the GC, that of calculation, graph, tables, sequences, the activities were oriented to solve reality problems.

The activity we present in the paper refers to the study of sequences both as functions in $n \in \mathbb{N}$ (figure 1) and as recursive functions (figure 2).

Using the recursion menu, the behavior of a sequence was analyzed both from a tabular and graphical point of view and it was possible to compare the two models on the same screen to deduce the behavior of the sequence. The visualization of the trend of a sequence allowed to introduce the

concept of limit by observing it both from the point of view of values calculated in the table and as a sequence of points on a graph with behaviors that depended on the defined mathematical relationship. The teachers felt that the meaning of convergent, divergent and indeterminate sequence can be easily translated into the observations of the behavior of the graph or table.

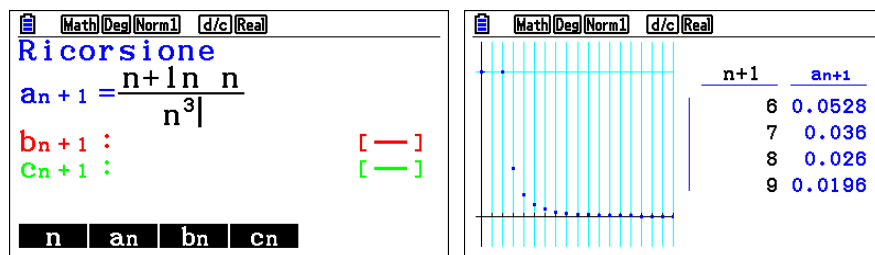


Figure 1. An example of the activity on recursion

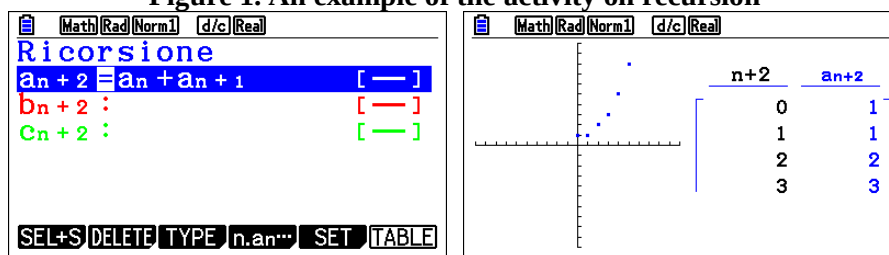


Figure 2. The recursive Fibonacci sequence

THEORETICAL FRAMEWORK

The Italian school reality has not so far officially provided for the adoption of the TPACK model, a model that outlines the knowledge domains underpinning teaching/learning processes in which technology plays a substantial role and, as a conceptual reference framework in the definition of initial and continuous teacher training courses. However, the set of regulatory interventions that have taken place over time defines a theoretical and methodological horizon that is in fact compatible with the model itself. Technology is changing learning and teaching strategies and, among the many questions that this introduction raises, the one that investigates what knowledge a teacher must have to successfully manage a teaching experience that involves the use of technologies in a decisive way, occupies a central role.

The well-known TPACK model proposes an answer to this question, explaining that a teacher must be competent with respect to the intersection of three types of knowledge: pedagogy, disciplinary content and technology.

The teacher must therefore be competent not so much and not only in relation to the technology itself, to the pedagogy and to the specific content of his disciplinary field, but above all in relation to the intersections of these domains, or to those uses of technology that support appropriate pedagogical strategies in relation to his subject of teaching.

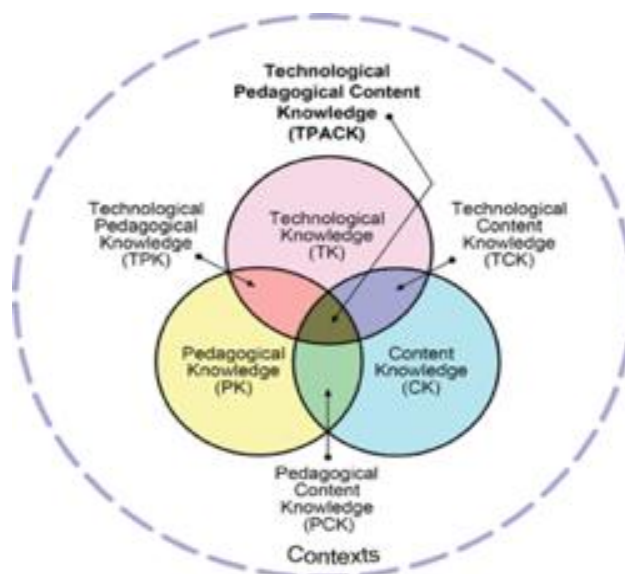


Figure 3. The TPACK model, © 2012 by tpack.org.

RESULTS AND DISCUSSION

Analysis of questionnaire responses led to the following results.

The teachers were asked to express with a vote from 1 (totally disagree) to 5 (totally agree) on the following aspects of the course:

- The activities were clear and organized, 100% voted 5
- experts stimulated participants' interest, 50% voted 5 and 50% voted 4
- created a positive atmosphere during the lessons, 100% voted 5
- Learning objectives were achieved, 25% voted 5 and 75% voted 4
- course content was organized and well planned, 100% rated 5
- course workload was appropriate, 25% voted 5 and 75% voted 4
- The course was organized to allow everyone to participate fully, 100% voted 5

Teachers were asked to express with a vote from 1 (very negative) to 5 (excellent), on the following aspects of the course:

- The level of commitment required by the course, 100% voted 3 (adequate)
- the organization with reference to the contents and methods of the course, 100% voted 5
- usefulness of course activities to improve academic performance, 100% voted 4
- the level of difficulty in participation, 50% voted 5 and 50% voted 4
- the interaction between the teacher of the course and the students, 100% voted 5
- the choice of topics addressed, 100% voted 5

To the open question on which aspects of this laboratory activity were most useful, the teachers replied that they had learned to use the graphing calculator and understood its potential especially from a didactic point of view. When asked to briefly describe a strong point of the course, the teachers answered the expendability and repeatability in class of the activities carried out and the acquisition of a method to make the students discover the calculator with an exploratory approach. No weaknesses of the course emerged. When asked how they suggested improving the workshop they had done, the teachers indicated that it might be useful to have a greater number of meetings to develop more different topics.

In interviews with teachers, it emerged that they had never previously used the graphing calculator to carry out teaching activities in the classroom as they did not know the tool and therefore did not feel masters of the potential to be able to replicate and teach them to students, so they did not want to be unprepared in front of questions on instructions or use to which they could not give answers. They expressed their concerns earlier in the course about the risk of students relying entirely on the results of the calculator by not developing critical thinking and stated that they changed their opinion during the course because, using it, they understood that the calculator can help students observe and deduce properties but does not replace the study of mathematics that remains necessary. It has also emerged how GC can be a valuable teaching aid for students who have learning difficulties.

In the focus groups we discussed the theoretical, pedagogical and methodological effectiveness of the GC focusing on the strengths and weaknesses of a teaching in which students use technologies and on the engagement of students in laboratory activities with GC instead of classic math lectures.

The teachers highlighted that GC can be particularly effective for an approach to mathematical contents according to the various learning styles, especially for those areas in which there are "dynamic" trends in the study, such as in sequences in which graphic and tabular observation helps to understand the behavior of the sequence and introduces in a natural way to the concept of limit. The teachers also underlined how the natural teaching with the GC develops in small groups as it happened during the laboratory and how they think they can replicate it in the classroom as in a

Peer Education collaboration helps students to overcome mutual difficulties in a comparison also thanks to their natural predisposition to the use of technological tools.

CONCLUSIONS

For the qualitative analysis of data through the lens of TPACK, a positive attitude towards the GC emerges, in fact teachers in the interviews and in the focus groups sublined positively the use of GC designed with a teaching perspective and the acquisition of new technological skills useful to project classroom activities tailored on students' needs.

With reference to the TPACK Theoretical Framework, the teachers and participants in the laboratory activities understood how the component of technological knowledge of the GC is not an end in itself but intervenes and collaborates in relation to the pedagogical and content area.

During the course the teachers experienced that their competence in technology must not simply be a validation of experience and skills but must be integrated and must be at the service of the disciplinary contents that are proposed and the pedagogical tools chosen to develop them in the classroom. Technology therefore becomes an adequate support for pedagogical strategies and theoretical insights.

From the participants' comments emerged that for teachers the GC is an effective tool to promote conscious exercise and convey new mathematical contents bypassing the mere calculation, being able to focus exclusively on theoretical development and exploration. The teachers, who had never used GC before their participation to the project, underlined the positive impact in terms of active participation and recognized the didactic effectiveness of the tool GC that can promote and empower laboratory activities of teaching mathematics. Furthermore, the activities can help to build skills such as problem-solving, teamwork and creativity, which can all be beneficial in other aspects of life.

The experimental laboratory enabled the participants to actively engage with the material, as well as to practice their problem-solving and analytical skills. The interaction between participants and knowledge was mediated by the GC tool, providing an innovative way to explore mathematical objects through a variety of stylistic approaches, allowing for a heuristic exploration of mathematical concepts. In addition, this tool provides a platform for participants to better understand the abstract concepts that form the basis of mathematical knowledge.

Ultimately emerged that the engagement in these activities can help to create a more meaningful and favorable approach also for students with learning difficulties.

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ABSTRACTION OF MOTION IN PATTERNS ENHANCE ALGEBRAIC GENERALIZATION IN MATHEMATICS CLASSROOMS

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In this paper we present results of a study that investigated year 8 (ages 12-13) students' construction of meanings for algebraic generalisation. The students worked in groups in the classroom using a specially designed microworld (eXpresser) to explore a task that linked the realistic situation of a bee hive to figural patterns. In order to trace student's meaning making for algebraic generalization, we used abstraction in context to analyse a classroom focus group and the mathematical progress of classroom community. Analysis allowed us to follow the evolution of construction of meanings for the concept of variable. It also indicated the critical role of the teacher and available tools in knowledge shifting when students generalise while modelling a realistic situation. We focus here on the students' knowledge transformations about the concept of variable through motion as they worked in groups or the whole class setting.

Keywords: patterns, generalisation, variable, abstraction, classroom

INTRODUCTION

Generalisation of patterns is considered a way to introduce algebra (Radford, 2018). The existing literature indicates that the use of a visual aid in presenting problems involving the search for patterns can lead to the application of different approaches to achieve generalisation, either of visual or nonvisual nature. However, teachers traditionally tend to mainly emphasise the numerical patterns and not the visual ones. In many situations, this can be problematic when trying to help students reach an algebraic expression which generates any term of the pattern (Barbosa & Vale 2015). Also, even though students can identify and predict patterns (Mason, 2005), they are unable to articulate a general relationship in natural language or in algebraic symbolism. Existing research around patterns has shown that this difficulty perhaps arises because patterns are presented mainly in a static way through images that show some of the early stages. Yet, students are asked to construct near and far generalisations for any stage or figure number in the pattern through questions such as: How many are here? How many are there? And 'How many are there in general?' (Rivera, 2007; Radford, 2018). Specific studies (e.g., Mavrikis et.al., 2013) have reported that students aged 12–13 usually resort to counting in order to answer 'How many' questions, especially if the thing being counted is static.

Another reason for this weakness could be most students' superficial understanding of the concept of variable that lacks multiple representations. Variable is the main component of algebraic notation and used to express mostly unknown quantities. Researchers (Mavrikis et.al., 2013) have indicated that the process of algebraic generalisation requires recognising a relationship between quantities and variables and expressing it through general statements. In recent research, teachers' predominant teaching approach centred on the treatment of 'letters as unknowns' rather than as variables. Additionally, teachers used algebraic patterns to support students' developing appreciation of an algebraic variable as a dynamic concept (Clark-Wilson & Hoyles, 2017). An approach through patterns, especially in a figural form, has been proposed (Rivera, 2007) as a fruitful way to introduce students to the notion of variable. Also, the combination of figural patterns and real-life contexts seem most suitable for supporting students' developing knowledge of variables and ability to express generalisations symbolically (Maj-Tatsis & Tatsis 2018).

As regards the use of digital tools in pattern generalisation, recent research approaches have suggested microworlds' reliability. This is because when a mathematical microworld is placed within a realistic context which is meaningful for the learners, it can build a basis for progressive algebraic generalisation (Mavrikis et.al., 2013). The realistic scenario microworld designers choose most often is that of motion in the sense of animation. This is because motion is not just an illustration but a paradigm for most of the part of mathematics that examines change. This evolution from static to dragging, sliders and animation has presented new challenges. These concern specifically the role of teacher and researcher claims that the dynamic motion context supports teachers in rethinking their own mathematical knowledge and developing more precise mathematical responses to the meaning of a variable provided by the dynamic mathematical technology (Clark-Wilson et. al. 2017). Also, applying an embodied perspective to the design of animations facilitates understanding of a dynamic system and be enhanced by asking learners to re-enact or follow movements through gesturing (de Koning & Tabbers, 2011). However, many teachers lack confidence in using dynamic affordances in their teaching. This creates a difficulty in studying interactions in the complex classroom environment with digital tools and the role of the teacher (Clark-Wilson et. al. 2020). For instance, there have been studies over the last decade based on eXpresser: a specially designed microworld with explorative tasks aimed at linking figural patterns to algebraic relations. The Geraniou & Mavrikis 2015 study indicated the need for further studying how the available software structures, the teacher and classroom community mediate students' generalisations as abstractions. Also, despite some successes, difficulties remain, and these tend to coalesce around the need for significant pedagogic support from the teacher to provide a bridge to algebraic symbolism, especially variable and generalisation.

Our literature review in the field revealed that over the past twenty years, there has been a plethora of studies and research findings on algebra that focused on algebraic thinking through patterns with or without the use of technology. Despite that, we still know very little about the evolution of meaning making for algebraic generalisation in the classroom environment. For this reason, in this study we investigate year 8 students' construction of meanings for algebraic generalisation. The students collaborated in groups using the exploratory microworld eXpresser to create the pattern of a bee hive by expressing it through repeated building blocks of square tiles and developing the rule underpinning the calculation of the number of tiles in the pattern. The microworld allows students to use iconic variables to reproduce their constructions for a different number of repetitions, to express generalisation and to check their correctness through appropriate feedback. In this paper, we focus on the role of teacher, classroom community and available tools to students, proceed to algebraic generalisation and then consolidate the concept of variable through motion.

THEORETICAL BACKGROUND

A theoretical framework for investigating processes of constructing abstract mathematical knowledge is Abstraction in Context (AiC) (Dreyfus et.al., 2015). AiC offers a way to describe how meanings are constructed by shedding light on their connections through three epistemic actions: recognising (R), building-with (B) and constructing (C). Recognising an already known mathematical concept occurs when a student recognises it as inherent in each mathematical situation. Building-with involves combining existing knowledge elements to solving a problem or justifying a solution. Constructing is carried out by assembling or integrating previous knowledge elements to produce a new structure. The model suggests constructing as the central epistemic action and constructing actions are at times nested within more complex constructing actions. Those constructs that are nested in others are called sub-constructs. This progressive, familiar epistemic action is called consolidation. It expresses the increasing ability to recognise and of building-with. The model is called the "RBC+C-model." In it, the second C is used for Consolidation (Dreyfus et al., 2015). The RBC methodology begins with a priori analysis of the task, in terms of the knowledge

elements that students should construct. Those knowledge elements are then used as milestones in the analysis of the processes of construction and the constructs that will emerge for the students during their engagement with the task.

According to Radford, learning occurs when the cultural knowledge has been transformed into knowledge for students, through students' and teacher' joint activity (Radford, 2022). In our attempt to contribute to the research about generalisation of patterns and to see what happens in the evolution of students' meaning making for algebraic generalisation, we examine this joint activity that Radford called "joint labour", as a path through the lens of AiC. To shed light on the transition process we consider the construction of knowledge as an abstraction process where construction corresponds more with the notion of knowledge transformation and less by knowledge construction. So, in short, we make a systematic investigation of the joint labour by analysing it with epistemic actions, and by identifying knowledge elements in algebraic generalisation of patterns. Influenced by the discussion of the transformation of knowledge through classroom interaction (Dreyfus et.al., 2015) and its contributions to current research on the role of the teacher, our paper focusses on how the teacher, classroom community and available tools contribute to the creation and synthesis of those processes so that students can reach algebraic generalisation and consolidate the concept of variable through motion.

EXPRESSER

eXpresser is a mathematical microworld designed to support 11–14 years-old students in constructing figural patterns using coloured square tiles. More precisely, includes a set of actions that can be carried out so that students can build patterns and test their generality through animation (Mavrikis et.al., 2013). Some key features of eXpresser are visible in Figure 1. It consists of two main areas: (a) My Model (a work area, on the right of the screen); and (b) Computer's Model (on the left).

In My Model, students can use building blocks of square tiles to make patterns that can be combined to models. EXpresser allows students to work with icon variables so as to reproduce dynamically (i.e. to 'animate') their patterns for different numbers of repetitions, express generality through semi-algebraic relations, and test the validity of these relations for random values of repetition through appropriate feedback. By default, all numbers in eXpresser are constants and it is possible to change their values by "unlocking" them to become variables. Students can convert a constant number to a variable in My Model while in Computer's Model, the variables take random values. Students can construct a model rule for the total number of tiles. If the pattern is correct, it will be coloured. Otherwise, as an indication of error, the pattern appears with no colours. Finally, when students find the general rule in Model Rule, the sign "✓" appears and the face icon on the central toolbox turns green and "smiles". From our perspective, it is crucial to see how this microworld, which can provide motion (in the sense of animation) to patterns, the setting of a classroom in coordination to the realistic context of a hive coexist and shape knowledge transformations with which students can reach algebraic generalisation of patterns.

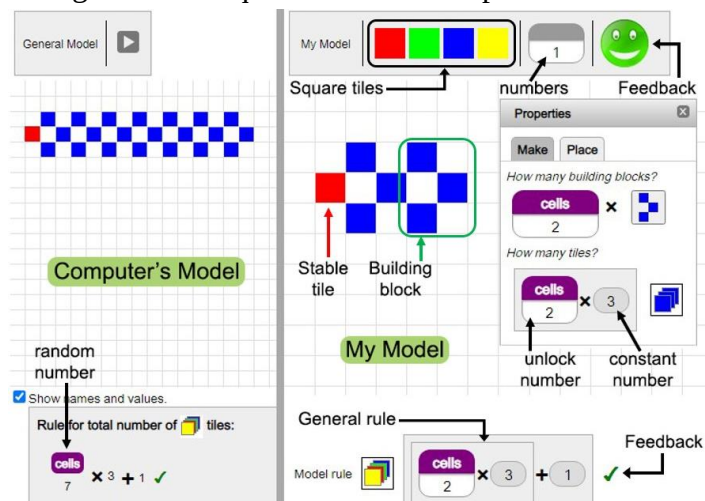


Figure 1. Pattern on eXpresser

METHODOLOGY

The data analysed here come from a larger study in a lower secondary experimental school in Athens. Our research approach is informed by the idea of “design” in learning (Prediger et al., 2015) and task design aims to link patterns to a problem associated with the realistic context of a hive (Figure 2). Thus the need for algebraic generalisation was expected to arise as part of a beekeeper’s problem solving. 24 Students (year 8) worked in six groups of four in 25, one-hour sessions over two months. Analysis in this paper focussed progressively on students’ meanings making for the concept of variable through motion. The class teacher implemented the task and acted as facilitator, challenging the students with specific reference to the authentic context. A researcher was a participant observer who handled four cameras and six recording devices for data collection and specifically captured the focus group and the whole classroom experimentation. The data were fully transcribed for the analysis. Episodes were analysed through the lens of AiC. Classroom’s utterances that appeared to be crucial in students’ conceptualisation and expression of generalisations were coded through the RBC+C model. The task “Beekeepers” requires the exploration of a two-dimensional pattern consisting of several non-linear patterns. This task is designed to let the students experience patterns with two variables: the length and width of a honeycomb. The aim is to find the most economical pattern of waxes, so students are asked to compare triangle, square and hexagon in relation to the economy of perimeter and surface coverage. Students have to explore the building blocks through manipulatives, so the teacher gave them cardboard squares and tire-up sticks (Figures 3, 4) to recognise the pattern’s building block. Then students construct those patterns in eXpresser and find an algebraic formula to describe the total number of honeycomb walls. The expected formula is $2xy+3x+y+1$, where x is the number of cells in a row and y the number of series of cells vertically.

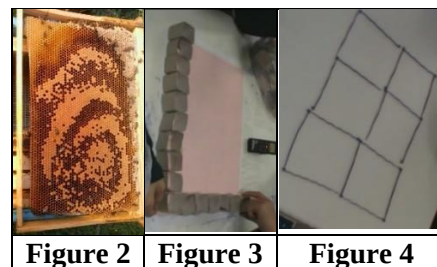


Figure 2 **Figure 3** **Figure 4**

A PRIORI ANALYSIS OF THE TASK

From a priori analysis, the main knowledge elements in this task were: the construction of knowledge elements C_{bb} – building block, C_{un} – unlock number, C_{mr} – model rule, where sub-constructs were nested in C_{un} : C_{un2} – 2nd unlock number, C_{mo} – motion and in C_{bb} : C_{bb2} – 2nd building block. The main difficulty of this task is that the hive pattern consists of two different non-linear patterns.

RESULTS

Our results came from one small group’s work and a whole class discussion in our attempt to show how the concept of the variable transformed through students’ and teacher’ “joint labour”. We analyse focus group’s work on the hive pattern (Episode 1) and close our analysis by a whole class discussion. In the latter, the teacher highlights the motion that two separated variables provide, and students consolidate the concept of variable (Episode 2). Episode 1 took place during the 17th teaching hour where while working in eXpresser, students recognise the need for the 2nd building block (C_{bb2}) for two different patterns series of squares (Figure 5). As students’ arguments still seem weak, during the 18th teaching hour the teacher called for a whole class discussion by inviting the students in the focus group to share their findings with the whole class. All names used in the dialogue of the two episodes are pseudonyms.

Episode 1: Recognise the need for two variables.

Although students subsequently construct the concept of two independent building blocks (C_{bb2}), they have not yet recognised the need of two independent variables (C_{un2}). For this reason, the teacher, (1) tries to mention the need of independent number of columns and series in the honeycomb, as they work in groups.

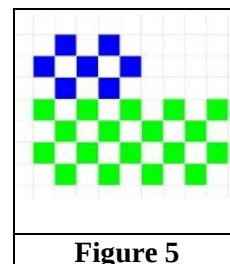


Figure 5

- 1 Teacher: Could you show me a hive with 10 columns and 3 series?
- 2 Chloe: No, we have “walls” [unlock] everywhere, so all change simultaneously.
- 3 Teacher: But we want a different number of series and columns [show to the right/down]
- 4 Kate: So, we don’t want the “walls” everywhere, lets create another one, unlock number named “toi”!
- 5 Teacher: Find a name that makes more sense. What does it mean “toi”?
- 6 Chloe: Just a way to say walls. So, unlock number is the number of cells in the first series, let’s name it cells! [**Connection C_{un2} – C_{na1}**]
- 7 Teacher: And now what is at 3? [The teacher changes unlock number from 4 to 3.]
- 8 Kate: Its series... it’s the number of the series that consists of 2 walls each time, so, let’s name it “series”. [**Connection C_{un2} – C_{na1}**]

Here, students have to deal with handling two separated unlock numbers (C_{un2}) and their names, something they (Chloe 2) seem to recognise (R/C_{un2}). After the teacher's interventions (Teacher 1, 3), they (Kate 4) also demonstrate building with (B/C_{na2} - C_{un2}) through naming. Next, the teacher helps students construct the 2nd unlock number (C/C_{un2}) by requesting students to give meaning to the name (Teacher 3). Also, the teacher indicates for the first time how to manage two different directions (Teacher 3): one for the columns and one for the series. The teacher pays attention to the motion of the pattern pointing right and down with their hands (R/C_{mo}). Students through name negotiation, came up with two names, cells for the first unlock number (Chloe 6) that controls the number of cells in each series, and series for the second (Kate 8) that controls the number of the series. In this episode we see how students cooperate with their teacher to approach the creation of a hive with different width and length. In short, the group developed a pattern of square cells with only one unlock number (C_{un}) that simultaneously controls the rows and columns (Chloe 2). Through the discussion with their teacher students recognise that they need more unlock numbers (C_{un2}) that at the same time means different things (Chloe 6, Kate 8). It is worth mentioning that as students are building with (B/C_{na2}) – the name of the 2nd unlock number – they then construct (C/C_{un2}) – the meaning of the 2nd unlock number – (Chloe 6). When they construct the name of 2nd unlock number (C/C_{na2}), they then consolidate the meaning of 2nd unlock number (C/C_{un2}).

Episode 2: Consolidate two separated variables that moves.

In episode 2, students consolidate the concept of variable through the motion that eXpresser provides and discuss in the whole class the need for two separated variables. This is an episode that shows us the crucial role whole-class discussion plays in consolidating the concept of variable.

- 9 Teacher: Unlock number determines how many building blocks I use. Give it a name!
- 10 Students: Cells.
- 11 Teacher: Cells well, and there, in model rule, what should we fill in there??
- 12 Kate: Cells multiply by 3, plus one for the front square to close the first cell.
- 13 Teacher: Ok, now the second pattern of the series. Would this be the same pattern?

- 14 Mia: No, there are already walls above so we need 2 extra walls each time. Put 2 lime [-coloured] squares for the new building block and unlock the number.
- 15 Teacher: Yes, but now we have 4 cells above and 5 below. How grow them together?
- 16 Kate: We have to replace unlock number of the second series with the “cells”.
- 17 Teacher: And what do we have to insert in the model rule now?
- 18 Mia: Cells multiply by 3 plus 1, plus cells multiply by 2 plus 1.
- 19 Teacher: Ok, but we have only two series. How can we create more downwards?
- 20 Kate: Make a new pattern with the 2nd series, unlock a number and name it “series”!
- 21 Student: No, “columns”!!!
- 22 Teacher: The columns grow vertically [moves hand to the right] and series horizontally [moves hand downwards]. What change do we make now in model rule so all series are coloured?
- 23 Chloe: “Cells” multiplied by “series” multiplied by 2 because each cell has 2 walls.
- 24 Kate: So, cells used to move horizontally, while series move vertically!
- 25 Chloe: Move them to check. See, when you change “cells,” the blue pattern moves right together with the lime [-coloured] one, and when you change “series,” the lime [-coloured] pattern moves down.
- 26 Mia: Unlock number series is down and cells is right! (**Connection Cmo– Cv**)
- 27 Teacher: So, can you describe the total number of walls with a formula?
- 28 Kate: $2 * \text{series} * \text{cells} + 3 * \text{cells} + \text{series} + 1$, where $2 * \text{series} * \text{cells}$ refers to the whole lime part of the pattern, $3 * \text{cells}$ to the first row of blue squares, series to the column of lime squares that closes the lime part of cells and the 1 for the square that closes the blue row of cells. Series is the number of rows and cells is the number of columns. (**Connection Cmr– Cbb2– Cun2–Cmo– Cv**).

In this episode, two noteworthy ideas are developing. First, that unlock number describes the number of building blocks that are the cells. Recall that in episode 1 (small group work), this idea was put forth as a claim. Now in the whole class, it appears as data. More precisely, it is interesting that unlock number is renamed (B/C_{na2}) from “walls” in the previous group work to “cells” in the whole class discussion (Kate 12). Moreover, in this episode we see that many knowledge elements found in the group work were on the building with phase. Now in the whole class discussion, the knowledge elements are constructed or consolidated through the presentation. For example, students consolidate (C/C_{bb2}) the 2nd building block (Mia 14) and construct (C/C_v) – the concept of variable – using the same unlock number in order to create the same amount of cells in the two different patterns of the 1st and 2nd series (Kate 16). The second noteworthy idea developing here is the merging of the two interpretations for unlock number “cells”, namely that of the number of cells in a row and the interpretation of unlock number as a tool which gives motion to the right (Chloe 25). Here we also have corresponding interpretations about unlock number “series” that controls the number of series to the motion down (Mia 26). Regarding the RBC model, the teacher once again calls attention to the motion with by making corresponding hand gestures. This helps students recognise (R/C_{mo}) this extra feature of unlock number (teacher 22). Students demonstrate building with (B/C_{mo}) this new feature (Kate 24) by associating each variable with a separate move and construct (C/C_{mo}) it (Chloe 25) by confirming moves in software by manipulating variable sliders. In addition, Mia (26) consolidates it (C/C_{mo}) by describing the connection of unlock number with motion in reverse when she says ‘Unlock number “series” is down and “cells” is right’. Last

but not least, we see the gradual building with (B/C_{mr}) of model rule (Kate 12, Mia 18), then its construction (C/C_{mr}) (Chloe 23) and finally, its (Kate 28) consolidation (C/C_{mr}). As a result, the students finally achieve representing a semi-symbolic formula. This is possible because, the concepts of 2nd building block, 2nd unlock number and concept of variable had been consolidated.

CONCLUSIONS

As shown in our analysis, by using the RBC model, we found that constructs corresponded to the knowledge element of variable. For example, we found that the unlock number controls the rate of change that features a variable with motion. The key element was the “unlock number” of eXpresser. The concept of the variable (C_v) seems to be built-with and be constructed through the manipulation of the unlock number (C_{un}) and linking it with the other software elements simultaneously via the different task features, such as the number of cells and rows. Furthermore, students tried to express their thoughts through contextual task elements and build links between prior knowledge constructs. For example, we observed an evolution in the way of giving name to unlock number (C_{na}). At first, the names were given by chance, but then evolved to be more related to the task activity. In the task, the main difficulty was that students had to manipulate two independent unlock variables (C_{un2}) with independent names (C_{na2}). This added another feature to the concept of the variable (C_v), the property of motion (C_{mo}), that students conceptualised as the way to move to different directions, to the right and down. Finally, the consolidation of the concept of variable came about because of an attempt to connect all the previous consolidations of nested sub-constructs that are motion, 2nd unlock number, 2nd name, and 2nd building block to create the model rule (C_{mr}).

The teacher contributed by going back and forth between small group work and whole-class discussion and highlighted crucial knowledge elements when it was necessary. These included the need for different building blocks, motion through unlock numbers and names of variables related to context of the hive. The motion context, supported by the animation, enabled the teacher to make the important connections between knowledge elements. Moreover, the teacher indicated the need for more than one unlock number, focused the students' attention on finding relationships between the unlock numbers, and find meaning in each software element. It's worth mentioning that within the group work, we observed that after the teacher commented, the result was: students tended to connect some knowledge elements, recognise others' knowledge elements, and/or push meaning making of those elements higher. The analysis provides evidence of the importance of whole-class discussion, which was an occasion for participants to develop ideas beyond what had been constructed in small groups. More precisely, as students tried to share their ideas with the whole class, ideas gradually be condensed through discussion, as for example we observed in the students' attempt to present the model rule. Additionally, many knowledge elements present in the group work were in the building-with phase. Then in the whole class, these knowledge elements became constructed or consolidated through the presentation. We observed this, for example, in the students' attempt to describe what happens in the pattern where they reached to consolidate the model rule. As this paper has shown, the diversity of available tools and the classroom community are bearers of great potentiality. They can make the learning processes, that mediate until generalisation becomes visible, emerge. We saw that the construction of the meaning of the concept of variable initiated through different collaborative and individual social settings, including the classroom where students moved between working alone, group work and whole-class discussion and technological tools supporting the process of abstraction. In summary, we saw that algebraic activity is characterised by actions of representing and manipulating and because of its abstract character, algebra is not an easy subject to teach without any context. Although, in this paper we have provided some evidence to the evolution of student's meaning making for the concept of variable through motion, both the critical role of the teacher and available tools in knowledge

shifting when students generalise while modelling a realistic situation still deserves more investigation.

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Fostering mathematical collaboration and communication

DESIGN OF DIGITAL ENVIRONMENTS AIMED AT FOSTERING ASYNCHRONOUS WORKING GROUP ACTIVITIES: EMERGING CATEGORIES OF STUDENTS' COLLABORATIVE PROCESSES

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In this study we focus on the collaborative processes developed within an asynchronous small group activity carried out through an instant messaging platform. To investigate the ways students communicate and collaborate during the asynchronous activity, we refer to the theoretical lenses provided by the dimensions introduced by Weinberger and Fischer (2006) to study the processes realised within computer-supported collaborative learning environments. In particular, we develop a combined analysis that takes into account both the quantity of students' interventions and their social modes of co-construction. This analysis has revealed two main macro-categories of collaborative processes activated by the groups of students who participated in the study, which mainly differ in the level of sharing and comparison within the groups and in the aim that guided their work.

Keywords: Asynchronous discussion, collaborative processes, instant messaging communication

INTRODUCTION

The pilot study presented in this paper is part of a wider research project aimed at investigating the potentialities of digital environments in supporting mathematical discussions conducted in an asynchronous way. Asynchronous discussions are characterised by a communication between participants that are separated in space and time (Andresen, 2009) and are structured as linked messages relating to a common discussion topic. They have become an emergent field of research in the last two decades (see, for instance, Johnson, 2006; Gao et al., 2013). Digital environments play a role in mediating asynchronous mathematical discussions in terms of both stimulating different modalities of students' engagement and communication, and of supporting the teacher in the orchestration of the discussions. In fact, the asynchronous nature of the discussions enables students (and teachers) to take their own time in reading others' posts and comments, preparing thoughtful responses and reflecting upon their contributions (Andresen, 2009). Moreover, by providing a transcript of the discussion and making all communication elements explicit, the use of a written form of communication makes the discussion more transparent for those who are involved within it, turning it into an effective support for students in both studying and reflecting (Meyer, 2004).

The digital environment investigated in our study involves the combined use of two different platforms, aimed at two main purposes: (a) supporting students' collaborative processes when working in groups in an asynchronous way, and (b) supporting the teacher in orchestrating effective asynchronous mathematical discussions (Bartolini Bussi, 1996; Cusi & Malara, 2013). In this paper, we focus on the role played by this environment in fostering purpose (a).

THEORETICAL FRAMEWORK: COLLABORATIVE LEARNING ACTIVITIES WITHIN THE CONTEXT OF SOCIAL MEDIA

The role of online social media for improving collaborative learning among students has been widely reported and discussed especially during the Covid-19 pandemic (Bakker et al., 2021; Biton & Segal, 2021). The discussion on the use of social media for teaching and learning purposes has

highlighted the potentialities related to the fact that learning can occur anywhere and at any time (Naidoo & Kopung, 2016), enabling students to take their own time in reading, reflecting and answering to other students' contributions and in giving them the opportunity to re-organise their knowledge and to express and communicate it in different ways (Biton & Segal, 2021). Moreover, research has shown the effective role played by social media in fostering students' engagement and in promoting their collaboration (Naidoo & Kopung, 2016). As regards this last issue, Weinberger and Fischer (2006) developed a framework to analyse different process dimensions of knowledge construction in computer-supported collaborative learning environments. They state that four dimensions should be taken into account when analysing the computer-supported collaborative learning environments: (1) the participation dimension, that consists of the quantity and the heterogeneity of participation; (2) the epistemic dimension, that refers to how learners work on the knowledge construction involved in the task they are working on; (3) the argument dimension, that concerns the construction of single arguments and the construction of sequences of arguments; and (4) the dimension of social modes of co-construction, that refers to how learners work on the task and formulate arguments referring to other learners' contributions or individually. The different social modes can be characterised according to the degree with which learners refer to the contributions of the other participants:

- (a) Externalization (EX), when learners make contributions without (implicit or explicit) reference to previous contributions to the discourse. It occurs typically at the beginning of a discussion or when a learner does not comment on any other message present in the discourse.
- (b) Elicitation (EL), when learners request information, feedback or specific actions from the other participants. It is important to observe that learners can rely on and abuse of elicitation, so it positively contributes to knowledge building only if the learner receives help but applies it on his own to the problematic situation.
- (c) Quick consensus building (QC), when learners accept other participants' contributions just in order to move on with the discourse and without taking charge of it. The acceptance can be explicit, such as "That's right!", or it can consist of a rephrasing of the contribution without changing its meaning.
- (d) Integration-oriented consensus building (IC), when learners' perspectives are taken over. It generally occurs when learners integrate their contributions during the discussion.
- (e) Conflict-oriented consensus building (CC), when learners do not accept the contributions as they are, but they replace, modify or supplement them.

DESIGN OF THE ACTIVITY

The activity implemented in the pilot study documented in this paper, called "Digital Mathematical Discussion" (DMD), is set within a digital environment characterised by a combined use of two digital platforms and it consists of two main phases. First, students, divided into small groups, work within chats of an instant messaging platform (Telegram in this study) to collaboratively face a mathematical problem. In each chat, the teacher is present only to answer students' questions on practical aspects, if needed. Students are given a few days (in our study, 4 days) to collectively solve the problem communicating only through the chat and to send to the teacher (by email or through a shared e-learning platform, such as, for example, Moodle) the written solution agreed by the members of the group. In order to limit additional interactions between the participants on the problem they have to face, it is advisable to set this part of the activity in a moment of the week within which students have a limited amount of presential classes. The second part of the activity consists in a collective discussion conducted on a collaborative web platform, Padlet

(<https://padlet.com/>) in this study. The discussion is designed by the teacher starting from selected excerpts from the groups' solutions (Cusi, Morselli & Sabena, 2017) and from significant interactions that emerged in the Telegram chats. The Padlet link is shared with students as soon as all the groups submit their solutions and students can insert their comments within a period of about 4 days (more days can be considered if needed). Finally, a collective discussion is conducted in presence to resume and deepen the reflections emerged on the Padlet.

Since research has documented pitfalls related to the distracting nature of social media, in particular when students work with mobile instant messaging platforms (Naidoo & Kopung, 2016), we decided to design the activity with an accompanying set of indications to regulate the use of the social media for educational purposes: (1) interact only on the Telegram chat and on the Padlet devoted to the activity, trying to avoid any other form of communication regarding the mathematical problem under discussion; (2) make all the thinking processes explicit, including the ones regarding the resolution of the problem, doubts, ideas, difficulties, individual explorations, etc. in order to let the thoughts become visible to all the participants; (3) share all the used materials, such as pictures of what is written (if paper and pencil are used to develop the reasoning); (4) do not use vocal messages in order to maintain the communication written.

RESEARCH CONTEXT AND CHOICE OF THE TASK

The students involved in this study are preservice primary school teachers enrolled at the first year of the master-degree course "Primary education sciences" at Sapienza University of Rome. The activity DMD was implemented within a 48 hours course aimed at making students develop reflections, in an integrated way, on specific mathematical contents, mathematical processes and on specific pedagogical aspects of mathematics teaching-learning. This sample was chosen since, during the course, the students got used to a teaching methodology characterised by small group collaborative problem solving activities, followed by classroom discussions designed by the teacher educator (one of the authors) starting from groups' written productions and orchestrated with the aim of fostering comparison between different approaches and reflections on the thinking processes activated to face the given problems.

31 students, divided into 5 groups of 5-7 students, participated in this study. The task (Fig. 1) proposed to the groups within the Telegram chats was in tune with other problem solving activities faced during the course, which were aimed at making them experience the use of algebra as a thinking tool by means of tasks focused on numerical explorations, conjecture, argumentation and proof.

Observe carefully this table. Do you notice any regularities? After listing all observed regularities, try to justify them.	1	3	3
	2	4	8
	3	5	15
	4	6	24
	5	7	35

Figure 1. The task faced by students within the Telegram chats

RESEARCH QUESTIONS AND RESEARCH METHODOLOGY

The research question that guided the pilot study documented in this paper is the following: What kind of collaborative processes are fostered, within the phase of the DMD devoted to asynchronous small group activities, by the design of the digital environment where the DMD activity is set?

The data collected to investigate the collaborative processes developed by the groups of students within the instant messaging platform were: all the messages written by students within the Telegram chats; all the files uploaded by students within the chats; the written answers uploaded by each group of students within the e-learning platform of the course. To investigate the ways in which students communicated and collaborated with each other within the chats, we analysed these data by referring to the theoretical lenses provided by Weinberger and Fischer (2006). In particular, we focused on two of the four dimensions introduced by Weinberger and Fischer, that is the participation dimension and the dimension of social modes of co-construction. We developed our analysis by combining the quantity of students' participation with the qualitative analysis of students' interventions. The quantity of students' participation was investigated by counting the number of interventions that each student proposed within the chats. The qualitative analysis of students' interventions was developed by coding them according to Weinberger and Fischer's categories of social modes of co-construction. To better determine the category to which each intervention refers, we also distinguished between the interventions that are focused on content (the problem under investigation, the argumentations to be developed, the mathematical knowledge at stake...) and the interventions focused on organisational aspects (the roles to be played by each student, the ways in which to organise the answer to be sent to the teacher...). We also counted the number of students whose interventions refer to specific categories.

Moreover, in order to investigate students' perspectives on their experience of participating in the DMD as further evidence of our interpretation of the processes developed throughout the whole activity, we asked them to answer a written questionnaire. The questions asked in relation to the small group collaborative activity within the Telegram chats were aimed at investigating aspects such as students' difficulties in respecting the given guidelines for the activity, students' opinion about the number of messages written within the chat, students' feeling of being listened by their mates during the small group activity, the usefulness of having their mates' written messages at disposal, the usefulness of working in an asynchronous way, their reflections on the comparison between working in groups in presence or within the Telegram chats.

ANALYSIS

Table 1 exemplifies the ways in which we developed the qualitative analysis of students' interactions within the Telegram chats. The students' interventions within the transcript (column 1) are coded (in column 2) in terms of social modes of co-construction, distinguishing between EX (externalization), EL (elicitation), QC (quick consensus building), IC (integration-oriented consensus building) and CC (conflict-oriented consensus building). In column 2 the coding process is made explicit. We chose this specific transcript (belonging to group C's chat) due to the variety of categories of social modes of co-constructions that can be referred to students' interventions within the chat.

Excerpt from the transcript of a chat	Coding of the interventions according to Weinberger and Fischer categories of social modes of co-construction
11:08 I: Good morning! Have you noticed any other regularity beyond the ones already written? So, if you agree, in the meantime we can reformulate the regularities and write them in order, so that then we can focus on the missing generalisations	EL – Student I implicitly requests some actions from the group in order to organise the following work. The focus is on organisational aspects.
11:10 E: Good morning, I agree with the regularities you identified. I haven't found any different	QC – Student E agrees with the issues proposed by student I and reformulates her

regularity. So, we can stop here and put them all together.	message. The focus is on organisational aspects.
11:22 C: I agree, I would start from the more “trivial” observations, that is the relationship between the first and the second [columns], second and third [columns] and first and third [columns], then I would move to the more articulated ones, such as the difference between results, etc...	IC – Student C proposes her point of view on the ways in which the observed regularities should be listed in the collective document.
11:28 I: Perfect!	QC – Student I explicitly agrees.
11:29 I: How do we want to organise ourselves for the file we have to submit? Is one person writing it and then sharing it here in the chat?	EL – Student I asks her mates how to organise the writing of the collective document.
11:31 A: It can be done this way	QC – Student A explicitly agrees.
16:23 <i>I shares a document with the collection of all the regularities emerged in the group</i>	
16:23 I: Girls, I decided to write in a unique file all the regularities we found. Just generalisations are missing. Let me know what you think about it.	EL – Student I is leading the writing of the document, but she wants to involve her mates, by explicitly asking them feedback on the file she started creating.
16:25 A: Excuse me, according to me the first two regularities are not essential, but I don’t know	IC+EL – Student A is pointing out her perspective on the content of the document; the final “but I don’t know” can represent an implicit request for feedback.
16:26 I: Indeed, on the first [regularity] I had your same doubt because I wasn’t sure to write it	IC – Student I is boosting the discussion with A expressing her accordance and her point of view.

Table 1. Qualitative analysis of an excerpt of the Telegram chat of group C

We summarised the results emerging from our combined analysis within 5 tables (one for each group) with the aim of supporting the comparison between the collaborative processes that characterised each group’s interactions. A first comparison between these results enables to highlight two main macro-categories of collaborative processes developed by the groups of students involved in this study: (1) interactions characterised by a widespread lack of shared reflections on each other’s proposals and mainly aimed at producing a written answer to be sent to the teacher; (2) interactions characterised by a constant comparison between each other’s proposals and mainly aimed at developing a really shared answer to the given task.

Table 2 and 3 summarise the results of this analysis for two groups (group A and group C), belonging, respectively, to category 1 and category 2.

The interactions observed within the chat of group A (Table 2) are characterised by a predominance of QC and EL interventions focused on organisational aspects. Table 2 also highlights that the work is guided by few leaders, since EX and EL interventions are proposed only by 3 students among 7. What is not evident from the table, but was observed through the qualitative analysis of the chat, is that, although students do not propose reflections on what their mates write in the chat and limit themselves to agree or to add further proposals, there is an effort to take all these proposals into account to be integrated into the written document to be uploaded within the e-learning platform.

Types of interventions	Number of interventions	Number of interventions focused on content	Number of interventions focused on organisational aspects	Number of participants who propose this type of interventions
EX	1	1 (EX+EL)	0	1/7
EL	23	2 (1 EL+EX)	21	3/7
QC	28	4	24	7/7
IC	9	6	3	5/7

Table 2. Summary of the combined analysis of the interventions within the chat of group A

The fact that the interactions observed within the Telegram chat of group C belong to category 2 is evident both from Table 3 and from the results of the qualitative analysis of the excerpt presented in Table 1. In fact, even if the work is led by only 3 of the 7 students (lines 2 and 3 in Table 3), all the students (except one of them, who never participated) proposed IC interventions, mainly focused on the content under discussion.

Types of interventions	Number of interventions	Number of interventions focused on content	Number of interventions focused on organisational aspects	Number of participants who propose this type of interventions
EX	8	8 (3 EX+EL)	0	3/7
EL	16	6 (3 EL+EX, 1 EL+IC)	10	3/7
QC	25	5	20	6/7
IC	20	19 (1 IC+EL)	1	6/7

Table 3. Summary of the combined analysis of the interventions within the chat of group C

DISCUSSION

The results of the analysis presented in the previous section enabled us to highlight two main counterposed collaborative dynamics within the groups that interacted in the Telegram chats. On one hand, two groups activated effective collaborative processes, characterised by many integration-oriented consensus building interventions proposed by most of the participants and aimed at developing a shared solution to the problem (category 2). On the other hand, the interactions that characterised the collaborative processes activated by three groups proved to be less effective, since we observed a widespread lack of shared students' reflections on each other's proposals and a predominance of quick consensus building interventions focused mainly on organisational aspects (category 1).

Although the ineffective collaborative processes observed within some of the Telegram chats, students' answers to the final written questionnaire highlight a general positive perception of the work developed with their mates in the chats, due to the opportunity to use the flexible time at disposal to better reflect on each other's proposal and to the potentialities of focusing on a written form of communication as a way to make thinking more visible, as these two excerpts from the questionnaire testify:

“The fact that everything was written allowed me, first, to read all the proposed observations taking my own time and, then, to verify and interpret them calmly.”

“If others express their thinking or even a thinking that does not have a clear form, when it is written, this thinking becomes more visible and it is possible to develop it and deepen the work.”

We hypothesise that the reason why some students share a positive perception of their experience of group work within the Telegram chats even if the collaborative processes developed within their groups are not effective could be ascribed to the fact that, in some cases, the dynamics activated within the chats do not differ from those activated during in-person group work (especially when some students set themselves as leaders and the other participants limit themselves to follow what the leaders suggest). We plan to assess this hypothesis in a future step of our research.

Moreover, we think that the ineffectiveness of the collaborative processes observed in some of the groups could be also related to the difficulties due to the impossibility of looking at interlocutors during the interactions within the Telegram chats. Being not able to grasp typical aspects of para-verbal communication and the gaze of the group's mates when sharing their thinking within a chat prevents the participants from understanding the unwritten, making them possibly not realise the need of reformulating their thoughts to make them better understandable to others. This hypothesis has been confirmed by some students' reflections, who stress that within digital environments “there is less emotional involvement” and define the digital chats as “aseptic and detached situations”.

In spite of the difficulties related to these last aspects, most of the students stated that their participation and engagement in the DMD activity were higher than within in-presence activities implemented during the course. This result is in tune with other research studies, which showed that, within text-based computer-supported collaborative learning environments, the quantity of participation is higher than in traditional classrooms (Weinberger & Fischer, 2006). The students who declared that their engagement has increased during the DMD activity identified the greater amount of time at disposal for developing reflections as a factor that influenced their way of participating, as the following reflection testifies:

“Having more time at disposal for reading and formulating observations and answers to my mates encouraged me to participate”

Another aspect highlighted by the qualitative analysis of the interactions within the Telegram chats is the complete lack of conflict-oriented consensus building interventions. This is certainly due to the explorative nature of the task, which makes almost all the students' statements on what they observe meaningful and useful for the development of the activity. However, we hypothesise that this result could also be due to the ways of working and interacting within the chats, which prompts most of the students to meditate on their interventions for a long time before writing them. This is testified by the following reflection written in the questionnaire:

“I noticed that, differently from what happens in class, I thought a lot more before sending my comments since I was afraid of making mistakes, so I made sure that what I was about to send was correct.”

As a future step of this research, we plan both to implement a further study aimed at verifying the hypothesis we developed and to work on the re-design of the digital environment with the aim of trying to overcome the difficulties in developing effective collaborative processes highlighted by the analysis presented in this paper.

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EXPLORING THE POTENTIAL OF 3D PRINTING AT MATHEMATICAL LITERACY

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Three-dimensional (3D) printing refers to a technological procedure that turns computer-based digital files into solid objects. This study attempts to explore to what extent 3D printing could be used as a learning and educational tool, assisting students to understand mathematics and raise their environmental awareness. We claim that the use of 3D printing in education will revolutionize the methods and concepts related to education and will assist students to gain a better understanding of scientific and experimental concepts. In this paper, we present and discuss our findings from a two year project, run at three primary schools in Greece, Italy and Portugal. A total number of 512 students and 34 teachers participated in this experiment and as indicated by the research results 3D printing could modify mathematical literacy and improve the students' capabilities and skills.

Keywords: 3D Printing, Mathematics, Recycle, Educational Curricula

INTRODUCTION

Three dimensional (3D) printing is a process that turns digital files into solid objects using a desktop 3D printer. Once the digital files are created, either by using computer design software or a 3D scanner, they are sliced into sections and a 3D printer stacks the raw material into layers.

3D printing is a technology that enables hobbyists to turn imagination into reality. According to Lipson and Kurman (2012), this technology will close the gap that divides the virtual and the physical worlds. 3D printing offers the promise of gaining control over the physical world because it provides powerful new design and production tools.

Alpern (2010) notes that "... all you have to do is to load a file and you can replicate shapes that are not manufacturable through traditional methods. All this is called a 'flexible factory in the box'. 3D printing has a wide range of applications in medicine, education, culture, the clothing and footwear industry, as well as in the army. Items, such as body organs, upper and lower limbs, dentures, guns, toys, shoes, jewelry, and statues can be produced using additive manufacturing (Martorelli et al., 2013; Rankin et al., 2014).

This article describes the findings of a project, funded by the European Commission, whose duration was thirty months (November 2019 to April 2022). The main goal of the project was to support students to integrate mathematics into their daily lives, to overcome "fear" of applying mathematical concepts & operations and finally to understand relevant theories in more favorable terms. The Project aimed to enhance the imagination and the creativity of students and to strengthen their skills. Subsequently, they became familiar with the concepts of recycling and reuse of materials. The knowledge formed in the fields of mathematics, 3D printing & recycling, is transformed into educational material, which was used, in the 2nd phase, in order to address the identified needs.

The goal of the paper is to assist students to overcome difficulties at mathematical learning by using and playing with a 3D printer. Three different mathematical modules are examined in this paper: (1) fractions (2) combinations and (3) stereometry. In this context students print 3D objects like

pizzas, letters and numbers of different colours and 3D shapes in order to understand the corresponding concepts.

With reference to the structure of this paper: Section 2 presents a literature review of studies focusing on 3D printing in education, and a brief overview of Mathematical Learning Difficulties; Section 3 formulates our research questions and the methodology that was used; Section 4 presents the results of the implementation of the educational scenarios at the schools of the three participating countries and Section 5 summarizes the results and suggests areas for further research.

LITERATURE REVIEW

3D printing uses a new approach in education. Recently, schools and universities have provided students and faculty with new labs that are equipped with 3D printers and 3D scanners, while new educational frameworks have been developed that incorporate this new manufacturing technology into learning and teaching activities. Until recently, few students had access to this new technology due to cost related limitations. As 3D printers and scanners become more affordable, universities and schools are in a better position to purchase more digital manufacturing systems and to train their students in using this technology (Junk et al, 2015).

In the European Union, the mathematics curriculum for primary, secondary, and tertiary education aims to help students apply their mathematical knowledge and skills to solve problems and make decisions. Students usually learn more by “writing” mathematics and less by “thinking” as mathematician. This means that it is common for students to learn difficult techniques in order to solve equations, but they do not really understand how to utilize these concepts to solve real-life problems. Thus, there is a gap between the goals of mathematical curricula and the learning outcomes.

Many students face difficulties in mathematics. According to Shalev et al, (2000), this represents 3–6% of all children. Students in this group have comprehension problems and face challenges trying to solve arithmetic problems or problems associated with disordered numbering. This group of students is heterogeneous, and it encompasses many subgroups, each of which is characterized by a different learning disability (Bartelet, 2014). Rousselle et al. (2007) claim that students with MDL exhibit difficulties in connecting symbolic representations of numerical magnitude, such as Arabic numerals, with the non-symbolic quantities that they represent. Some studies have found a significant correlation between MLD and verbal working memory deficits in children (Roselli et al. 1997). Some children also find it difficult to do comparison tasks (Mundy et al, 2009). The numerical distance effect, determining the distance between closer numbers in comparison to the distance between numbers that are far apart, refers to the difficulty of determining the numeric properties of a set of items in the real world (Fayol et al, 2005). According to Prathana, et al. (2014), the lack of ability to read is the biggest obstacle to students’ comprehension. If students cannot understand the meaning of the text, they cannot proceed to solve the problem.

To summarize, students face a number of difficulties in learning mathematics due to: 1) the course syllabus, which is sometimes advanced, 2) the teaching strategy that teachers use in the classroom, 3) the difficulty of the assignments (Kavkler, 2014), or 4) the traditional way of teaching (Cabrita et al., 2015) in which students learn how to solve math equations without teaching them ways to utilize them in everyday life.

It is our belief that 3D printing could help students through engineering projects to convert a problem of building a house to a physical prototyping, something which aims them to understand mathematical concepts like straight and parallel lines, measurements, distance from a specific point. By printing in three dimensions’ students with arithmetic disabilities could sketch and print for example the Tower of Anoi and understand the different numbers and different scheme sizes. More

specifically, teaching through the use of 3D printing enables teachers to shift the students' interest and caution from mathematics to technology and better understand mathematical concepts.

RESEARCH QUESTIONS & METHODOLOGY

The current paper discusses the results of the 3D-ReMath project, whose goal was to assist pupils to better comprehend mathematical concepts and empower mathematical thinking, by using the innovative production technology of 3D printing. Another area of focus, apart from technology, was the protection of the environment through recycling. Moreover, the necessary material for 3D printing, called PLA (plastic), could be replaced by a special thread that would be created by students with recycled material, thus covering both needs. In this light, the application for the 3D-ReMath Project was based on the assumption that 3D printing could assist primary and secondary school students better understand mathematics and at the same time become familiar with the concept of recycling. Note that in this project we examine difficulties that students face with maths and not "disabilities".

The contribution of 3D-ReMath was based on pupils' participation in recycling initiatives. By collecting plastic that was later reused, students had the chance to explore a new world of solid objects and through the use of technology they managed to increase their learning ability in maths. Difficulties in mathematics produce discrimination among students. It is widely accepted that technology and its new applications motivates pupils.

3D-ReMath was totally aligned with STEM's (Science, Technology Engineering & Maths) philosophy (Do questions like Scientist, Design like a technology, Print (Build) something solid like an engineer, Deduce like a mathematician). The innovation of the program therefore lies in the fact that it managed to combine three different concepts, to create corresponding educational material and to train teachers and students with multiple and beneficial fields such as: 3D printing (cost reduction by creating a special thread instead of buying it), mathematics (introduction of an educational method to reduce inequalities) and ecology (raising awareness on reuse of materials and recycling).

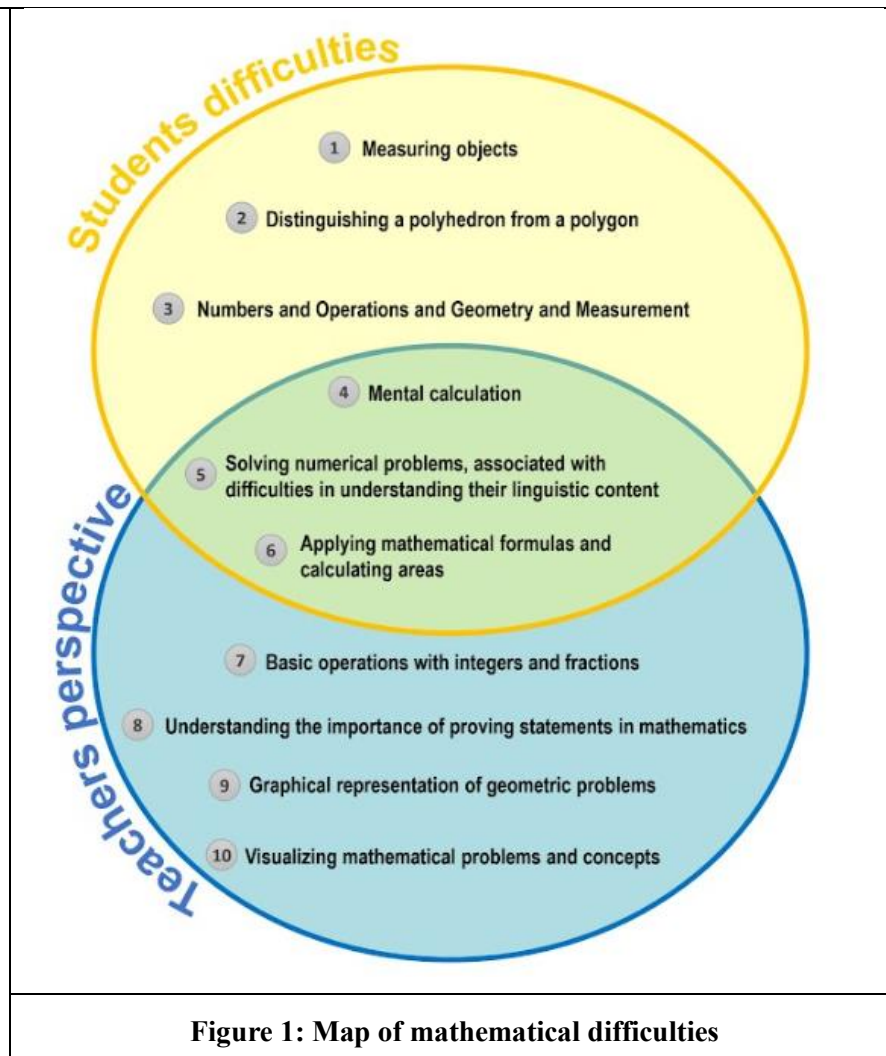
In order to meet the set goals, several steps preceded: (a) the development of a Map for math difficulties, (b) the development of curricula for Mathematics, 3D Printing and Recycling and (c) two rounds of pupils' activities in the three distinct modules.

A map identifying math difficulties was developed based on the teachers' experience and on a sample of students from all schools. Two questionnaires were developed: one for teachers who taught students aged 6 to 10 and one for teachers who taught students aged from 11 to 14. Online surveys were created on google forms and collected responses were imported into SPSS for data analysis. After analyzing the teachers' questionnaires and identifying maths difficulties from their perspective, these were grouped into categories. Then, each partner proposed several multiple choice questions for the students' questionnaires, Based on the students' population of the three schools, the size of the sample per gender and school was defined. The students' questionnaires were created on google forms and applied to this sample. Finally, based on the results of this study, a map for math difficulties was created in CMAP Tools. The conceptual map contained 10 examples of questions related to each identified difficulty. All partners actively participated with quality contributions for the elaboration of the difficulties map. Figure 1 presents the produced map.

Based on the difficulties, which were defined at the map, we proceeded to the development of three Lessons. Indicative structured Lessons for mathematics' module. Each lesson examined a different mathematical topic. The addressed issues were: (a) Fractions, (b) Combinations (c) Stereometry. Each lesson is separated in ten lectures of 2 hours teaching and 4 hours studying.

The structure of lesson involved the description of the relation between a real problem and a maths problem and Learning Objectives of each lecture. Websites, e-books, open source software, books, videos, tutorials were also employed in order to support these curricula. A series of activities and exercises for increasing learning ability were employed and practical exercises were solved by using open source software like Geogebra and corresponding 3D printing activities at the Laboratory.

Through this process it became clear that students were motivated to learn how to use a 3D printer and better understood mathematics.



Students were trained at the use of 3D printers and were acquainted with the 3D printing process, at designing 3D models using Tinker CAD and Sketch Fab. For the purpose of this project, XYZ DaVinci w+ 3D Printers were used and the method of Fused Filament Fabrication. This method turns digital files into solid objects by using thermoplastic material. Digital files are created, either by using computer design software or a 3D scanner, and once these files are uploaded to the 3D printer's software, they are sliced into sections and the 3D printer produces the object by stacking the raw material into layers. Throughout the pupils' training activities, we used PLA filament (plastic). In order to raise the students' awareness concerning recycling, we urged them to collect plastic water bottles and transform the plastic material to filament that could be used for printing objects for mathematics' module i.e. pies, numbers, letters, etc. Pupils were introduced to recyclable materials, such as plastic, paper and glass, and recognized the importance of waste management. Figure 2 presents the procedure of transforming plastic bottles to filament.

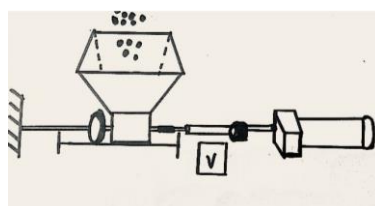


Figure 2: Mechanism of 3D filament

Using the above mechanism, after the plastic is collected and cleaned thoroughly, it is cut into small pieces, transforming it into a filament suitable for a 3D printer. The shavings are then placed in the

hopper and are melted at a very high temperature. The produced mixture then starts to freeze and is directed through a tube of small cross-section into the rotating tube and wrapped around it in order to proceed to the filament.

RESULTS

The students who faced difficulties in mathematics and participated in the project joined a specific educational pilot program in order to improve their understanding of mathematical concepts and eventually integrate these concepts in their daily lives, assisting them in solving everyday problems. To achieve this goal, they were trained in the use of 3D printing and became familiar with the concept of recycling and other ecological aspects.

As mentioned above, three different mathematical topics were examined: fractions, combinations, and stereometry. Concerning fractions, the learning outcomes were: (a) Compare and order fractions and mixed numbers (b) Represent equivalent fractions (c) Identify the division statement that represents a fraction (d) Apply techniques of converting fractions into other, equivalent ones, (e) Simplify the numerator and denominator of a fraction by dividing them with the same divisor and (f) Add and subtract fractions having like and unlike denominators.

Through the use of a 3D printer, students printed a pan of pizza, of four, six and eight parts (pieces) as depicted on Figure 3 (a).

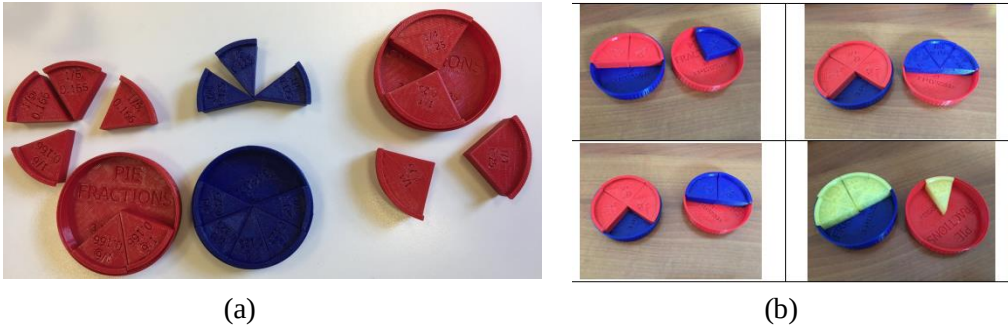


Figure 3: Printed Pizzas and compare fractions

They compared fractions like 2/4 and 3/4, or 1/8 and 2/4 and also used the pizza pieces to perform calculations, i.e. addition or subtraction of fractions (Figure 4).

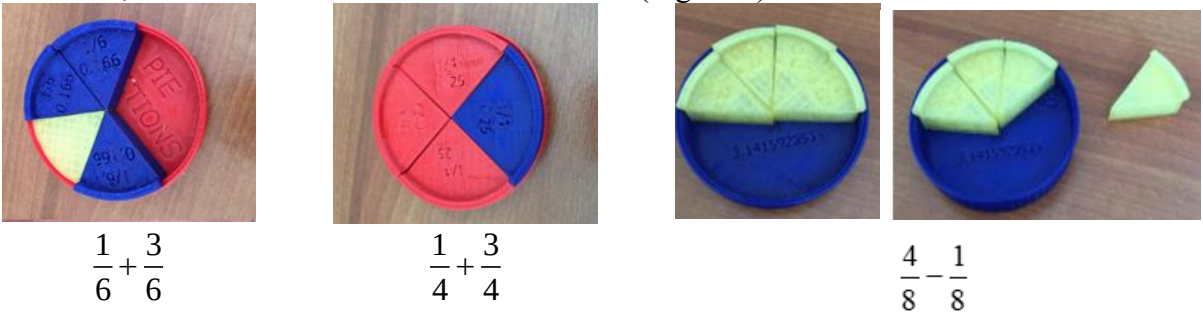


Figure 4: Calculations



Figure 5: Sets and Groups

Students were also taught about sets, subsets and combinations by printing numbers and letters. They also learned the concept of grouping objects based on a criterion e.g. color, name, etc. In terms of mathematical achievement, students learned to (a) understand the concept of Sets and Combinations (b) distinguish objects based on a criterion and (c) identify differences between sets and combinations of objects (Figure 5).



Concerning the combinations, students printed cubes in different colors, red, green, black and white. They used these four cubes and the noted down the ways in which they could be used, when repetition was allowed.

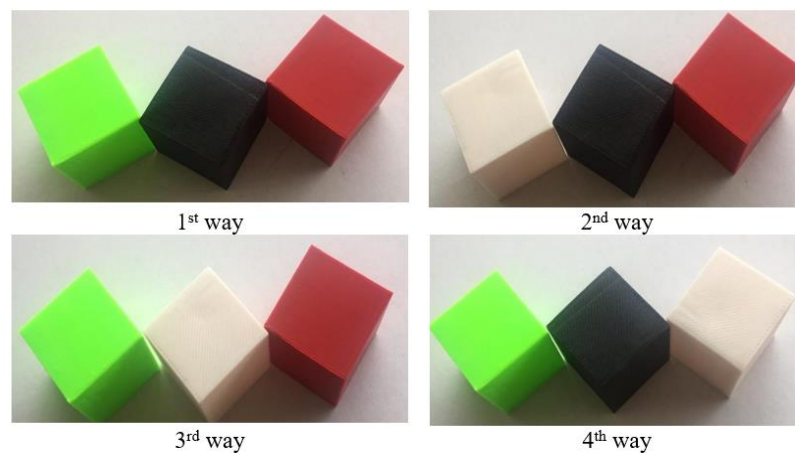


Figure 6: Combinations

Finally, the Learning Outcomes of Stereometry's curricula were to (a) develop spatial visualization, (b) distinguish among different criteria, (c) realize and apply mathematical formulas (d) Perform simple calculations. In terms of mathematical achievement, at the end of the training activities, students were able to (1) understand the difference between 2 and 3 dimensions (2) distinguish 2D and 3D shapes (3) calculate perimeter and area of 2D shapes (4) calculate surface area and volume of 3D shapes. At the Lab, students printed the followings shapes and then attempted to match the 2d shape with the corresponding 3d shape. (Figure 7).

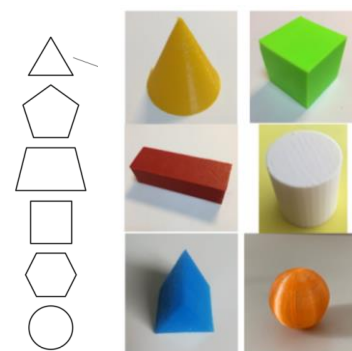


Figure 7:3D shapes

EVALUATION- DISCUSSION

Both the project and the proposed activities were evaluated through the use of questionnaires regarding: a. the above Curricula and b. their effectiveness in addressing the identified educational needs.

The students' questionnaire contained 10 questions, from which 8 questions referred to their learning experience, as well as the relative level of their satisfaction, while the last 2 questions concerned demographic issues. Out of the total number of 512 students that participated in the activities, we choose randomly 197 students, through the use of stratified sampling method. The two used data were "age" and "mother-country". The questionnaire was filled by 197 students, half of which were male. Concerning the involvement of each country in the project, 17% of the students came from Greece, 49% from Italy and 34% from Portugal. The collected data was analyzed through the use of IBM SPSS software. In this section we present the results concerning the pupils' satisfaction from their participation in the program, their belief on the contribution of 3D printing to mathematical understanding and their perception about the usefulness of the training program.

Based on the analysis, 88% of participating students enjoyed strongly their involvement in the program. For Greece and Italy, the level of satisfaction was rated from "enough" to "very much".

Specifically, from the students that responded that they enjoyed the training activities very much 14% came from Greece, 45% came from Italy and the remaining percentage from Portugal.

Students were asked if they believed 3D printing helped them to better understand Mathematics. We used a four -scale Likert answer (“Not at all”, “Not much”, “Enough”, and “Very Much”). Out of the 197? students 43% answered “very much”, 40% answered “enough” (30%), 14% answered “not much” and 3% answered “not at all”. The majority of Portuguese students (83%) gave positive feedback about the use of 3D printing technology in overcoming difficulties in maths. Additionally, 71% of Greek students believe that 3D printing helped them a lot (Enough”/ “Very Much”) to understand Mathematics, while 35% of Italian students answered that 3D printing helped them “very much”, 43% answered “enough” and the remaining percentage of 22% gave negative feedback.

Finally, the perception of pupils on the usefulness of the training course is overall positive as 74% of students answered, “very much” and 23% “enough”. Only 3 negative responses (“not at all”) were recorded out of the 197 students, two of which came from Greek students and one from a Portuguese student. The corresponding question was: “Do you think that the topics covered and the teachings you have received in these courses will be useful for your future?” and a Likert scale of 3 items was used “Not at all”, “Enough” and “Very much”.

Regarding the distribution per country, there replies of the three countries seem to be equally divided with most students answering that the educational topics were very useful for future reference and use. Specifically, 57% of the students from Greece found the project’s content very useful for the future, while 13% of them didn’t find it adequate enough. The other students considered the program useful enough for the future. Additionally, 72% of Italian students claimed that the subject was very useful, while 2 out of 10 stated that it was adequately useful for the future and only 8% students did not consider the training program enough for further future use. Concerning the Portuguese team; 38% of students found the training program useful enough, while 57% believe that they will refer in future to the addressed topics of the program.

Based on the overall evaluation of the actions in which students participated, it appears that they helped them to achieve the purpose of supporting them in mathematics while becoming acquainted with the world of recycling and 3D printing.

CONCLUSION

In this paper, we briefly present a pilot training program based on the axes of education and the use of modern technology with a dominant feature of innovation. In particular, we introduced an alternative educational methodological model aimed at eliminating difficulties in mathematics. A method that utilized innovative manufacturing technology (3D printing), in order for students to understand mathematics, to increase their learning ability and at the same time their knowledge about recycling. Students exploring technology were able to improve their learning ability in mathematics while their awareness was raised regarding the protection of the environment through the reuse of materials and recycling.

One potential area for future research could be that of mathematical difficulties of students with special needs. One aspect that cannot be overlooked is the limitations associated with the challenges of familiarizing teachers with 3D printing technology and the high cost of the equipment that universities and schools incur when purchasing 3D printers and required materials (filaments). Setting aside these limitations, the paper tries to shed light on how traditional educational approaches will be transformed by technology and especially by this innovative technology of 3D printing technology.

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‘INTERNAL RECORD’ AS A CATALYST: FOURTH GRADERS’ PROBLEM-SOLVING PRACTICES WITH the PROGRAMMING ROBOT EMIL

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This research aims to explore the synergy between internal records of the representation and problem-solving practice while fourth graders use Robot Emil to solve a mathematics-related task. The participants are three pairs of pupils from a primary school in Norway. The data comes from pupils’ video-recorded group work in which they discuss the solution to a task in the context of a mathematics lesson. We view the data with a focus on the problem-solving aspect of computational thinking alongside the novel construct of an ‘internal record’ of the representation within the context of the task. The analysis revealed that the internal record of the representation in the Robot Emil environment played a catalyst role in conveying pupils to devise an initial path, evaluate the completed steps and re-devise a path to complete the task.

Keywords: Computational thinking, internal record of representation, problem-solving, Robot Emil.

INTRODUCTION

The synergy between the use of computational thinking (CT) tools as a mediational practice to promote mathematical thinking and developing 21st-century skills (such as *computing*) has opened a door to a new plea. Linking CT practices to learning mathematics has been a major interest of educators due to the interrelationships between CT, programming skills and mathematical thinking (Hoyle & Noss, 2020; Kalaš et al., 2022; Stephens & Kadijevich, 2020). Consequently, CT has been positioned as a new digital literacy that has motivated many countries to refresh their curriculums and policies. Regarding classroom practice, the block programming tool *Scratch* (<http://scratch.mit.edu>), has been extensively researched within primary-level mathematics (Benton et al., 2017; Bråting & Kilhamn, 2021). Recently in Norway, and similar to other countries such as England and Sweden, CT and programming have also been regarded as emerging notions in primary and lower secondary school curricula (Utdanningsdirektoratet, 2019). In this paper we report research from a project that adopts a different CT tool, the Robot Emil environment, which we describe in the next section.

The background and design of Robot Emil not only provide pupils with a motivating context for learning the basics of programming and computing (Blaho et al., 2019), but also promote conjecturing, exploring, and communicating in primary mathematics (Turgut et al., 2022). The tasks in the Robot Emil environment have been designed with different features within the representation and with different levels of pupil control (for a complete discussion, see Kalaš et al., 2018). One such representational feature is the provision of an internal record of the pupils’ mouse clicks (or finger touches) that enable pupils to see the completed steps of the on-screen Robot Emil’s path on the one hand, and to estimate the further steps needed by Robot Emil to proceed, on the other. In this paper, we focus our analysis on this representation of the *internal record* in relation to the problem-solving aspect of CT (Kallia et al., 2021) while fourth graders program Robot Emil for the purpose of solving a mathematics-related task.

PROGRAMMING ROBOT EMIL

Robot Emil is a programming environment for the teaching and learning of programming and computing for grades 3 and 4 (Blaho et al., 2019). It is a combination of ‘unplugged’ (away from

the computer tasks completed in a workbook) and ‘plugged’ activities (through an app on a smartphone, tablet or computer, and workbook). In this study, we worked with Robot Emil for grade 3, which consists of three worlds, each of which has different levels or units categorised by (eight) letters from A to H. These units include progressive tasks in which important ideas in programming and computing skills are embedded for the pupils to discover themselves. Robot Emil can be controlled by clicking (or tapping) on the squares on the ‘stage’ (the screen in view), but Emil only moves *vertically* or *horizontally* and collects the visible objects along his way. In the first world, Emil collects different kinds of objects, such as fruit, coins, or letters, which appear in a tray on the right-hand side of the screen, to show what has been collected. Figure 1 provides an exemplary task G4 (the fourth task in unit G) from the first world.

Help Emil to buy (1) two buttons for 10 kroner⁷, (2) three buttons for 6 kroner, and (3) four buttons for 11 kroner. Prepare two similar tasks for your learning partner.



(a) The initial look of G4 (screenshot from App)

(b) Appearance of G4 after three clicking/moves

Figure 1. Task G4 from the first world of Robot Emil

In unit G of the first world, a specific feature appears, which can be seen on the left hand side of the screen in Figure 1a, the white number strips that are called “bookmarks”. This feature serves to limit the number of clicks or taps allowed. For example, in G4, only five clicks (i.e., five moves by Emil) are possible.

The Emil Robot tasks guide pupils (who work in pairs) to collaborate, communicate and explore the proposed situations together. Pupils are expected to discuss and record their solutions in the workbook. The environment has been deliberately designed to *not* provide any instructional feedback to pupils while they complete the tasks or when they are finished with a task. Therefore, the teacher is expected to initiate classroom discussion to ensure shared understanding and meaning within the class (Kalaš et al., 2022).

CONCEPTUAL FRAMEWORK

CT in Mathematics Education

CT, which is now being considered as an umbrella term (Kallia et al., 2021) for several sophisticated skillsets, originates in Papert’s (1980) seminal book about Turtle geometry and the LOGO programming language in which he aimed to create a dialectic between programming and cognitive development regarding mathematical learning. Later, Wing’s (2006) definition went beyond and underlined the role of CT in different levels of education by stating CT ‘... just like reading, writing, and arithmetic, should be added to every child’s analytical ability’ (p. 33). As a consequence, CT is now considered a (new) digital literacy (Stephens & Kadjevich, 2020). There have been many

⁷ The *kroner*, plural *kroner* is the Norwegian currency, with coins of value 1, 5, 10 or 20. “10-kroning” is a usual way to name the 10-kroner coin.

attempts to describe components, dimensions and aspects of CT in mathematics education (see Hoyles & Noss, 2020 for a broad discussion regarding components of CT). Recently, a particular characterization of CT in mathematics has been provided by Kallia et al. (2021), which highlights three aspects (p. 179): (i) *problem-solving*: understanding a problem, hypothesizing a solution strategy and performing the strategy; (ii) *cognitive processes*: abstraction, decomposition, pattern recognition, algorithmic thinking and evaluation of solutions and strategies; and (iii) *transposition*: phrasing the solution of a task in such a way that it can be transferred to another human and/or machine.

Kallia et al. (2021) note that these aspects are not exhaustive of all CT practices within educational settings. Additionally, in our study we only focus on the *problem-solving* aspect of CT due to the very early stage of programming addressed in the first world of the Robot Emil environment (Kalaš et al., 2018). Hence, we reformulate the above description for this context in Table 1, which we use to analyse the interrelation between the internal record of the representation (dimension) and the pupils' problem-solving practices.

<i>Steps</i>	<i>Description</i>
1. <i>Understanding the problem</i>	This step involves understanding the proposed task, by considering the required inputs and outputs, and how these can be accomplished.
2. <i>Devising and performing a solution path</i>	This step involves envisaging a solution path and/or a computing strategy and performing it by controlling Emil.
3. <i>Evaluating the (solution) process</i>	This step involves evaluating the process. Pupils collaborate, discuss and assess their progress by revisiting the screen and workbooks.

Table 1. Description of Problem-solving Steps in the (first world of) Robot Emil Context

Dimension of the representation in Robot Emil Environment

Within the design of Robot Emil, Kalaš and colleagues focus on the cognitive complexity of different levels of control alongside the dimension of the representation (Kalaš, 2016; Kalaš et al., 2018). Within the first world and task G4, indirect manipulation is the level of control and the representational feature is the 'internal record'. The pupils' level of control is their clicks (or taps) on a certain position to give a signal to Emil to move/float there. For example, by clicking on one button (positioned in the middle of the first row of the square in Figure 1a), Emil will float there and, on his way, will collect two buttons and one 10-kroning. The latter, the *internal record of the representation* refers to representing the performed clicking/tapping. Each click colours the square grey and any previously numbered and greyed positions cannot be reselected. Additionally, the maximum number of clicks is visible as numbered *bookmarks* (Figure 1a), and on the clicked squares (Figure 1b). This representation of the internal record encourages pupils to think in advance and estimate a path to proceed, whilst also providing an opportunity to test this estimated path. This process (step 2 in Table 1) informs our research question: *What is the synergy between an internal record of the representation and problem-solving practices while fourth graders use Robot Emil to solve a mathematics-related task?*

METHODS

Our case involves three pairs of pupils, grouped by the teacher according to their communication skills and mathematics performance, who were video-recorded whilst using Robot Emil to solve task G4. We focus on data first reported by Turgut et al. (2022), to which we include further data sets. The participants, (pseudonyms: Per and Ane, Ida and Mia, and Lise and Hanne) were all fourth-

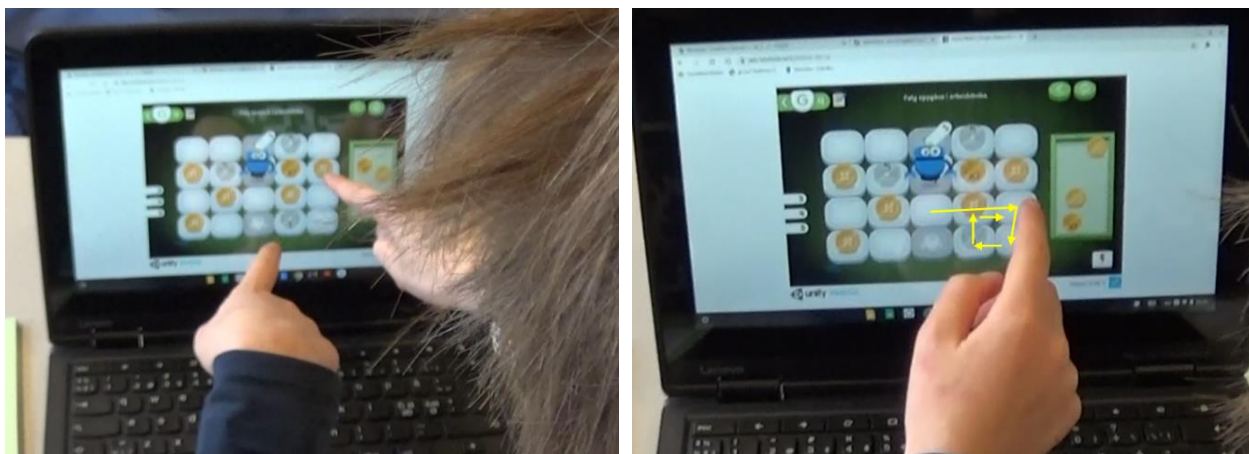
grade pupils when the data was collected in November 2020. The data includes video recordings of pupils' work with researcher/observation notes. The videos were later transcribed and translated from Norwegian into English by researchers. Each case was analysed by one author and controlled and discussed by another author with a particular reference to the problem-solving aspect of CT. This was later combined through the lens of the representation of internal record by the team.

Task G4, in our previous report, was formulated as an optimisation task due to the dimension of the representation. To describe Robot Emil's positions and movements within the context of our analysis, we apply an imaginary coordinate system where the first number represents the row (from top to bottom) and the second represents the column (from left to right). For example, in Figure 1a, Emil is located at [4,3]. Following the internal record of the representation in Figure 1b, Emil floated this path: [4,3] → [4,4] → [2,4] → [2,2]. We note that questions (1), (2) and (3) of task G4 are independent of each other.

FINDINGS

The Case of Per and Ane

Per and Ane started to discuss the first question of the task. Initially, due to the bookmarks provided in the task, Per thought that they had five possible clicks and that they should collect as many buttons as possible. Later, Ane read the task again, understood the problem and immediately devised a possible path by tracing her pencil on the screen: [4,3] → [4,1] → [3,1] → [3,2]. Then, Ane suggested they collect 10-kroner on [3,3], which was a correct solution without performing the devised path. After evaluating Ane's path, Per suggested an optimized solution by clicking on [1,3] and stated '*It is 10 kroner and two buttons*'. After having two solutions, Ane explored what to do with the remaining moves (pointing on the screen with a pencil, at bookmarks for moves on the left-hand side), and they focused on the third question of the task (4 buttons for 11 kroner). Ane refreshed the screen and devised a path [4,3] → [1,3] → [3,3] → [3,5] by pointing at the screen and Per noted the steps and suggested additional movements by clicking on [2,3] and pointing at [2,5] (Figure 2a).



(a) Per's suggestion to proceed

(b) Ane's (an)other devised path

Figure 2. Per and Ane's discussion to complete the task

Ane tried collecting more buttons. However, she realized that she could not, because they could not click on a position that Emil had already visited (i.e., a greyed square) (Figure 2b). The pupils then discussed the situation; Per suggested focusing on four buttons for 20 kroner since two buttons cost 10 kroner. Ane reminded him that four buttons are 11 kroner at first (which confirmed her understanding of the problem), later she moved to the second question of the task by stating '*or maybe three buttons which are six kroner? If we go ...*'. Ane devised a path and performed it by

$[4,5] \rightarrow [1,5] \rightarrow [1,3]$. Two buttons and 6 kroner were in the tray. Per evaluated this solution and realised that something was missing. Ane then clicked on $[2,3]$ to collect one more button, which completed the second question of the task. Per then expressed *'it should be possible to get one button'*, which is most likely due to the one bookmark that remained.

The Case of Ida and Mia

Ida and Mia started to solve the third question of the task G4 (4 buttons for 11 kroner), having already solved the first two questions. Emil was on $[1,3]$ and after Mia's click, he floated to $[1,5]$, so there were 2 buttons and 15 kroner in the tray. Then Mia instructed him to move to $[4,5]$ and Ida tried to devise the next move by pointing with a pencil at $[4,4]$. However, Mia performed her strategy and clicked on $[4,1]$. At this moment 4 buttons were collected, and the girls started to plan how to collect 11 kroner. Ida stated: *'And so we need eleven kroner, but we have fifteen ... five here ... and this /she pointed with a pencil at 5 and 10 on $[2,2]$ and $[2,4]$ / will be fifteen'* (Figure 3a). Ida and Mia didn't understand the problem correctly, as they focused on buttons and coins separately. In the next attempt, they tried a different path $[1,3] \rightarrow [1,2] \rightarrow [4,2]$. Then Mia counted 3 buttons in the tray and Ida suggested to Mia (who was controlling Emil) to instruct Emil to go to $[4,5]$. Mia noticed they had only one move left, however, Ida suggested going to $[2,4]$ (Figure 3b). Mia evaluated the steps and expressed the impossibility again: *'Do you think it is possible to go there when we only have one move left?!'*. Ida understood the issue and came up with a new strategy: *'I had an idea! Because if you had gone here /she pointed with a pencil at $[4,4]$ /, we could have gone like this /showed the path $[4,4] \rightarrow [2,4] \rightarrow [2,5]$ /'*.



(a) Ida counts the left coins on the stage

(b) Ida's suggestion to Mia to complete the task

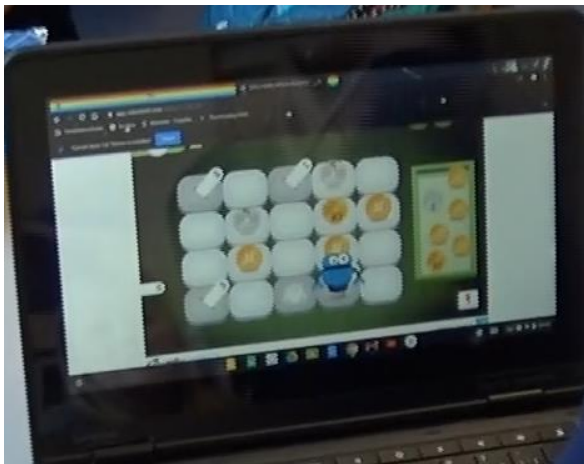
Figure 3. Ida's and Mia's discussion to complete the G4 task

Mia assessed Ida's strategy, confirmed it, and Ida added that this strategy also involves collecting 11 kroner. However, to perform it, they would still need six moves, although the last move to $[2,5]$ is unnecessary because they would have had 4 buttons and (in their way of thinking) 11 kroner if Emil got to $[2,4]$. After refreshing the screen, Mia instructed Emil to perform the path $[1,3] \rightarrow [1,2] \rightarrow [4,2] \rightarrow [4,4]$ and then the girls discussed the last move, whether it should have been to $[1,4]$ or $[2,4]$. At last, Mia sent Emil to $[1,4]$ and Ida confirmed, by counting in the tray, that they collected 4 buttons. Then Mia started to count the value of all coins in the tray, yet Ida convinced her that the solution was correct: *'Look here! Eleven. /she pointed with a pencil at 10 and 1 coins, back and forth three times/ So then we have eleven kroner! I think we managed it.'* After accepting the solution, the girls drew it in the workbook. Despite the fact that the internal record of the process was on the screen, the girls tried to recall from memory which path Emil followed.

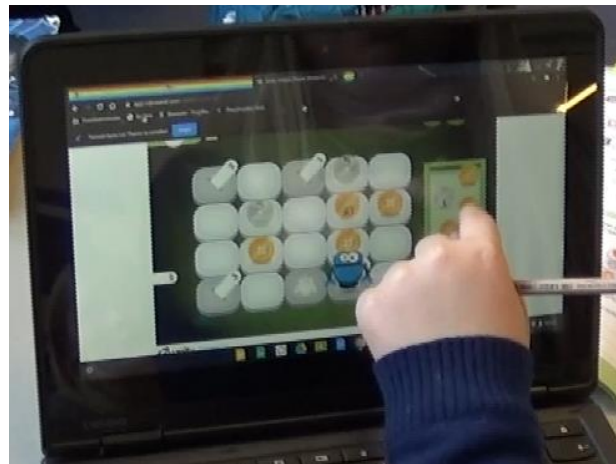
The Case of Lise and Hanne

Lise and Hanne completed three questions in task G4 (noted in their workbooks). For example, they performed a path $[1,3] \rightarrow [1,1] \rightarrow [4,1] \rightarrow [4,4]$ to complete the third question (four buttons for 11

kroner) (Figure 4a). Lise and Hanne discussed different possibilities more in the last part of the task, which was about preparing two similar questions for each other. Lise proposed two subtasks, “seven buttons for 1 krone” and “two buttons for 15 kroner” for Hanne. Before doing this, Lise checked these questions by counting the buttons on the screen (Figure 4b). We observe that Lise purposefully proposed the first task with an awareness that it was not possible to solve because the internal records of the representation had given her a good understanding of the task situation.



(a) Lise and Hanne’s solution for the third question



(b) Lise counts (possible) buttons

Figure 4. Lise and Hanne’s solution to the third question in G4 and Lise’s counting of buttons

Hanne evaluated that it was impossible to collect all the buttons without picking up more than 1 krone as she had devised a mental path by considering the number of bookmarks. Later, Lise updated the task to “seven buttons for 11 kroner”, Hanne refreshed the screen and tried to solve this new task. Hanne started to think and devised a new path on the screen, $[1,3] \rightarrow [1,1] \rightarrow [4,1] \rightarrow [4,4]$. Later, Hanne pointed at the tray for a while and evaluated the situation, ‘*this is not possible Lise*’ and by explaining further, ‘*one could only get ... one, two, three, four, five ...*’, she expressed that it was impossible to collect more buttons. Lise replied that she would check it, which means that Lise wanted to think once more about it. Then Lise updated the task to “three buttons for 11 kroner”. Hanne easily devised a path and completed the task. Further, regarding “two buttons for 15 kroner”, Lise stated ‘*this is actually very easy*’, which showed us Lise had – at least – one solution in her mind. Hanne refreshed the screen and solved it easily by $[1,3] \rightarrow [1,4]$.

Lise solved Hanne’s first task (four buttons for 15 kroner) by instructing Emil to perform five moves $[1,3] \rightarrow [1,2] \rightarrow [3,2] \rightarrow [3,1] \rightarrow [2,1]$. Later, Hanne asked for “three buttons for 10 kroner”. Lise tried to solve this by tracing a devised path, $[3,3] \rightarrow [4,3] \rightarrow [3,3]$. Lise changed her mind, as the greyed squares prevented her proposed actions. Later, Lise devised a new path and performed $[4,1] \rightarrow [1,1] \rightarrow [1,3]$, then waited and pointed on $[4,2]$ and $[3,3]$. Then Lise evaluated her finding and stated, ‘*it is not possible*’. So, we conclude that Lise overviewed the internal record on the screen and realised that in this situation it was impossible to collect the targeted amount without picking up more buttons. After encouragement from Hanne, Lise devised a new path and instructed Emil to float the path $[2,3] \rightarrow [1,3] \rightarrow [1,1] \rightarrow [1,2]$.

CONCLUSIONS AND DISCUSSION

In this paper, we focused on the synergy between an internal record of the representation in the Robot Emil environment and fourth graders’ problem-solving practice. Our overall conclusion is that the internal record of the representation acted as a catalyst for pupils’ understanding of the task, devising, refreshing, and performing a path, evaluating the completed steps and re-devising a path

to proceed. We observed three main features of the internal record of the representation: *bookmarks*, *greyed squares* (Kalaš et al., 2018), to which we add the *tray* as our examples evidence how, pupils revisited the tray several times to count the collected items and compare these with the remaining bookmarks. The items in the tray cannot be removed and the tray displays the record of what has been collected synchronously. Figure 5 provides an overview of pupils' problem-solving practice through the lens of the internal record of the representation.

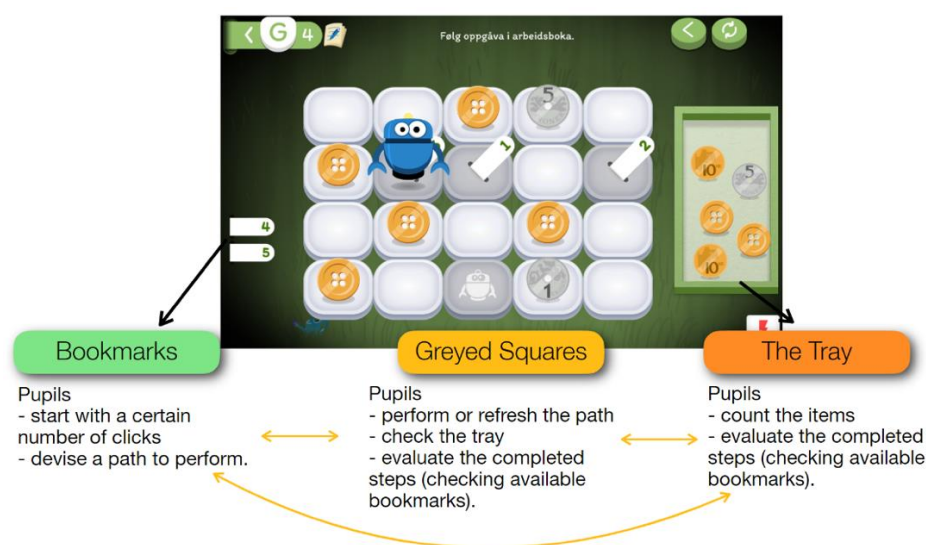


Figure 5. Summary of fourth graders' problem-solving practice through the lens of the internal record

Figure 5 summarises that, in our case, the synergy between bookmarks, greyed squares and the tray conveyed pupils' problem-solving processes, i.e. starting with a certain number of clicks, considering this and devising a path. This follows performing or refreshing the path and checking the tray. The next step is evaluating the completed steps (checking available bookmarks), counting the items in the tray and refreshing the screen, if needed. We note that the tasks in the Robot Emil environment become increasingly complex. For example, in the subsequent worlds, there exist the ideas of direct and *computational control* (Kalaš et al., 2018), where the user needs to program Emil by considering future moves and actions before Emil carries out these steps. These tasks are relevant to explore a number of specific CT components, such as pattern recognition and generalisation (Hoyles & Noss, 2020) and algorithmic thinking (forms of different mathematical reasoning, like spatial or geometric reasoning) (Stephens & Kadijevich, 2020). Therefore, further research could explore the interrelationship between, for example, spatial reasoning and pattern recognition and developing, testing, and refining algorithms through Robot Emil.

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A PROPOSAL OF MIXED METHOD ANALYSIS IN MATHEMATICS EDUCATION USING FUZZY COGNITIVE MAP

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Qualitative research is often identified as idiographic: a research focused on cases and avoided generalizations. Whereas quantitative research is identified as nomothetic by proposing the determination of general laws. Most of the time, quantitative and qualitative research should not be regarded as two paradigms, as two different worldviews are unable to communicate. In this paper, we first focus on the main characteristics of the two kinds of research analyses. Then, an approach based on Fuzzy Cognitive Maps is proposed as a mixed method analysis to be used in mathematics education to combine qualitative and quantitative research results. Finally, some examples of applications are discussed.

Keywords: qualitative research, quantitative research, Fuzzy Cognitive Map, mixed methods.

INTRODUCTION AND MOTIVATIONS

The empirical research line in Mathematics education has two major traditions: one commonly defined as quantitative, oriented towards numerical quantification with nomothetic intentions, and one commonly defined as qualitative with idiographic intentions. The nomothetic approach starts with a hypothesis and, thus, a prediction. Having studied situations, it is possible to recognize them and predict what will happen when the same circumstances recur. In the idiographic approach, an explanation of the studied episode is obtained after it has happened: we speak of a retrodiction. The two traditions have often focused on different salient features to the extent that they are perceived as opposing ontological, epistemological, and methodological levels. However, in today's research landscape, the idea is increasingly gaining ground that in quantitative research, there must be moments in which the concepts to be quantified are rigorously defined, and this requires a prior qualitative analysis of these concepts that can make use of epistemological and methodological facilities typical of qualitative approaches. Similarly, in qualitative research, there is the problem of how to generalize "unique cases," and therefore, conceptual categories on which quantification operations can be carried out must be regularly defined. In several cases, it is important to recognize that qualitative and quantitative research methods in mathematics education are not mutually exclusive. Many researchers use both approaches in their studies to gain a more comprehensive and nuanced understanding of the phenomena they are studying (Johnson et al., 2007). Rather than being seen as two separate paradigms, qualitative and quantitative research could be viewed as complementary approaches that can be used to address different aspects of a research question. Kelle and Buchholtz (2015) point to the limitations of a purely qualitative approach. They critically review the continuing dispute about qualitative and quantitative research methods that overshadow research in mathematics education. These authors question the restriction to either quantitative or qualitative methods, which they find particularly striking in research on teacher knowledge. They argue how a "mixed methods design" can enrich mathematics educational research in this domain. Hart and her colleagues (2009) examined 710 research articles on mathematics education in six prominent educational journals during the period 1995-2005, finding that 50% of the studies used qualitative methods only, 21% used quantitative methods only, and 29% mixed qualitative and quantitative methods in various ways. In this study, an approach based on the use of Fuzzy Cognitive Maps (FCM) (Kosko, 1986) as a mixed-method analysis is proposed. Starting from qualitative data, it uses an appropriate categorization of the observed elements to provide the

quantitative data of an educational phenomenon. The authors have already applied this methodology in some studies in the field of mathematics education (Capone & Lepore, 2021; Capone et al., 2022) and in some studies of humanistic informatics (D’Aniello et al., 2021). Some examples of applications of FCM in Mathematics Education are shown here, highlighting how it can be generalized to other studies.

QUALITATIVE RESEARCH

Qualitative research is often used to explore the experiences and perspectives of students and teachers in specific contexts. This approach typically involves collecting data through observations, interviews, and other qualitative methods to develop a deep understanding of the phenomena being studied (Bikner et al., 2015). Qualitative research is often associated with idiographic methods, which focus on individual cases’ unique characteristics and features (Cobb, 2007). In 1974, at the assembly of the American Psychological Association, Donald T. Campbell argued that qualitative knowledge “precedes” and guides quantitative knowledge and that the second depends largely on the first. In the same assembly, Lee J. Cronbach proposed reversing the priorities followed until then in experimental studies: from generalization to individual cases, exceptions, uncontrolled conditions, personal characteristics, and events that occur during the study. Qualitative research is observational-interpretative and favors the approach of textual analysis, aiming to understand the meaning of the actors’ behavior and the texts produced based on detection (observation) or by them (interviews, documents). The approach may be Hermeneutic, which is based on interpretation aimed at reconstructing the meaning of the set of an actor’s actions or of a text, and Phenomenological, i.e., based on the recognition of certain themes within sets of actions or within texts that identify themes or events common to different referents (uniqueness). To this line of research belongs, for example, Anne R. Teppo (2015), who introduces grounded theory as a methodology; Vollstedt (2007), that identifies different types of personal meanings and describes conditions of their emergence, which constitute significant elements of a theory of personal meaning in mathematics learning. Other scholars focused on perspectives regarding the reconstruction of social interaction and argumentation, e.g., Götz Krummheuer (2015), Christine Knipping (2008), also following Toulmin’s theory. Other scholars highlighted a methodology based on the Vygotskian perspective (Radford et al. 2005). Furthermore, various qualitative research methods that have emerged in mathematics education over the past three decades are analyzed (Bikner et al., 2015). Some examples of qualitative analysis came from Tommy Dreyfus, Rina Hershkowitz, and Baruch Schwarz that presented *Abstraction in Context* (2020) as a theoretical-methodological approach for researching students’ knowledge constructions. Moreover, Arthur Bakker and Dolly van Eerde (2015) introduce design-based research as a specific methodological approach in realistic mathematics education. Michèle Artigue (2009) considers didactical engineering in the French tradition as a case of design research.

QUANTITATIVE RESEARCH

Quantitative research in mathematics education typically involves collecting and analyzing numerical data to identify patterns and make generalizations about larger populations. This approach is often associated with nomothetic methods, which aim to identify general laws or principles that can be applied across different contexts (Shaughnessy, 2007; Watson & Callingham, 2003). The use of quantitative data analysis methods based on statistics became widespread in the early 20th century, thanks to Karl Pearson, Udny Yule, and Ronald Fisher, who defined numerous indices and procedures. The advent of the personal computer in the 1980s lowered the data processing cost, allowing anyone to process large volumes of data. In those years, the computer generated a real euphoria for the quantitative, with an often uncritical application of data collection and analysis techniques. Research based on quantitative methods stems from a realistic ontological view and generally pursues nomothetic aims, aiming at identifying relationships between factors. It

uses highly formalized data collection and analysis procedures to describe a given educational reality based on the given statistical parameters (Bolondi et al., 2019). Quantitative methods usually explain a factor based on other factors (e.g., identifying the relationship between the use of computerized teaching tools and the skills acquired by students in certain subjects). The goal is to identify and detect specific factors that leads to the use of procedures that produce highly structured data (Lee et al., 2022).

MIXED METHODS

In recent years, many scholars have used mixed methods in mathematics education (e.g., Kelle & Buchholtz, 2015; Lahdenperä et al, 2022). Following Johnson et al. (2007) definition, “mixed methods research is the type of research in which a researcher or team of researchers combines elements of qualitative and quantitative research approaches (e.g., use of qualitative and quantitative viewpoints, data collection, analysis, inference techniques) for the broad purposes of breadth and depth of understanding and corroboration. [...] A mixed methods study would involve mixing within a single study; a mixed method program would involve mixing within a research program, and the mixing might occur across a closely related set of studies”. (Johnson et al., 2007, p. 123). The key is to synergically use qualitative and quantitative approaches at different points in the same research to find better answers to the research questions. The question then arises of how to integrate the two approaches without overlapping them, how to find ways to integrate them, and how to interpret the results in the light of a unified framework. Some strengths of mixed methods research include simultaneous generation and testing of theory, the possibility of answering a broader extent of research questions and added value of additional knowledge for theory and practice. In contrast, a researcher has to be firm and confident in applying multiple research methods (Kelle & Buchholtz, 2015; Radmehr & Goodchild, 2022).

FUZZY COGNITIVE MAP

In this study, Fuzzy Cognitive Analysis is proposed as a methodological framework that, starting from a qualitative analysis and appropriate categorization, attempts to give a quantitative answer to an educational phenomenon through Fuzzy Cognitive Maps. A Fuzzy Cognitive Map (FCM), as introduced by Kosko (1986), is a symbolic representation based on a fuzzy graph useful for representing causal relationships. It can symbolically describe complex systems/environments, highlighting events, processes, and states. An FCM consists of an interconnection of nodes through weighted edges: a graph node is called a concept, and an edge is called weight. The edge allows for implementing a causal relationship between two concepts (nodes), and the weight represents the strength of the influence of the relationship, described with a fuzzy linguistic term (e.g., low, high, very high, etc.).

A FCM can be formalized through a 4-tuple (N, W, A, f) , where:

1. $N = \{N_1, N_2, \dots, N_n\}$ is the set of n concepts which are represented by the nodes of the graph.
2. $W: (N_i, N_j) \rightarrow w_{i,j}$ is a function $(N \times N \rightarrow [-1,1])$ which associates the weight $w_{i,j}$ to the edge between the pair of concepts (N_i, N_j) .
3. $A: N_i \rightarrow A_i$ is the activation function which associates to each concept N_i a sequence of activation values, one for each time instant t : $\forall t, A_i(t) \in [0,1]$ is the activation value of the concept N_i at time t .
4. $A(0) \in [0,1]^n$ is the initial activation vector containing the initial values of all the concepts; $A(t) \in [0,1]^n$ is the state vector at a certain instant t .
5. $f: R \rightarrow [0,1]$ is a transformation function with a recursive relation $t \geq 0$ between $A(t+1)$ and $A(t)$:

$$\forall i \in \{1, \dots, n\}, A(t+1) = f\left(\sum_{\substack{j=1 \\ j \neq i}}^n w_{ji} A_j(t)\right)$$

Different functions can be used as $f(x)$, such as the sigmoid function, the bivalent function or the linear function. FCM can be used to make a what-if inference, starting from a given initial activation vector $A(0)$, to understand what will happen next to the modelled system/environment.

A Fuzzy Cognitive Map is developed by integrating existing experience and knowledge related to a system. This can be achieved by using a group of experts to describe the structure and behaviour of the system under different conditions. With FCM, it is usually easy to find which factor needs to be changed, and being dynamic modelling tools, the resolution of the system representation can be increased by applying further mapping. FCMs can be used for various purposes, such as underlining agents' behaviour, understanding the reasons for their decisions and actions taken, and highlighting any distortions and limits in their representation of the situation (explanatory function). They can also be useful for predicting future decisions and actions (forecasting function) and for helping decision-makers reflect on a given situation's representation (reflexive function).

BUILDING A FUZZY COGNITIVE MAP

Before choosing FCMs as a tool to summarize the qualitative data of research quantitatively, an in-depth study of the scientific literature was conducted, evaluating different fuzz-type intelligence computing approaches proposed by the community to describe structures and behaviour of agents and different dynamic situations. With respect to other fuzzy approaches, the use of FCM provides us with some advantages. For example, FCMs are based on causal cognitive mapping, which provides an efficient way to elicit and capture the knowledge of the experts of the domain and provide an intuitive way to represent such knowledge that can be easily managed and updated by such experts.

Typically, an FCM results from a construction process involving several experts. The approach used by the authors is presented here. The designed FCMs, proposed by the authors in their works, resulted from a consensus process in which a team of four education experts worked. Starting from one or more methodological frameworks typically used in mathematics education and considered useful to describe ongoing research, each expert has proposed a FCM to identify the causal relationships and weights between the available concepts. The weights are represented by several linguistic terms, for example they could be no impact = 0.00, very low = 0.165, low = 0.335, medium = 0.50, almost high = 0.665, high = 0.835, very high = 1.00. Then, the experts aggregate the different maps proposed to obtain one FCM. When some differences arose between the relationships and weights proposed by the experts, they discussed these differences. They tried to find an agreement until they reached a sufficient consensus. The authors proposed FCMs can be considered organized in different layers. The lowest level usually contains the concepts that represent the atomic variables, allowing a numerical correspondence with the observed phenomenon to be clearly established. The numerical values provided as input could be made available by software data collection tools or simply by a human operator consistently assigning a numerical value based on the observed phenomenon. For example, suppose we are interested in modelling the engagement level of an undergraduate mathematics student, a low-level concept might represent the number of completed assignments. The activation levels of these “leaves” concepts represent the value of each variable. When these variables' values change, the other concepts of the FCM are influenced according to the causal relationships between them. The middle layer usually contains the nodes composing the top-level concepts of the model they are intended to represent. For example, if the high-level concept to be modelled is motivation, the intermediate-

level concepts contributing to its definition could be intrinsic motivation, extrinsic motivation, and social motivation.

USING A FUZZY COGNITIVE MAP

In several research studies, we sought quantitative confirmation of what emerged from the qualitative study. For example, we created a situation model using a customized Fuzzy Cognitive Map capable of mapping the students' cognitive processes in the activities proposed during the research, referring to specific methodological frameworks of mathematics didactics. Specifically, using a suitably designed Fuzzy Cognitive Map allowed it to transform qualitative analysis into quantitative analysis and compare different approaches and situations. To provide concrete evidence of this approach, two different examples of the application of FCMs that we built in our research are given. Specifically, are presented a custom FCM to assess the impact of changing environment on undergraduate mathematics students’ status, and a custom FCM to assess whether the use of Augmented Reality (AR) can improve the conceptualization of specific mathematical objects. The two custom FCMs, apart from the different goals they are intended to achieve, differ mainly in the theoretical frameworks they use and how the numerical values of the low-level concepts are set. We start by describing the motivations that led to the building of the first map, then we look at its structure, and finally, we sum up the major findings.

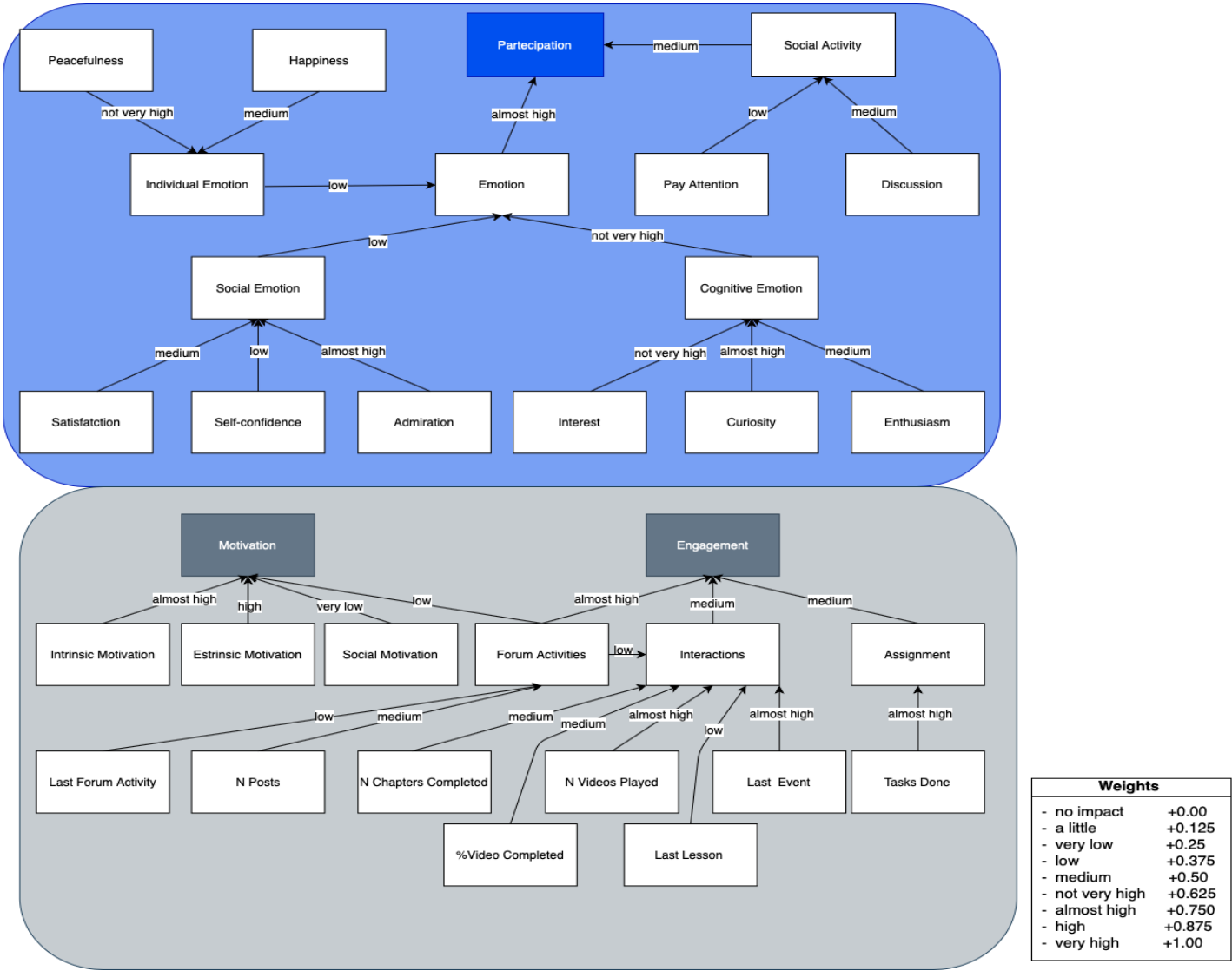


Figure 1: FCM describing student’s status in terms of engagement, motivation, and participation

Mathematics teaching with STEM undergraduate students has evolved over the past three years; it has moved from blending to fully distance learning during the pandemic. We were interested in how students reacted to changing environments from a cognitive point of view. Data collected by our situation were e-learning platform were summarized through engagement, participation, and motivation, which described students' status, calculated through an ad hoc fuzzy cognitive map. The values of the low-level map concepts were provided directly by the implemented software platform. The FCM that is being referred to is shown in figure 1. As can be seen, the top-level concepts are engagement, motivation, and participation, and they are the roots from which all other nodes branch out to the leaves. An accurate description of concepts that make up the map can be found in (Capone & Lepore, 2021). The results of the engagement, motivation, and participation levels through the years and experiences highlighted the positive and negative aspects of using technology in mathematics teaching. Thus, as a methodological implication of this research, we proposed a teaching method (called Integrated Digital Learning) that integrates moments of distance teaching with activities carried out in the presence, classroom, or other university environments. A mix of styles, a fluid flow of knowledge between the physical classroom and the virtual classroom.

As a second example, an FCM to assess the utility of Augmented Reality is given. Our research focused on designing and implementing teaching strategies using GeoGebra Augmented Reality to improve conceptualizing specific mathematical objects (Capone et al., 2022). Referring to Duval's theory of semiotic representations, our research hypothesis is that AR facilitates students' transition between different semiotic representations, boosting their motivation and engagement and improving their mathematical conceptualization process. We describe a teaching sequence on paraboloids, framed by the Method of Varied Inquiry and Marton's Variation Theory, which involved thirty undergraduate students in Mathematics. The research methodology was not limited to a qualitative analysis of video-recorded activity. Still, it was also based on a further quantitative analysis of the learner's status, described through the top-level concepts of conceptualization, inquiry, and emotions, using a custom Fuzzy Cognitive Map we reported in Figure 2.

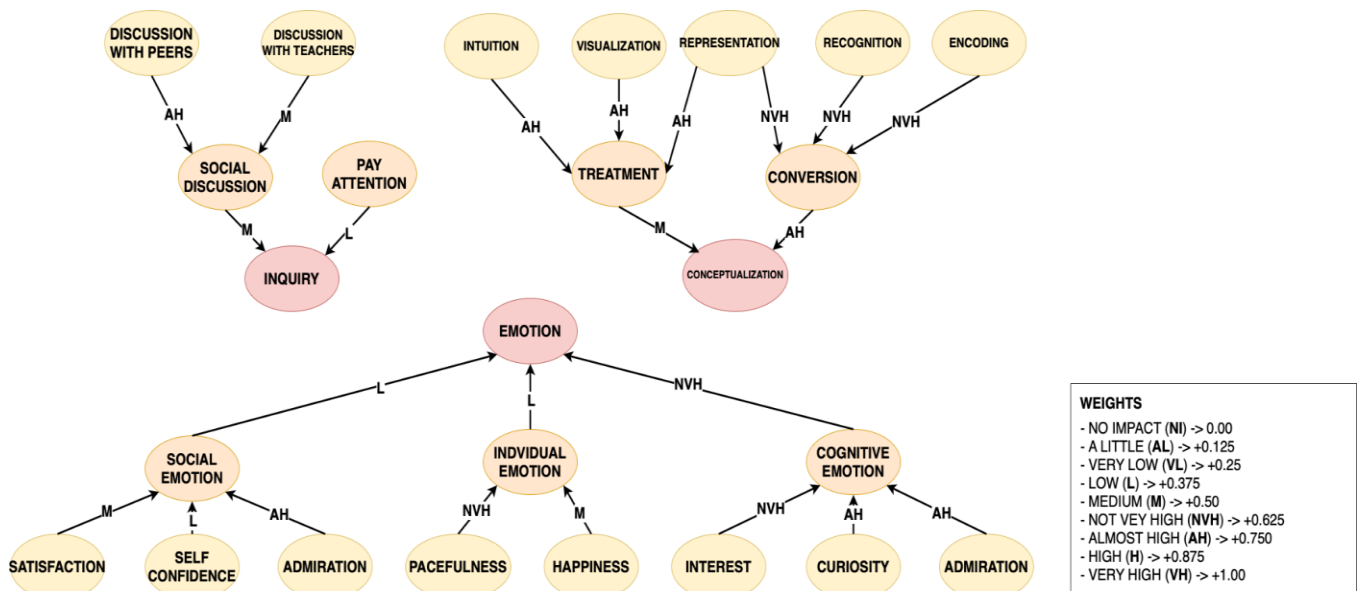


Figure 2: FCM for student's emotion, inquiry, and conceptualization evaluation

The low layer contains *discussions with peers* and *discussion with teachers*, associated with the *top-level inquiry* concept, which help describe the amount of student interaction with peers and teachers, respectively; *intuition*, *visualization*, *representation*, *recognition*, and *encoding*, associated with the *conceptualization* top-level concept, which has been used to express the students' level of ability of treatment and conversion, according to Duval's theory; *satisfaction*, *self-confidence*, *admiration*,

peacefulness, happiness, interest, curiosity, and excitement, linked to *emotion* top-level concept, which were chosen as the most important feelings to describe the student's emotional state while performing the activities. In this research, a human expert operator estimated numeric values for the low-level concepts from the available material acquired during the experimentation.

The results show how the AR activity successfully engaged and guided students to characterize paraboloids by easing the transition between semiotic representations, from the graphical to the analytical and vice versa.

CONCLUSIONS

To sum up, Fuzzy Cognitive Map (FCM) is widely used in various fields, including mathematics education. FCM is a graph used to represent knowledge and its causal relationships. It allows the visualization of the complexity of a system and the identification of the most relevant factors. In mathematics education, we used FCM to conduct a mixed-method analysis using qualitative and quantitative data. The methodology involves categorizing observed elements, which are then represented in the FCM. Some examples of the application of FCM in mathematics education include the analysis of students' problem-solving strategies, the identification of the most effective teaching methods, and the evaluation of the impact of technology on learning outcomes. The use of FCM in mathematics education is not limited to these examples and can be generalized to other educational phenomena. Overall, using FCM as a mixed method analysis in education provides a valuable tool for researchers and educators to understand better and improve educational phenomena. As future developments, we are working on defining a mathematics education framework of data analysis based on describing students' cognitive processes concerning the literature's main and most widely used frameworks.

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E-COACHING METHODOLOGY IN TEACHING LINEAR ALGEBRA AND CALCULUS

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“E-coach: Towards e-coaching – the first step to build trust with a digital coach” is an Erasmus+ project developed in international partnership between Białystok University of Technology in Poland, Leipzig University in Germany, The Technical University of Crete in Greece and Tampere University of Applied Sciences in Finland. The goal of the project has been to develop a methodology for online coaching pedagogy in a university setting and a handbook and course material for both university teachers and students to help them move towards e-coaching. In this paper we present descriptions of our pilot distance classes with students, focused on selected material from linear algebra and calculus. We discuss methods, tools and summarise impressions from students.

Keywords: E-coaching, pedagogy, e-learning, linear algebra, calculus

INTRODUCTION AND HYPOTHESIS PRESENTATION

In the application of the project “E-coach” we presented the analysis of the situation. The COVID-19 crisis brought organizational and educational challenges for universities throughout Europe. All teaching content had to be converted to digital formats at short notice. The Consortium of the Project (Białystok University of Technology (BUT), Leipzig University (ULE), Tampere University of Applied Science (TAMK) and Technical University of Crete (TUC)) conducted in-depth research on the needs of the academic community in connection with the pandemic. The research results are based on over 5,000 student surveys and over 1,000 surveys among lecturers carried out at Partners universities after "digital" semester. They show general experience and perception of both groups about conversion. In the overall view of the survey results, digitization was largely successful. Most of the teachers confirmed that they were able to conduct all courses online and most students had to postpone only a few planned courses due to the situation. What do students like in distance learning? In the first place: availability, mobility and possibility to work at home. In the second place - I do not like anything! What do they dislike the most? In the first place - lack of contact and help from the teacher. In subsequent positions - technical challenges, the chaos, lack of coordination, lack of direct contact with the teacher, lack of direct contact with the group.

Nearly half of surveyed ULE students (46%) indicated that they found it difficult to participate in courses. A lack of motivation (61%), lack of social contacts (60%) was cited as the main reasons for this. Many lecturers (72%) also stated that communication with students did not always run smoothly. During online education, it is very difficult to involve students to actively participate in the classes. And the biggest challenge is to build trust with digital educators. Over 70% of BUT students expect that in the near future the mixture of the traditional approach and the form of distance education will be implemented.

We have stated the thesis that using coaching methods in distance teaching so including e-coaching in teaching process, particularly at higher mathematics problems, we increase motivation and satisfaction among students gaining from remote lectures/meetings. To expertise the hypothesis we

run 6 distance meeting, 3 on linear algebra, 3 on calculus with students from Bialystok University of Technology. We recorded these meetings, created films, see the links to YouTube ([The links to films from meeting on linear algebra], n.d.; [The links to films from meeting on calculus], n.d.), collected impression from students by simple questionnaires after each meeting organised in mentimeter and discussions with them after the meeting and during the local dissemination conference, see the link to the film ([The link to the film with student's impressions], 2023).

COACHING IN EDUCATION

There are many different definitions of coaching, but one that has influenced our opinion of online coaching is Flaherty's definition (Flaherty, 2014), which focuses on the following effects of coaching: 1) long-term good results - coaching does not focus on short-term results. 2) self-correcting of their performance (by students) as individuals and as a team independently of the trainer, and 3) self-generating (learning) through practice in a way that continuously improves performance. In coaching, it is necessary to go beyond the idea that students are somewhat passive recipients of teachers' didactic interventions. The trainer creates the necessary atmosphere of mutual understanding and trust, openness to new ideas, indicates alternative ways, encourages students to create and implements independent decisions, ideas and projects. Coaching as a learning style can be used in any form of the educational process: during seminars, colloquiums, research, classes. Today, it is one of the most important directions, next to the use of new information and communication technologies in education (Fullan & John, 2011; Anatoliivna et al., 2019). The term coaching is increasingly used in the context of teaching at universities, e.g. as guidance integrated with teaching. As a result of the change in teaching culture that the Bologna reforms required, from instructing to guiding student learning.

THE IDEA OF E-COACHING

The COVID-19 crisis brought organizational and educational challenges for universities throughout Europe. All teaching content had to be converted to digital formats at short notice. E-coaching refers to coaching that takes place through electronic means, such as email, video conferencing, instant messaging, or other online communication tools. It is a form of coaching that is conducted remotely, allowing coaches to work with clients from anywhere in the world. The concept of e-coaching can be defined as the practice of coaching through technology (Kamphorst, 2017). For example, a human coach uses technology as a mode of communication to get information about a coachee's behaviour or to give feedback. Or more specifically according to (Ribbers & Waringa, 2015), as: "E-coaching is a non-hierarchical developmental partnership between two parties separated by a geographical distance, in which the learning and reflection process is conducted via both analogue and virtual means."

Based on the extensive coaching literature available and the wide range of experiences available within the project team, we have selected four main themes that function as cornerstones of our e-coaching methodology: 1) trust, 2) dialogue, 3) ownership/responsibility, and 4) co-creation.

The cornerstones indicated in Fig. 1 form a whole system, because they support each other, and the coaching approach is built on them. Without any of these cornerstones, the online coaching framework remains only partial. Teachers and students often find it easier to use the traditional delivery method. Especially if they feel pressure to minimize the extra workload.

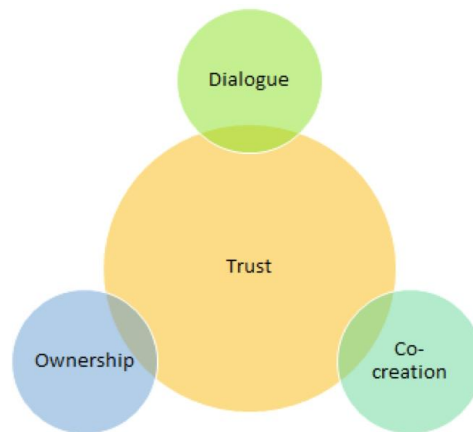


Figure 1. The four corner stones of e-coaching.

The basic building block of coaching is trust, both in face-to-face and online learning situations. In general, students and teachers can rely on two types of trust: trust that others will act the way they said they would and, more importantly, trust based on vulnerability (Lencioni, 2017; Edmondson, 2018).

Dialogue can be defined as a collaborative exploration of experience and thinking, see (Isaacs, 1999). In a given team, this is needed to understand the common situation and how it is perceived and understood differently by each team member. Dialogue is also great for building trust and even friendship between people with different cultural and social connections and personalities.

Ownership creates engagement and motivation. Ownership here means that students are the primary owners of their personal and team learning processes and the work they do together. This means a sense of responsibility for the results, but also an awareness of the impact that one's own actions and behaviour - including interactions with other people involved - have on the joint work. Facilitating a sense of ownership increases students' sense of autonomy, competence, and teamwork, which helps build and maintain their intrinsic motivation (Ryan & Deci, 2020) and initiative.

Co-creation in e-coaching means that the team engages in a creative process where they create something of value together. It can be a product, a service, a prototype, a solution to a complex problem, or an experiment they are involved in together. Co-creation is a collaborative mode of working that provides a context for coaching. Co-creation creates opportunities for all the other cornerstones of e-coaching: there is a clear need to build and maintain trust between the people involved, there is a clear need for dialogue to make sense of what is co-created and how, and building something together is one of the best ways to build a sense of ownership over the process and its end products, but also over the entire team.

STUDENT STATEMENTS DESCRIBING THE INDIVIDUAL CORNERSTONES OF E-COACHING

The following statements may initially be used by the teacher as a heuristic to think about the student's experience ("How would students respond in my course?"). Each cornerstone is associated with 3-4 statements that affect the cornerstone positively and one that affects it negatively.

Trust

- (+) I feel that I can safely share my thoughts, experiences, feelings and even insecurities with the group.
- (+) I can talk about the mistakes I've made or my insecurities without fear of being punished or ridiculed.

- (+) The teacher, by example, encourages you to build trust by sharing your own mistakes, doubts or uncertainties from the past.
- (+) The teacher shows a high level of confidence in our ability to learn and succeed in our joint efforts.
- (-) I wouldn't feel comfortable sharing my weaknesses or weaknesses in the group.

Dialogue

- (+) I feel safe sharing and talking about my own experiences and thoughts. I feel that others listen to me and respect me.
- (+) Together, we explored, in a spirit of open dialogue, the different ways in which we think about the most important issues and problems.
- (+) I actively take responsibility for the quality of my interactions with others to support the collaborative learning process.
- (-) I feel that the teacher is not interested in how we think about the matter.

A sense of ownership

- (+) I feel responsible for the topic and learning targets. The topic of the classes is related to my life goals.
- (+) We have jointly established learning targets and quality criteria for performance.
- (+) We can plan and implement our own ways of learning in a way that best supports our learning.
- (+) We can set the pace of our work together keeping in mind the overall learning objectives and expected results.
- (-) The teacher retains unilateral control over the work we have to do.

Co-creation

- (+) Together we create solutions to complex problems.
- (+) There is a place to practice creativity and innovation together.
- (+) No one is left alone against their desires to deal with difficult problems or work that needs to be done.
- (+) Everyone's intelligence and effort are appreciated and used.
- (-) The course and success in it are mainly based on individual effort.

EXAMPLES FROM PILOT CLASSES

The three online linear algebra pilot meetings focused on the following topics:

1. Introduction to complex numbers.
2. Systems of linear equations and their applications.
3. Application of linear equations and inequalities.

In the math analysis class, we tested the new methodology in meetings about:

1. Applications of derivatives of functions of one variable.
2. Applications of definite integrals.
3. Selected applications of first order ordinary differential equations.

We cooperated with students of the Bialystok University of Technology from the following faculties: applied mathematics, computer science, computer science and econometrics, electronics and telecommunications. The students were in their first semester of studies. At each meeting, the

group of students consisted of about 12-20 people. Then we divided the group into smaller subgroups of about 3-5 people. Three teachers attended the meetings. Each of us looked after one or two small groups of students. We used the following tools:

- MS Teams or any other application where you can create rooms for small groups.
- Online boards (MS Whiteboard) both in the main meeting and in virtual rooms.
- Other tools used by participants (unlimited).
- Cameras (if possible).
- Mentimeter or any other app for quick survey creation (modern, mobile, dynamic response).
- Additional materials, in particular in advance, were also posted on MS Teams.
- The description of the scope of the tasks performed can be found in the relevant courses on the Coursevo platform <https://ecoach.coursevo.com/>, developed by the Technical University of Crete, Greece.

HOW DID WE BUILD THE TRUST?

For example, by using the Mentimeter on menti.com to jointly build a word cloud as answers to the following questions (examples, results presented on Fig. 2):

- What's your superpower?
- What do you think about e-coaching?
- When are you most stressed?
- Who is your biggest inspiration in life?
- Other.

E-coaching methods used at meetings (dialogue, co-creation)

- Buzz Groups, i.e. discussion in small groups
- assigning a task/topic for discussion (for a limited time);
- providing a place to take notes (e.g. MS Whiteboard, OneNote, Etherpad, etc.);
- appointing a person in the group responsible for reporting;
- view the shared document and summarize the discussion;
- setting a short time frame for any questions.



Figure 2. An example of words cloud to build the trust (from menti.com).

Selected methods of e-coaching

- The muddiest point of trust and co-creation;
- What? So what? What now? about ownership, dialogue;
- Peer instructions for ownership, co-creation, dialogue, trust.

Most of the methods were included in a teacher training course and a separate course for students on e-coaching methodology on the e-coaching Coursevo platform.

EXAMPLES OF OUR CLASSES

Algebra 1. Introduction to complex numbers

It was a kind of discovery lesson. Working in groups students tried to solve a quadratic equation with negative discriminant. Then they discuss the issue of algebra in a set of complex numbers. They define how two complex numbers can be added, multiplied and divide. We summarize each session inside small groups on a general channel. Then they were asked what set it is created when a complex number $z=x+iy$ is multiplied by its conjugation is equal 1.

Algebra 2. Systems of linear equations and their applications

The aim of the course was to apply systems of linear equations by students to solve specific practical tasks. Students decided on the existence of solutions to posed problems related to the application of systems of linear equations. In small groups students tried to solve some exact problems.

Algebra 3. Application of linear equations and inequalities

The aim of the course was to describe the given optimization problem using linear equations and inequalities. In the considered decision problem there will be two variables, so one of the possibilities of solving the linear programming problem will be the graphical method. Before the classes, students were asked to familiarize themselves with the advance material available on the Khan Academy. We propose to solve in small groups/rooms in MS Teams some exercises. Students discussed it at white board, shared ideas, built the model and then present in general room before other groups.

Calculus 1. Applications of derivatives of functions of one variable

Before the classes, students were asked to familiarize themselves with the advance material provided in the form of videos available on the Khan Academy website (Applying Derivatives to Analyze Functions, n.d.).

The first question we posted for students to be solved in buzz groups was the following.

Calculus 2. Applications of definite integrals

After reading the material preparing for the classes in the form of videos available on the Khan Academy website and materials presented in class, students support each other in groups and solve given geometric problems. Then we use an anonymous quiz with Mentimeter to check pre-reading material and after that we came with groups to work over the given problems.

Calculus 3. Selected applications of first order ordinary differential equations

During the classes, participants received selected problems that are related to first-order differential equations. After reading the material preparing for the classes in the form of videos available on the website (First Order Differential Equations | Math, n.d.) and materials presented in class, students supported each other in solving problems in groups, which can be described by ordinary differential equations of the first order. As problems we chose typical description of RLC or RC circuits.

HOW DID WE SUMMARIZE THE CLASSES?

At the end of each meeting, we talked with students about their impressions, both directly on the general channel and also anonymously through the use of menti.com and mobile devices. See Figure 3 for an example of an automatic class summary. At almost each of 6 meetings we received similar answers from different groups of students. We conducted our meetings with students from Applied Mathematics, Computer Science and Econometry and Electrical Engineering and they had similar impressions. After the automatic survey and direct discussion we observed that students found interesting working in rooms in more than 90%, they also think that they learn more in such way that in traditional e-learning session (85%), they are in 73% sure that they are co-authors of the solutions.

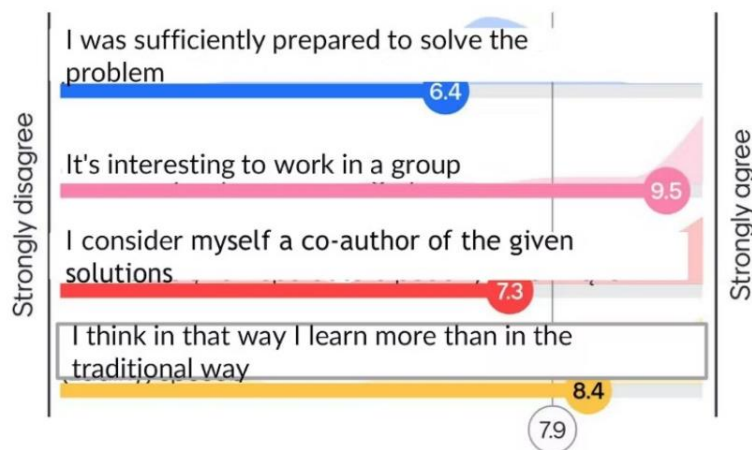


Figure 3. An example of the summary of impressions using sliders on menti.com.

SUMMARY

The pilot classes conducted using the e-coaching method concerned selected topics from the indicated subjects. In addition, the students involved were volunteers who agreed to participate in online meetings and to record individual sessions. Recordings from the pilot meetings described in this article, together with the English translation, will be available on YouTube of the Bialystok University of Technology. The E-Coach project (2020-1-PL01-KA226-HE-095455) was implemented with the support of the European Commission.

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Understanding the value of Analytics and AI in Mathematics Education

HELP-SEEKING IN AN ONLINE MATHS ENVIRONMENT: A SEQUENCE ANALYSIS OF LOG FILES

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In online mathematics environments feedback is often provided to help students make progress. However, the extent to which students make use of such feedback, so-called ‘help-seeking’, depends on numerous instructional variables, including the design of the online platform and individual student characteristics. Furthermore, student behaviour in such platforms are not independent events: the order in which tasks are completed matters, and we therefore have to study sequences of such events. This study used student data from UK students in grades 3 to 5 with at least 100 lesson records in the academic year ’18-’19 (N=1,799), totalling 1,048,575 records between December 2010 and January 2019, from an online mathematics platform. Sequence analysis was applied to the data to uncover patterns of help-seeking. The results showed that help, task difficulty and precision interact, demonstrating the usefulness of learning analytical approaches like sequence analysis.

Keywords: help-seeking, online mathematics platform, sequence analysis, feedback

BACKGROUND

The recent global pandemic has shown how in a relatively short time, online provision has become essential in the learning experiences of school pupils. This, however, is not without its challenges. Utilising computers to aid tutoring is a relatively recent development in education and coincides with the development of computers. Whereas human tutoring has been used in schools for 2,500 years—or for as long as schools have existed—computer tutoring is largely a product of the past half century (Kulik & Fletcher, 2016). In this period numerous reviews have appeared that tried to summarise the efficacy of such technological developments. Already, in the 70s and 80s computer tutoring systems appeared that tried to combine artificial intelligence concepts and cognitive theory. Such systems guided learners through each step of a problem solution by giving hints and feedback. The first-generation computer tutors are now often referred to as CAI tutors (computer-assisted instruction), the more modern variations are usually called Intelligent Tutoring Systems, or ITSs (VanLehn, 2011), which distinguish themselves from CAI tutors because they can, in addition to providing feedback on student answers, also provide feedback on the thinking that goes into individual answers. ITSs in that sense are systems that present feedback and/or hints (but they in essence also are feedback) to learners as to assist the learning process. For Mousavinasab et al. (2018) this also includes adaptive guidance and instruction, which I take to be guidance that can vary depending on the learner’s context. As reported by Du Boulay (2016), there have been a number of recent positive reviews in support of the effectiveness of ITSs (Kulik & Fletcher, 2016; Ma et al., 2014; Steenbergen-Hu & Cooper, 2013, 2014; VanLehn, 2011). Thus, it is well known that well-designed ITS can successfully complement and substitute other instructional models at all educational levels and in many common academic subjects (Ma et al., 2014). Kulik and Fletcher (2016) in their review of Intelligent Tutoring Systems concluded that “ITSs typically raise student performance well beyond the level of conventional classes and even beyond the level achieved by students who receive instruction from other forms of computer tutoring or from human tutors” (p. 70). One of the working ingredients of these platforms is the provision of feedback. In their review of effects of feedback in computer-based learning environments, Van der Kleij et al. (2015) showed that elaborated feedback (EF; e.g., providing an explanation) produced larger effect sizes (0.49) than feedback regarding the correctness of the answer (KR; 0.05) or providing the correct answer (KCR; 0.32). EF was particularly more effective than KR and KCR for higher order learning

outcomes. The larger effect sizes were found for mathematics compared with social sciences, science, and languages. Appropriate help-seeking behaviour in relation to feedback use, can positively influence student learning gain (Tai et al., 2016). Help-seeking influences students' learning when interacting with ITS as such systems often provide answers to students' requests. The role of metacognitive skills and cognitive factors is important in help-seeking behaviour (e.g. Aleven et al., 2003, 2016; Aleven, 2013; Roll et al., 2011), as the use of help can also be harmful for learning. Help-seeking in tutoring systems has been associated with students whose main objective is to finish the activity at hand and not to acquire knowledge. Although help-seeking in ITS is not a new issue, log files of student behaviour have not often been looked at in terms of sequences of activities and associated help-seeking related to performance in an online maths environment.

METHODOLOGY

Online mathematics platform

Maths-Whizz is an intelligent online tutor for 5 to 13-year-olds. It comprises more than 1200 learning objectives which have been organised into 22 topics and sequenced within each topic based on a curriculum map developed by educationalists. Maths-Whizz is being used by hundreds of schools in eight international territories and serves over 150,000 students worldwide. The platform has been used in several studies (e.g. Clark & Whetstone, 2014; Schatten et al., 2014, 2015; Mavrikis et al., 2018; McIntyre, 2022a; 2022b), but to the best of my knowledge not with an emphasis on help-seeking. An example of feedback for a 'Shape and Space' lesson is provided in Figure 1, with the feedback at the top given if an incorrect answer is given [i].

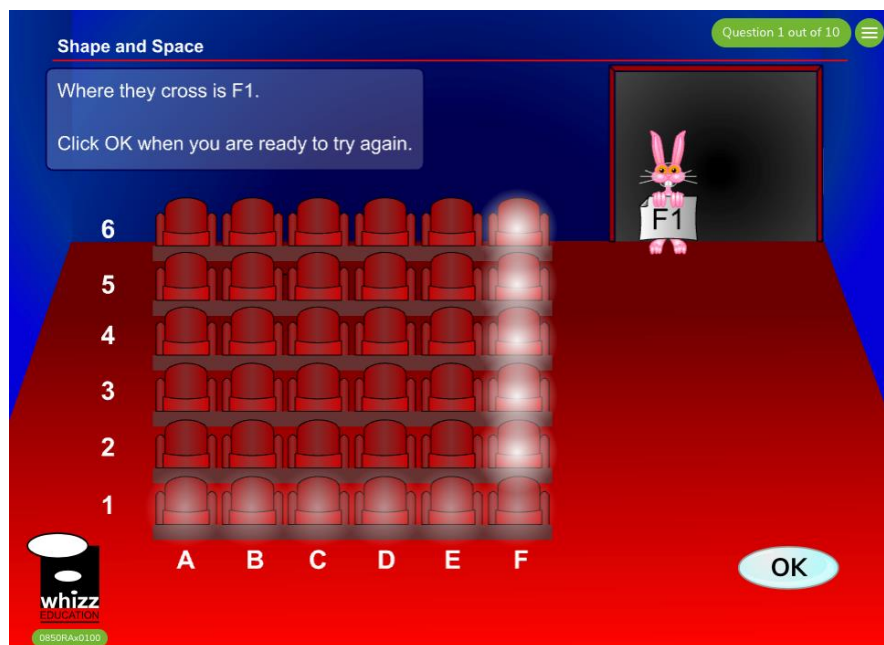


Figure 1. Feedback example for an incorrect answer in a 'Shape and Space' lesson.

Sample

The data sample included all records of school users in the UK in Years 3-5 who had at least 100 lesson records in academic year '18-'19 (N=1,799). Number of records ranged from 100 to 4970 entries, totalling 1,048,575 records from between December 2010 and January 2019.

Dataset

The dataset included 1,048,575 records, with each row corresponding to a single attempt at a lesson. Each lesson has a criterion-referenced measure of difficulty. Maths-Whizz lessons usually consist of an exercise (interactive, animated, scaffolded helps) and a test (modelled on paper tests). The dataset contains every attempt at a completed exercises and tests. Table 1 presents the variables in the dataset, as well as derived variables.

Variables	
id	A student id.
diff	The average difficulty of all exercises done by the student.
qtime	The total time the student spent completing exercises (sum of all the times).
nquestions	The total number of exercises a student completed.
totalscore	The total score for all the questions done.
totalhelp	Total help used by the student.
nlessons	Total of number of lessons (each having a number of exercises).
start	First date of Maths-Whizz usage.
end	Last date of Maths-whizz usage.
dob	Date of birth stored as integer.
gender	Categorical variable: 1=male, 2=female.
Derived variables	
range	Last date of usage minus first date of usage (end-start), in days.
precision	This was the total score divided by the number of questions (totalscore/nquestions).
intensity	This was the number of lessons per time unit (nlessons/range).

Table 1. Variables included in the dataset.

Finally, for every student there was a sequence of ‘run modes’. The length of the sequence could differ per student. The seven ‘run modes’ could be: p=test, x=exercise, j=exercise that was jumped (i.e. student was performing well and was given a pass before completing all question, a=assessment mode (i.e. student completed the test in the diagnostic initial assessment), b=exercise that was jumped backwards (i.e. student scored 0/4 and was assigned a fail), r=replay, and s=static (i.e. student scored between 30% and 70%; it is neither a pass or a fail and means they'll receive the same lesson when returning to the topic).

Data analysis

After presenting descriptive statistics, two analyses were applied to the dataset. Firstly, multivariate regression was applied with dependent variable ‘precision’, and independent variables ‘diff’, ‘qtime’, ‘intensity’, ‘totalhelp’, ‘dob’ and ‘gender’, to see if any of these were associated with precision. Secondly, sequence analysis was applied. Sequence analysis is a data-driven approach that can unearth sequence characteristics given a dataset with sequence element. Investigating behavioural logs “has a great deal of interest” (Boroujeni & Dillenbourg, 2018, p. 206) in educational data mining and learning analytics communities. The TraMineR package, an R-package for mining, describing and visualizing sequences of states or events, is used (Gabadinho et al.,

2011). In this paper we report the results of clustering to typify the types of sequences in the dataset, as well as the results of a discrepancy analysis. Such an analysis can represent the discrepancy between sequences and visualise the results in a regression tree, highlighting how the state sequences are related to one or more covariates (see Studer et al., 2011).

RESULTS

This section reports on the descriptive statistics, regression analysis and sequence analyses. Table 2 presents the descriptive statistics.

Variable	N = 1,799
precision	0.87
intensity	0.79
diff	861
qtime	1.01
nquestions	2,709
totalscore	2,288
totalhelp	369
nlessons	377
range	507
dob	39,933
Gender -	Male 961 (53%) – Female 838 (47%)

Table 2. Descriptive statistics of the variables.

Table 3 presents the results of the regression analysis.

	scale(precision)		
Predictors	Estimates	CI	p
(Intercept)	0.00		
diff	0.02	-0.03 – 0.06	0.402
qtime	0.42	0.36 – 0.48	<0.001
intensity	0.04	-0.01 – 0.08	0.112
totalhelp	-0.63	-0.70 – -0.57	<0.001
dob	0.03	-0.01 – 0.07	0.171
gender	0.03	-0.01 – 0.07	0.151
Observations	1799		
R ² / R ² adjusted	0.191 / 0.188		

Table 3. Results of the regression analysis.

The analysis shows that both the total time the student spent completing exercises and the total help used by the student are predictive of precision, with the other variables not being significant predictors. However, there where the coefficient for total time spent is positive, that for total help is negative, indicating that more help negatively impacts on precision. Note, though, that the study does not have an experimental design, and therefore the results are *correlational*. It also is sensible to assume that students with more precision (i.e. higher achievement) need less help. Sequences from 1,799 students were included in the sequence analysis, with the minimum sequence length being 100, as per the inclusion criterion under ‘Sample’, and the maximum sequence length being 4,970. For determining the optimal number of clusters, we used several methods (elbow, silhouette, gap), indicating a solutions of $k=3$. Figure 2 presents the three cluster solutions. It is apparent that the sequence length is a confounder for the cluster solution; in other words, students differ significantly in their engagement with the learning environment. However, it can also be seen that for Type 3, the ‘replay’ mode, is much more prevalent, possibly indicating help-seeking behaviour.

As some derived variables correlate with variables used to calculate them, not all variables were included in the discrepancy analysis, only diff, dob, gender, precision and totalhelp. If qtime would be included, it would be a large confounder, as visually clear from the clustering. Table 4 presents the results of the discrepancy analysis. Four of these are significant variables, but based on the pseudo R-squared totalhelp contributes most to the current model, far ahead to the second most contributing variable of precision. For the regression tree, see Figure 3, we only include these two variables, as visualisation of more variables would negatively influence readability. Just like the discrepancy analysis, we do not include qtime and intensity in the regression tree, as qtime and intensity are trivially associated with the sequences of exercises done. So, we included diff, dob, gender, precision and totalhelp.

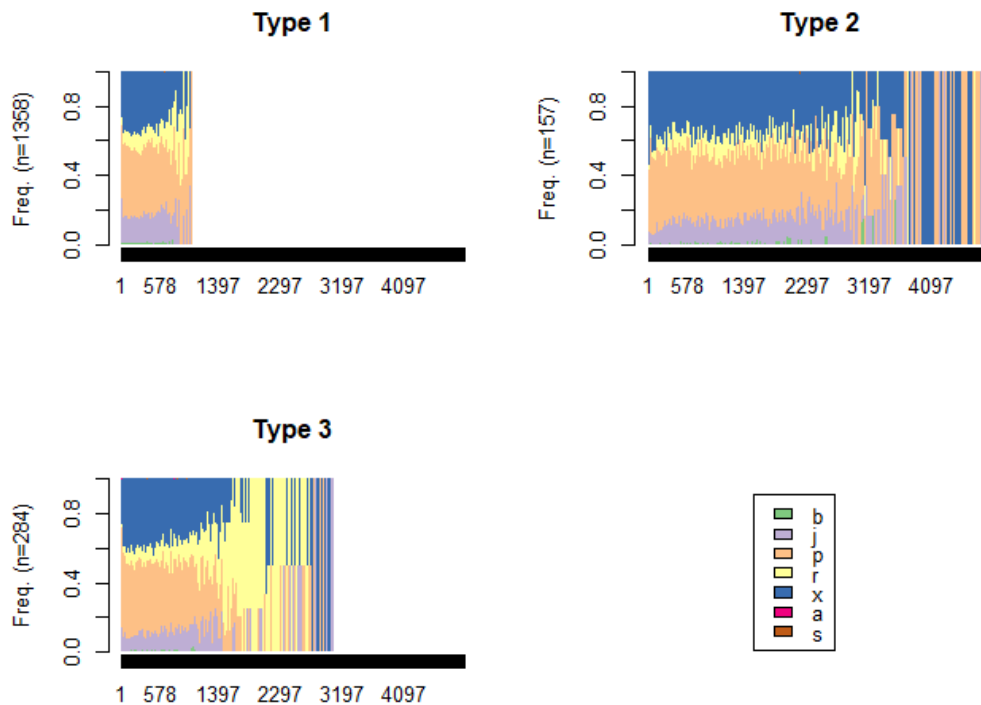


Figure 2. Three clusters of solutions.

Variable	PseudoF	PseudoR ²	p_value
diff	12.50181	0.005251	0.001
dob	18.15211	0.007625	0.001
gender	0.896066	0.000376	0.491
precision	44.77168	0.018806	0.001
totalhelp	553.9927	0.232703	0.001
Total	117.5366	0.246855	0.001

Table 4. Results of discrepancy analysis.

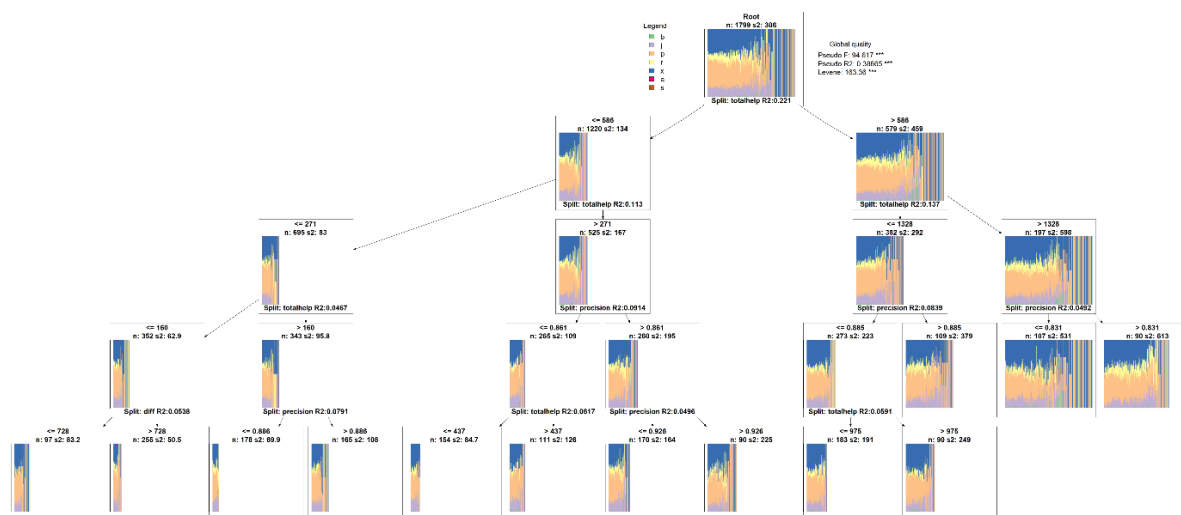


Figure 3. Regression tree for sequence analysis.

Figure 2 indicates that totalhelp and precision, as well as difficulty to a lesser extent, interact with each other. More help extends the sequence length and extends precision, but only up to a point. Furthermore, for too difficult questions (bottom left in the diagram) help does not increase the sequence. My interpretation here is that well-tailored help helps the student along without compromising difficulty. However, this is subject to a Goldilocks ‘sweet spot’: help-seeking does not contribute to precision if the learning content is too difficult or the student too proficient. Help is not needed if the learning content is too easy. Help-seeking might not be productive if a student is not proficient enough yet.

CONCLUSION

This paper looked at data from an online maths environment that can be considered an ITS: students can seek for help by making use of feedback provided by the platform. The results show that feedback and associated help-seeking may not be unequivocally positive or negative. Its usefulness depends on the way it is implemented, and interacts with numerous metacognitive and cognitive factors. These results, using a learning analytics approach through sequence analysis, echo those from Aleven et al. (2016) who adjusted their initial view of the importance of on-demand, principle-based help during tutored problem solving, to a view as “helpful under certain circumstances” (p. 205).

ACKNOWLEDGMENT

Great thanks to Junaid Mubeen, the staff at Maths-Whizz, and the students who worked with the platform.

ENDNOTES

[i] ¹ This example is from <https://whizz.com/year-3-maths-games/>

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SOLVING A PROBLEM WITH AI: CONSIDERATIONS ON ASSESSING LEARNING MATHEMATICS

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The application of artificial intelligence (AI) in STEM education and particularly in mathematics, as an emerging field, is confronted with a challenge of integrating diverse AI techniques and complex educational elements to meet instructional and learning needs. It is well known that assessment is a key point in educational ethics. In addition, central elements of assessment are fairness and validity. On one hand, unfair use of artificial intelligence (AI) by students could invalidate the assessment. On the other hand, teachers' behaviour when faced with AI helps us to construct their identity. It is urgent to investigate how students' use of AI could invalidate the assessment. We discuss a possible use of ChatGPT for investigating whether teachers recognize the use of AI.

Keywords: artificial intelligence, ChatGPT, assessment, mathematics education, computer science.

INTRODUCTION AND THEORETICAL BACKGROUND

Since January 2023, ChatGPT (accessible at the URL <https://chat.openai.com/chat>) has entered massively into the communication of major newspapers (Ayдын & Karaarslan, 2022; Qadir, 2023), even though the chatbot in question (in the first beta version) was released by OpenAI manufacturing company on November 30, 2022. After remarks by Microsoft CEO Nadella in Davos, it is known that ChatGPT will be integrated into many Microsoft products, primarily Bing, the company's flagship browser- and Bing's possible use in education is emphasized in Microsoft communication. The impact undoubtedly of the new tool has been strong, the final product is awaited.

The impact will involve all sectors of our society, but for now the most direct seems to be the training and cheating effect. ChatGPT, at the moment, is not certified and does not cite sources where it gets its information (Yu, 2023). A central and strategic question arises: what will be the impact on educational systems, specifically in the teaching of science disciplines and mathematics in particular? Can teaching not take into account the new perspectives and challenges that AI poses, is one prepared for this “challenge”? Our focus will be on a particular but key aspect of educational processes, i.e. *classroom assessment*. Ethical issues are involved in classroom assessment at several levels (Green et al., 2007); The EU recently drafted an interesting document “Ethical guidelines for educators on the use of artificial intelligence (AI) and data in teaching and learning” (European Commission, 2022).

In the past, artificial intelligent projects like “Eliza” (Weizenbaum, 1966), “Student” or “GPS” (Russel, 2010) attempted to produce agents which can solve general mathematical problems. One soon realized that this goal was too ambitious, especially when “problems” means “open ended tasks” and in particular “argumentative tasks”. Not only because of technical hurdles, but also because one had to realize that many problems are computationally hard and cannot be settled in reasonable time. While this slowed the momentum of AI developments in education in the last decade, it did not stop it, and the emergence of new tools is likely to start a new era.

Teachers' identity is looked at as a system of beliefs (Ferretti et al., 2021) and these beliefs affect classroom practices and values. Mathematics education research has traditionally looked at a

teacher's identity as a system of beliefs. Teachers' beliefs have become a focus of research in mathematics education on the agreed assumption that beliefs affect school practices and behaviors. The idea of system thinking is based on the assumption that the concept of belief can be better understood in the relation to other constructs, such as attitude, affect and values, where the different parts are not separable from each other and are situated in a specific context within any person or group.

The issue of defining beliefs has been rephrased, and beliefs are now seen as part of a larger set of constructs regarding teacher identity and classroom interaction, where teachers are "sensible systems that act in a coherent way" (Hannula et al., 2016). For the purpose of our study, we would like to outline a specific and operational definition of teachers' beliefs and convictions, which embeds their identity and thereby scrutinizes the interplay between identity and assessment.

An unavoidable starting point when we want to identify beliefs and convictions related to mathematics teaching and learning is the notion of epistemology, as it has been discussed by Brousseau (Brousseau, 2006). We therefore need to cast teachers' beliefs and convictions in epistemology and the related notions of cultural knowledge and personal knowledge. An epistemological conception is a set of convictions and pieces of knowledge that tell us what the knowledge of individuals and groups of individuals is: its functioning, its forms of validation, its acquisition and its influence on teaching and learning. Brousseau introduces the concept of school epistemology to outline the combination of convictions, beliefs that circulate in the school system regarding the nature of knowledge, methodology, assessment, teaching and learning and the social and political role of mathematics. The noosphere is the joining link between the school system (the teachers' choices included) and the broader social environment it belongs to (Godino, 1993).

The purpose of this paper is to describe an experiment whose goal is to bring out teachers' beliefs, and attitudes (McLeod, 1992) in an assessment situation in which AI is involved. Assessment is definitely an important element in defining teacher identity, and the role of AI in assessment processes is a central issue. In addition, comparing teachers' behaviors and attitudes in these assessment situations allows us to investigate their beliefs and attitudes toward AI.

This may help in exploring how far students' use of AI can invalidate assessment. In particular, we use ChatGPT to investigate whether teachers recognize the use of AI, whether this could be useful to bring out beliefs and practices of teachers. Comparing teachers' behaviors and attitudes in these assessment situations allows us to investigate their beliefs and attitudes toward both AI and assessment. Our general problem may be stated as follows: how AI's answers to a mathematical task interface with teachers' assessment practices and beliefs?

METHODOLOGY OF THE STUDY AND ITALIAN CONTEXT

The study involved 23 middle school teachers and a task on prime numbers from the Italian national standardized assessments (INVALSI). The tests are administered every year at census level and student results are provided to each school institution. Results and questions of the INVALSI tests are considered as a resource also for researchers in the field of mathematics education (Garuti & Martignone, 2015; Bolondi & Ferretti, 2021) and are used in national and international research (e.g. Spagnolo et al., 2021).

The SNV Framework is designed taking into consideration the Italian National Guidelines, in which argumentation is considered between the competence goals for every school grade (Garuti & Martignone, 2019). For this reason it is possible to find mathematical tasks that require the recognition or production of a justification, as the tasks we selected for our research design.

For this research, results at the national level were used as a benchmark of difficulty and significance at national level; specifically, nationwide in Italy, 37,1% of grade 8 students answer correctly,

41,4% of grade 8 students get it wrong, and 21,6% of grade 8 students do not answer. The task required production of argumentation to support a statement or its denial. Teachers had to assess 5 answers to the task, taking in consideration accuracy of content, accuracy of language, coherence to the task, coherence to the claim, completeness in calculation, and completeness in argumentation. After their assessment, it was disclosed to them that one of the protocols had been produced by AI ChatGPT-3 in answer to the task. None of the protocols (not even the one produced by ChatGPT-3) contained an answer that was considered correct according to standardized assessment standards. After that, teachers tried to identify the answer produced by ChatGPT and to give explicit reasons for their choice. In a second phase, teachers were involved in focus groups and individual interviews to compare criteria and ways in which they had assessed and addressed choices.

In particular, the tasks chosen in this study are represented in Figure 1; the choice of INVALSI task has been made because it allows us to have some extra information that provides a further background for our study, such as the performance of Italian students (which is related to task peculiarities). In our study, we focused on grade 8 INVALSI task, which represent the last year of middle school in Italy.

We identified an INVALSI task (Figure 1) for grade 8 and selected some of the typical answers that grade 8 students provide.

D15. n is a natural number. Consider the statement, "If n is even then $n+1$ is a prime number." Is the statement true or false?

Choose the answer and complete the sentence.

☐ The statement is true because

.....

.....

☐ The statement is false because

.....

.....

Figure 1. Task D15, Math INVALSI Test Grade 8, 2016, www.gestinv.it.

The same task was proposed to ChatGPT-3 and the response was as follows.

ChatGPT-3: The statement is false because, for example, when $n=2$, we have that n is even, but $n+1=3$ which is a prime number. However, if we take $n=4$, then n is even, but $n+1=5$ which is a prime number.

So, it is not possible to say that "if n is even then $n+1$ is a prime number" in general.

We asked 23 middle school teachers to assess all selected answers (including that of ChatGPT-3). The assessment was done according to the following criteria: accuracy of content, accuracy of language, coherence to the task, coherence to the claim, completeness in calculation, completeness in argumentation. Then it was revealed to the teachers that one of the solutions was made by ChatGPT-3, asking them to recognize it.

DISCUSSION AND PRELIMINARY RESULTS

The data of the national Italian sample are gathered, analyzed and released by INVALSI. Data can be found at www.gestinv.it. Following, we report the national results of the task shown in Figure 1.

This task aims to assess the ability to use of some typical forms of mathematical thinking (conjecture, argue, verify, generalize); in particular supporting their convictions by bringing appropriate examples and counterexamples and using argumentation to justify their choices.

From a quantitative point of view, the percentage of correct answers of the Italian national sample is 37,1%, while the percentage of incorrect answers is 41.4% and of unanswered questions is 21.6%.

Below we show 4 categories of student answers (Figure 2).

Student a	Student b
n è un numero naturale. Considera la seguente affermazione "se n è pari allora $n + 1$ è un numero primo".	n è un numero naturale. Considera la seguente affermazione "se n è pari allora $n + 1$ è un numero primo".
Scegli la risposta e completa la frase.	Scegli la risposta e completa la frase.
☐ L'affermazione è vera perché	☐ L'affermazione è vera perché
☒ L'affermazione è falsa perché <u>se aggiungiamo 1 non è più un numero primo</u>	☒ L'affermazione è falsa perché <u>non tutti i numeri primi sono dispari</u>
Student c	Student d
n è un numero naturale. Considera la seguente affermazione "se n è pari allora $n + 1$ è un numero primo".	n è un numero naturale. Considera la seguente affermazione "se n è pari allora $n + 1$ è un numero primo".
Scegli la risposta e completa la frase.	Scegli la risposta e completa la frase.
☒ L'affermazione è vera perché <u>se ad un numero pari pari aggiungiamo 1 è normale che il risultato sarà un numero 1.</u>	☒ L'affermazione è vera perché <u>se ad un numero pari aggiungiamo uno è normale che il risultato sarà un numero primo</u>
☒ L'affermazione è falsa perché <u>non è vero che il risultato sarà per forza un numero pari.</u>	☒ L'affermazione è falsa perché <u>non è vero che il risultato sarà per forza un numero primo</u>

Figure 2. Examples of student answers.

The transcription of the 4 responses (Figure 2) to the task in Figure 1 follows.

Student a: The statement is false because if we add 1 it is no longer a prime number.

Student b: The statement is false because not all prime numbers are odd.

Student c: The statement is true because if to an even number you add 1 it is normal that the result will be a number 1. [Answer deleted]

The statement is false because it is not true that the result will necessarily be an even number.

Student d: The statement is true because if to an even number you add one it is normal that the result will be a prime number. [Answer deleted]

The statement is false because it is not true that the result will necessarily be a prime number.

At the same task ChatGPT-3 answers by giving examples (not counterexamples) and without maintaining consistency with the request of the task and its own claim.

All 5 protocols have been assessed by 23 middle school teachers. After assessing them, we asked them to identify the ChatGPT one. Only 19% of teachers recognized the protocol by ChatGPT-3, while the most chosen protocol was that of Student d (that is “The statement is false because it is not true that the result will necessarily be a prime number”). The percentages are shown in Figure 3: 0.1% of teachers chose Student a answer, 19% chose Student b answer, 14,3% chose Student c answer, 47,6% chose Student d answer, and 19% identified the protocol produced by ChatGPT-3.

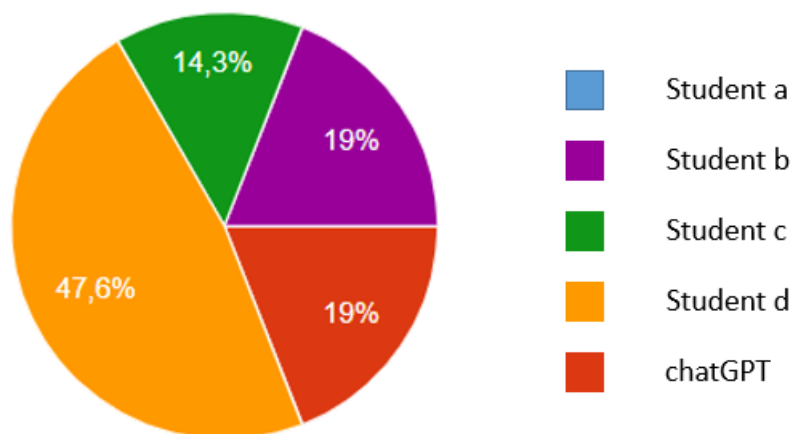


Figure 3. Percentages of teachers' answers to the question: “One of the 5 protocols was created by ChatGPT. In your opinion, which one?”.

Then, focus group and interviews have been conducted with the purpose of letting teachers share and make explicit reflections about motivations of their choices. Teacher motivations of their justifications have been analyzed from a qualitative point of view.

We provide examples of answers given by teachers explaining how they try to identified the protocol that they thought had produced ChatGPT-3:

Teacher 1: I chose by exclusion, because no answer seemed to me that could be from ChatGPT.

Teacher 2: I thought it was a trick question and there was no ChatGPT answer.

Teacher 3: I expected ChatGPT should necessarily answer correctly, so I chose the answer that seemed the less wrong.

From this it emerges that interviews allow us to make explicit some aspects of teacher's thinking about AI. For example, these motivations, bring out some teachers' beliefs, such as the idea that AI always gives a correct answer. Results represented in Figure 3 together with in-depth interviews allowed us to hypothesize the characterization of some aspects of teachers' perceptions of AI.

Specifically, 19 out of 23 of teachers interviewed explicitly stated that they had been in trouble trying to recognize ChatGPT's answer because they expected a *correct* one. The idea that AI always replies correctly is a misconception that can affect us as we work with it (whether we use AI to do an assignment or assess something AI has done).

At the end of the interviews, teachers tested ChatGPT-3. Another element that surprised some of the teachers was that the ChatGPT's answer changed radically if the teacher changed formulation of the question slightly, and ChatGPT's answer also changed if the same question was asked twice in the same way. One teacher during the interviews claimed "[...] if every time we ask the same question to ChatGPT it rephrases the answer it might be really difficult for teachers to identify students who use it for cheating!".

Qualitative analysis has clarified some aspects related to the comprehensibility of teaching situations, and some of these can also be used in teacher professional development. We believe that the analyses, put in relation with the motivations of the teachers, will bring out some teachers' beliefs, such as the idea that AI always gives a correct answer.

CONCLUSION

The irruption of AI into school practice creates new situations that can be considered stress tests that can be used by researchers (Wardat et al., 2023). One of the purposes is to gather information regarding these new tools in mathematics education (Rahman, & Watanobe, 2023). It also becomes important to move toward responsible AI in education, aware of the challenges and implications for research and practice (Mhlanga, 2023).

Questionnaire answers and interviews with teachers highlight some of the criteria they use when assessing student argumentation in mathematics. In particular our analysis shows teachers' beliefs about the role of AI in mathematics education, particularly the idea that AI always gives a correct answer.

It is clear that good and specific questions must be asked in order to obtain more and more significant answers to gain knowledge about the object of the research. If the question is generic, the answer is generic, often so much so that it is incorrect (Megahed et al., 2023).

This suggests that the fears surrounding the use of ChatGPT by students, and which have led some U.S. colleges and universities to ban its use, are overblown: if the output of ChatGPT is complex, articulate, and robust, there is accurate, detailed research behind it, grounded in good, surgical questions.

This is one of the fundamental goals of teaching action, and that is, to help students ask good questions in order to actually access the knowledge that is only theoretically available to everyone now.

So two sticking points for students: learning to ask good questions and checking the reliability of the information obtained. It seems to us that this is enough to ensure the need for the presence of competent teachers capable of helping students grow as knowledgeable, critical and ethical subjects. AI can become a formidable ally of the teacher in his or her many activities, the figure of the teacher assumes, in this new context, a strategic role even at a social level. This is a necessary and different approach to knowledge.

Finally, we believe that this study also allows to highlight some teachers' beliefs, such as the idea that AI always answers correctly.

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Poster presentations

A DIDACTIC PROPOSAL FOR TEACHING MATHEMATICS TO PRIMARY SCHOOL CHILDREN

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Mathematical education in Greece has stagnated for several decades using traditional curricula and outdated practices. The present project proposes a new model for game-based teaching intended for elementary education classes with mathematical content, allowing the flexibility of distance learning. We introduce pupils to basic concepts from graph theory and combinatorial optimization, utilizing 3D animations and interactive games. Our approach provides a student-friendly learning framework that is pioneering for the Greek educational system and promotes constructive thinking and induction rather than mimicking techniques.

Keywords: Game-based learning, Primary education, Problem-solving, Mathematical education.

INTRODUCTION

In this context, the University of Piraeus initiated the project entitled as KIDEDU (Play Create Learn), which is intended to reform the traditional classes in mathematics and economics and addresses pupils aged between six and twelve. To this end, a series of interactive games involving 3D animations is created and seeks to equip pupils with the fundamental concepts of basic operations as well as to familiarize them with conceptual tools borrowed from the areas of graph theory, combinatorial optimization and decision making.

DIGITAL GAMES BASED LEARNING IN PRIMARY EDUCATION: EVERYDAY ARITHMETIC

This part of the game guides pupils to construct and solve a system of equations that lead to the answer. One of the sentences reveals the number of oaks collected by a squirrel and the rest of them connect one squirrel's gathering to another's. Thus, the specific setting allows pupils to formulate basic equations and conceptualize the broader framework of a system of equations.

In the third stage of this game the pupils are asked to calculate the number of oaks each squirrel ends up with after pooling and sharing, assuming that (i) the oaks are equally divided among the squirrels, (ii) each of the two baby squirrels gets half of the oaks a squirrel gets and (iii) the leading oak-collector gets a bonus of 4 oaks. The specific setting helps pupils develop their skills in formulating elementary equations and practice the basic operations with special attention to that of division.

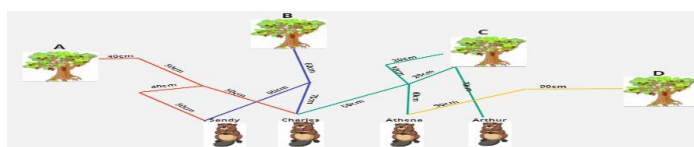


Figure 1: Routes to the oak trees



Figure 2: Statements leading to equations



Figure 3: Oak sharing in the group

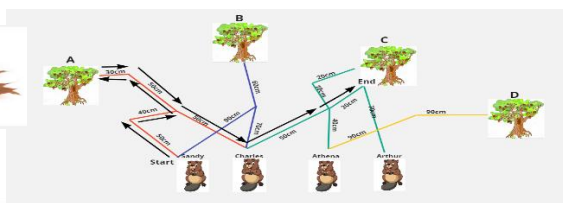


Figure 4: Map finding and route tracing

In the final stage of the game the pupils are asked to help the main character of the game find the map he is seeking. The map has been hidden by the squirrels in one of the oak trees. The squirrels guide the main character to identify the longest path from the starting point to one of the oak trees and subsequently the shortest path from that oak tree to another, in order to locate the map he is looking for. Figure 4 traces the route the main character has to follow to find the map.

CONCLUDING REMARKS

The specific game borrows tools from the field of applied mathematics in order to familiarize pupils, aged between six and eight, with elaborate concepts typically employed in engineering and decision making problems. We develop an IT-based learning mode that is both appealing to younger individuals and agile enough to tailor our teaching aspirations to the needs and preferences of each pupil. This endeavor combines the principles of differentiated teaching and guided discovery learning in order to provide an effective learning mode. Furthermore, this is an innovative approach that seeks to transform the mathematics curricula of the Greek elementary education, which have received considerable criticism on the grounds of their conventional and outdated practices.

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A FRAMEWORK BASED ON ECOLOGICAL THEORY FOR ANALYSING SPATIAL LEARNING IN AUGMENTED REALITY GEOMETRY

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Spatial skills are a crucial ability in reasoning, problem-solving in general and mathematics in particular. Augmented reality (AR) technology has been used to support the development of students' spatial skills, but how students develop their spatial skills while using AR has not been fully explained nor much researched. This paper proposes a framework for analysing students' spatial behaviours in AR based on ecological theory. We describe how this framework will be used for analysing the data generated during the learning process to gain insights into students' spatial learning.

Keywords: spatial learning behaviour, augmented reality, ecology.

BACKGROUND AND RESEARCH AIM

As defined by Tartre (1990), spatial skills refer to mental skills related to understanding, manipulating, reorganising, or interpreting relationships visually. They require the mental and physical processing of spatial information and play a crucial role in mathematics learning. AR technology has been regarded as an enhancer to help students develop their spatial skills in mathematics tasks (Özçakır & Çakıroğlu, 2022). However, due to the difficulties of analysing the primary learning behavior data, few studies provide further interpretations of the process of spatial skills learning with the support of AR learning activities. A more in-depth analysis of students' spatial learning behaviours is required to address this issue. This paper presents a study that aims to develop a framework for analysing students' spatial learning behaviours based on ecological theory, allowing for a deeper understanding of the development of their spatial skills.

A FRAMEWORK FOR ANALYSING SPATIAL LEARNING BEHAVIOURS

From an ecological perspective, an individual's spatial skills depend on their perception of affordances - that is, possibilities for action provided by the environment (Fajen & Phillips, 2013). Therefore, it can be argued that spatial perception and action are mutually stimulating, with the perception of spatial information guiding and facilitating actions, while actions provide feedback to perception, enhancing the acquisition of spatial information (Gibson, 1979). Drawing on this ecological understanding of spatial information processing, this paper proposes a framework based on the “from click to constructs” learning analytics framework proposed by Wise, Knight and Shum (2021) to illustrate the transformation process from raw data to spatial learning behaviours for analysing students' spatial learning, as depicted in Figure 1.

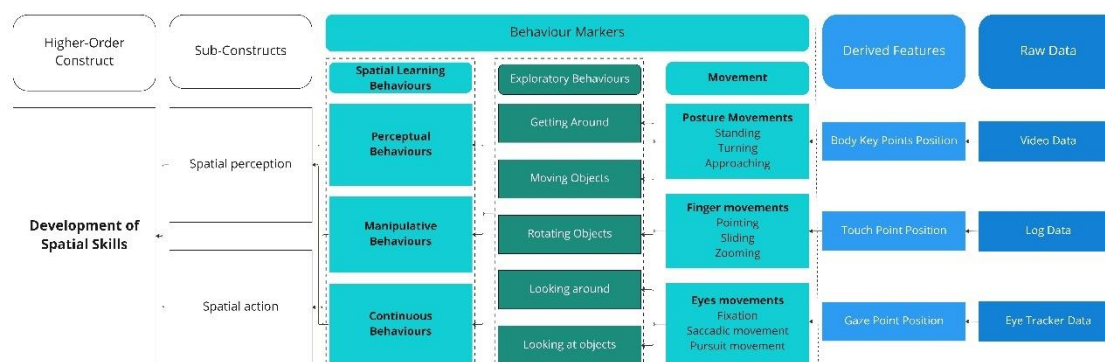


Figure 1. a framework for analysing spatial learning behaviours

The higher-order construct is the concept to be accessed and can be conceptually represented by sub-constructs, which can be refined further into different levels of behaviour markers. The basic behaviour markers, movements, are single observable behaviours modelled by derived features from automatically captured behavioural or physiological data. To analyse students' development of spatial skills, data will be collected on their interaction with the AR learning environment, including log data, video data, and eye tracker data. The extracted features of students' movements, such as changes in posture, gesture, and gaze, can provide insight into their exploratory behaviours: getting around, looking around, looking at objects, moving objects, and rotating objects, all of which are derived from Gibson's description of information pickup and object motions (Gibson, 1979). By analysing exploratory behaviours, it is possible to distinguish between perceptual behaviour, manipulative behaviour, and continuous behaviour, which respectively involve perceiving the environment, changing relations among objects to achieve goals, and synergistic and continuous actions to enable real-time cognitive processing and movement adjustment. Students often alternate between these three behaviours when using the AR system to learn. For example, a student exploring geometry in an AR system might walk around, rotate objects, create a new cube, and then move it to different positions before finally placing it. Perceptual behaviours such as walking and rotating support the acquisition of affordance and lead to manipulative behaviour such as creating a cube. Continuous behaviours, such as observing while moving the cube, also contribute to the task. Based on these spatial learning behaviours, the processes of students' spatial perception and spatial action can be presented for understanding of how their spatial skills are put into actions.

DISCUSSION AND CONCLUSION

In this paper, a framework for analysing spatial learning behaviours in the AR environment is proposed. According to ecological theories of the acquisition and application of affordances, students' spatial learning behaviours can be extracted from raw data and formed into three types of behaviours. This framework provides a perspective to analyse student behaviour in AR environments and gain insight into their learning processes.

In the future, we plan to use this framework to analyse students' spatial learning behaviours based on the digital data generated during the learning process. By doing so, we aim to explain how AR can help students develop their spatial skills and provide feedback to enhance their learning.

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INSTRUMENTAL GENESIS OF CHATGPT IN A CHALLENGE-BASED COURSE INVOLVING MATHEMATICS

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ChatGPT is a new technological tool with the potential to impact education. We present the case of a student group who discovered ChatGPT during a Challenge-Based course and started to develop initial utilization schemes of the tool. We consider the case a positive example of how ChatGPT can contribute to education and support the development of knowledge and competencies in students.

Keywords: ChatGPT, Instrumental Genesis, Challenge-Based Education.

INTRODUCTION

ChatGPT is a chatbot based on AI technology which has been trained to provide a detailed textual response to a user prompt or question (OpenAI, 2022). Quickly after its release in November 2022, discussions arose about the potential benefits and risks of the use of ChatGPT in education (e.g., Baidoo-Anu & Owusu Ansah, 2023). A process of *instrumental genesis* takes place (Rabardel & Bourmaud, 2003) when students start using a new tool such as ChatGPT, and find out how it may contribute to their cognitive development. In this process the students gradually develop different utilization schemes of the tool. Not much is known yet about student schemes development regarding ChatGPT in specific situations and understanding it may provide insights into the educational potential of AI technologies such as ChatGPT.

During a study on student learning experiences in Challenge-based education at a university of technology, we interviewed Mario's group on the resources they had used and how these had contributed to their learning. Mario and his group mentioned ChatGPT as an important resource for their work and elaborated on its use. We answer the research question:

What utilization schemes of ChatGPT do the students begin to develop about the ways in which the tool can contribute to their knowledge and competences in different situations?

THEORETICAL FRAMEWORK

We take the perspective of the Instrumental Approach (IA; Rabardel & Boumaud, 2003; Trouche, 2004). According to IA, human activities are mediated by artefacts and the approach then considers how using technological tools may contribute to cognitive development. Student's developing interaction with technological tools (the instrumental genesis) involves two processes: (1) the instrumentation process, in which the affordances and constraints of tools influence student practice and knowledge; and (2) the instrumentalization process, in which students adapt the tools to their own needs. During the instrumental genesis, students develop a dynamic scheme of use of the tool, containing: situations in which it can be used; aims and rules regarding its use; concepts-in-action (concepts considered relevant), and theorems-in-action (declarative statements, held to be true).

CONTEXT: A COURSE ON THE PHYSICS OF SOCIAL SYSTEMS

The study took place during the second of a three course learning line on the physics of social systems, in the second year of a bachelor engineering programme. The course followed a challenge-based approach: student groups were given a general challenge ("optimize the efficiency of a train platform boarding process") from which the groups derived the particular problem they wanted to address. In the first course, students analysed actual passenger flow data from the train platform. In the second course, the students modelled the passenger flow, using methods based on physics and

mathematics, taken from the literature. Mario’s group had decided to create a model based on Markov chains. We thematically analysed the interview data in which Mario’s group elaborated the use of ChatGPT.

RESULTS AND CONCLUSION

Mario indicated that he and his group had used ChatGPT in three situations. Table 1 shows a summary of how we interpreted the students’ utterances in terms of developing schemes of use of ChatGPT.

Scheme Aspect	Students’ schemes development
Situations and associated schemes	s1) Determining a suitable model for crowd flow; s2) optimizing Python code (Markov chains); s3) working on visualisation for the final report.
Theorem-in-Action	Most of the details that ChatGPT provides are wrong (s1, s2, s3); ChatGPT is good to provide (first) ideas and when you start to learn something new (“but it quickly becomes like, not enough”) (s1, s2); more efficient than Google (depends less on correct search terms; s1, s2).
Concept-in-Action	It is like Googling, and getting the main ideas (s1, s2).
Rule-of use	Use the main results of ChatGPT to investigate further (s1, s2); don’t use it for final things (s1, s2, s3).

Table 1. Aspects of a developing scheme of use

The students said that they had started using ChatGPT in a playful way, and then more seriously for their project. After they had experienced its usefulness in one situation, it was “nice to like see that it could be useful for other stuff“. We claim that instrumentation and instrumentalization processes were at play and that these facilitated the students to develop initial schemes-of-use. The students did not mindlessly accept ChatGPT’s results: they were aware of the errors made by the software and developed a critical attitude. Hence, we consider this case a positive example of how ChatGPT can contribute to education. We conjecture that the students’ involvement in a Challenge-based learning environment has contributed to their critical attitude towards ChatGPT, as for their project they needed results that really worked. However, this conjecture needs further research.

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MALT2: A DIGITAL EXPRESSIVE MEDIUM FOR STUDENTS' AND TEACHERS' MATHEMATICAL ACTIVITY

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The use of dynamic geometry software (DGS) is important in educational environment, and it is more advantageous for learning mathematics comprehensively. DGS provides a valuable instrument for revising and expanding students' knowledge, as well as adapting learning tasks in geometry to a dynamic geometry environment for expanding and deepening geometrical and pedagogical knowledge and reasoning skills, allowing students to have an accurate reflection process. In this poster, there is a students' engagement with MaLT2 (Machine Lab Turtle-sphere) – an online environment that integrates Logo textual programming with the affordances of dynamic manipulation. This can foster the students' retention level, who are encouraged to go beyond memorizing formulae and instead grasp the concepts.

Keywords: Logo, Turtle Geometry, 3D Geometry

WHY DGS?

The use of Dynamic Geometry Software seems to be more advantageous for learning mathematics comprehensively, as it provides a valuable instrument for revising and expanding the math knowledge of students of any age/class. There is a significant difference between learners' mathematics performance after the intervention of dynamic geometry computer software (Adelabu et al., 2019; Segal et al., 2021).

WHY MALT2?

MaLT2 is an online Logo-based application that allows the creation, exploration, and dynamic manipulation of 3D geometrical models with textual programming. MaLT2 encourages students to develop intuitively new ideas, to express them by using symbolic language, to run them on MaLT2 environment and to directly observe their outcomes. Thus, pupils can compare their estimates with the results, control their pre-existing knowledge, experiment with new ideas and construct meanings about the concepts (<http://en.etl.eds.uoa.gr/digital-resources/authoring-tools/malt2-an-online-tool-of-symbolic-expression-in-mathematical-activity.html>).

HOW?

By using “half-baked micro-experiment” (fallible artifact) having one or more “buggy” behaviours built into them. The aim of these “bugs” is to provoke students to fix them – to improve the original artifact (Kynigos & Grizioti, 2018).

EXPECTED RESULTS

Students are expected to amplify their computational thinking skills in the mathematical context, improve their abilities to produce correct conjectures, identify reciprocal relationships between geometrical/mathematical concepts and create meta-representational competence.

PROBLEMS

Problem #1 – Fix a “half-baked” isosceles trapezium.

Fix one (instance of a) correct isosceles trapezium so, if change one of four variables (two parallel sides, leg and base angle) the shape remains (as the class of) an isosceles trapezium.

Problem #2 – Fix a “half-baked” trapezium with a right angle.

Same as above, but with a (fixed) right angle and three variables.

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PRODUCTIVE USE OF GENERATIVE AI LANGUAGE MODELS FOR MATHEMATICS TEACHING

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The poster provides information about possible uses of generative AI language models such as ChatGPT in mathematics education research.

Keywords: AI, ChatGPT, Artificial intelligence, mathematics education research

INTRODUCTION

In March 2023, the U.S. company OpenAI released the newest version of its generative AI language model GPT-4 (Generative Pre-Trained Performer), which will gradually be included in their application ChatGPT (accessible at the URL <https://chat.openai.com/chat>). ChatGPT is an artificial intelligence-based chatbot based on a reinforcement learning algorithm that understands and generates human speech with impressive accuracy (Buchholtz et al., 2023). Despite the high attention the application has received in the media, the use of generative AI language models in school educational settings is discussed controversially. In this context, the critical points criticized in the area of mathematical education are primarily the lack of mathematical skills of the models, which are essentially semantic language models and therefore cannot correctly process syntactic and semantic aspects of mathematics and therefore cannot (currently) generate reliable mathematical answers (Buchholtz et al., 2023). Also, the presentation of fictitious and incorrect information, which is inherent in its algorithm, is currently still a problem, as well as the lack of transparency in the generation of answers (known as the so-called *AI black box problem*) or the non-free availability due to the introduction of payment models. In the research project Unlocking the Potential of Generative AI-Language Models in Professional Workflows and Education of Mathematics Teachers (GenAI-MT), we as a group of mathematics education researchers are exchanging ideas about the possibilities for a productive use of generative AI language models for mathematics teaching and mathematics education research. This poster informs about our ideas.

INDIVIDUALIZATION OF MATHEMATICS LEARNING

For mathematics education research, we see one possible application in the area of developing and evaluating individual teaching and learning materials based on a diagnosis of students' difficulties in learning mathematics (Buchholtz et al., 2023). Generative AI language models function here in the role of a diagnostic tool that provides individualized learning support f.e. by formulating individual hypothetical learning trajectories (Simon, 1995) that can be integrated into learning environments for mathematics learning. Foundations for research and development here can be design-based approaches that simulate individual student prompts and collect, categorize, and rate the AI-generated responses. For example, Monir (2023) investigated which explanations ChatGPT provides for conveying basic ideas about the derivative concept by systematically entering prompts

from fictitious students with difficulties in understanding the concept of the derivative and typical tasks. She analyzed ChatGPT's explanations for their technical correctness and completeness as well as the appropriate use of linguistic forms of representations. Although there were identified mathematical errors in the answers of the generative AI language model, a colloquial and figurative explanation was often chosen for theoretical questions. Without having intended it in the prompts, also references to the everyday life of students were created within the explanations, metaphors were introduced or rough examples were attached. Her findings on deliberate prompt design confirm the advances of CoT-Prompting (Chain-of thought prompting; Wei et al., 2023) for mathematical prompts. Correct answers mostly could only be assumed if computation-based questions and theoretical questions were separated, since the simulated conversation suggested errors in combined, context-bound prompts.

CREATIVE INVENTION OF MATHEMATICS FOR TEACHING

We see another possible application in the creative generation of mathematical tasks, solution paths or mathematical problems for use cases, for example in the classroom or for systematic practice. Here, generative AI language models could act in the role of a lesson planning assistant at hand for mathematics teachers. This application possibility can also be extended to other aspects of lesson planning, such as arranging learning environments and sequences of tasks in individual lessons. However, the current lack of reliability of the technology is still a problem here as well. Cherian et al. (2022) report a study of 11 simple mathematical word problems in which each problem was presented to neural networks (including ChatGPT) ten times independently, with a request for a solution and explanation of the solution path. ChatGPT could not reliably solve the tasks correctly, and solution frequencies varied from 0% to 90% per task.

CONCLUSION

It cannot be excluded that further developed generative AI language models will reduce this error-proneness in the future. Plug-ins like Wolfram Alpha and GPT-4 help ChatGPT to solve mathematical tasks with much greater precision. Therefore, we positively assume that further developed models, if integrated into learning environments, may well provide assistance for the individualization of learning processes for students with learning difficulties, for example in the reduction of requirements through simple language or in the understanding of tasks or mathematical terms.

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SKILLSPRINT: GAMIFYING MATHEMATICS REPETITION

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This study investigates the potential impact of introducing gamification elements in repetition-based (repeating the course curricula content) mathematics learning activities for increasing motivation. The authors developed a web application called SkillSprint that integrates gamification elements such as leaderboard, points, streaks, performance graphs, and badges. The application was tested on 10th-grade students at a local school, collecting data by tracking events and task attempts from the app and using a questionnaire that measured motivation. Preliminary findings suggest that the application was well received, leading to a positive perceived learning outcome and increased participation and motivation. The authors suggest that introducing gamification elements in mathematics learning activities could help overcome the problem of repetition being perceived as boring and unmotivating while still being effective for learning.

Keywords: mathematics, repetition, gamification, web application, app development

INTRODUCTION

Studies show significant results in increased learning outcomes, participation, and increment in motivation when gamification elements are introduced (Buckley & Doyle, 2014). A study shows that many students find repetition in mathematics boring and unmotivating, thus leading to a negative feeling toward the subject (Bernero, 2000). This is an exciting finding because repetition in mathematics is proven to be highly effective (Dahlin & Watkins, 2010).

THE APPLICATION DESIGN AND CONTENT: SKILLSPRINT

A web application with gamification elements was developed to investigate how repetition in mathematics could become more motivating. It was based on the students' curricula from their respective books for 10th grade and provided the same tasks. The pupils can select tasks from the task overview page based on difficulty or category.

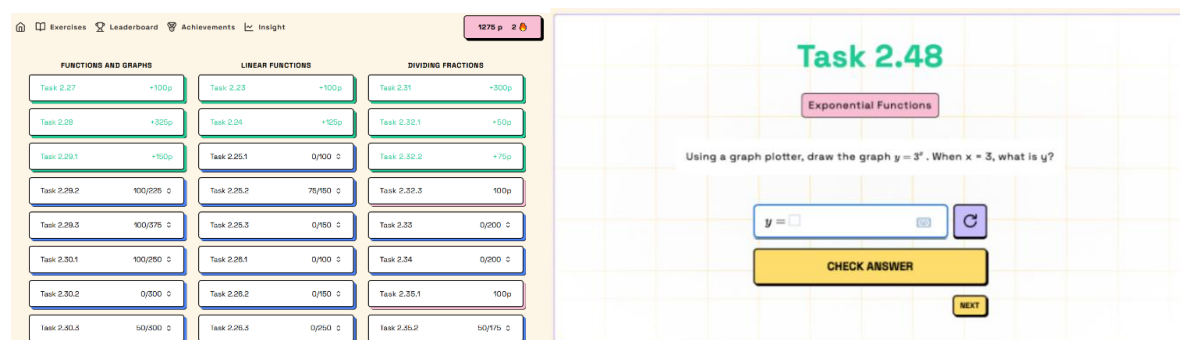


Figure 1. Left: Task overview page. Right: Task detail page

When a task is solved in the pupils' books, they provide the answer, which is automatically corrected. They are granted points, streaks, and progress toward achievements upon the correct answer.

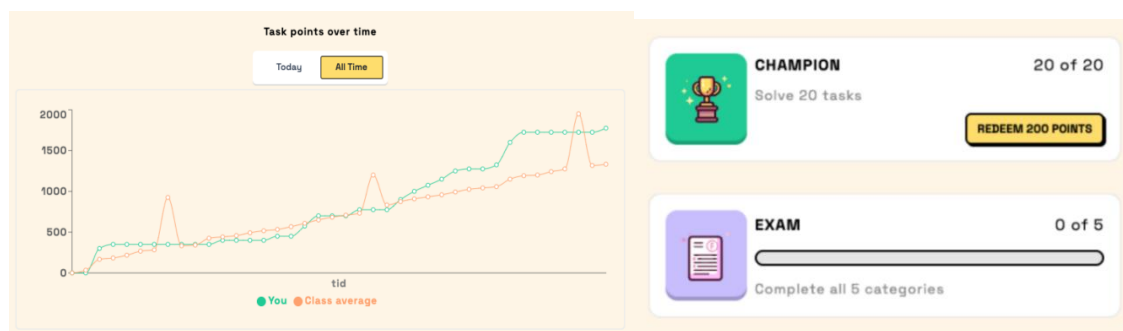


Figure 2. Left: Performance graph. Right: Achievements

Figures 1 and 2 show that the application is intended to be simple, playful, and engaging. The design is based on neo-brutalism, with bright colors and simple shapes that appeal to a younger audience. The application is developed with modern web technologies like React, Next.js, PostgreSQL, and TypeScript. The gamification elements, *leaderboard*, *points*, *streaks*, *performance graph*, and *badges*, were chosen after thorough research to engage different user types. Through a dashboard, the pupils can privately see their progress and how they compare to the other students.

STUDY USING THE WEB-BASED APPLICATION IN THE CLASSROOM

The application was tested on sixty 10th graders, split into two classes, at a local school. Both classes used the application for 60 minutes on two separate days with one week between. Before the first session, the attending pupils completed a pre-test survey gathering information regarding demographics, general interest in mathematics, and motivation towards repetition. After the second and last session, a post-test survey was completed, gathering information regarding perceived learning outcome, perceived motivation towards repetition in mathematics when using a gamified application, the various gamification elements' impact on motivation, and its overall usability.

Preliminary findings show that the web-based application was considered successful based on a quick overview and the positive feedback received verbally from both the teacher and the students. Most students reported a positive perceived learning outcome and that the motivational factors of *SkillsSprint*, helped them do more tasks than they usually would in a free practice session without the application. Even though the application was intended only to be used in the two test sessions, we can see in our logs that some pupils use the application on their own close to daily.

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THE ROLE OF CHATGPT IN MATHEMATICS INQUIRY-BASED LESSONS: A STUDY IN BRAZIL

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The evolution of artificial intelligence (AI) has given rise to new challenges in teaching and learning mathematics. While it has generated new opportunities for structuring students' mathematical work (Richard, Vélez & Van Vaerenbergh, 2022), there are concerns that students might not develop critical competencies. It seems evident that AI renders traditional approaches based on memorizing formulas and algorithms irrelevant. This is in stark contrast to mathematics inquiry-based lessons, where students are encouraged to undertake open tasks, explorations, and/or problem-solving (Alrø & Skovsmose, 2002). Yet, in this context, what changes does AI introduce to mathematics inquiry-based lessons? This question has gained renewed relevance with the popularization of the ChatGPT platform [1], an AI-based tool capable of responding to queries through a conversational interface. It has been trained to interpret human questions and provide relatively reliable responses. The opportunities offered by artificial intelligence have been the subject of research for a long time (Balacheff, 1993). Only recently, however, has there been a heightened focus on this topic within the field of Mathematics Education (Richard, Vélez & Van Vaerenbergh, 2022). Our understanding of the changes AI might cause in mathematics inquiry-based lessons remains limited. In particular, the free availability of ChatGPT since the end of 2022 has raised numerous questions about its impact on various sectors of society, including education, and more specifically, the mathematics education we practice in schools. In this study, I aimed to illuminate the changes wrought by ChatGPT in mathematics inquiry-based lessons. To do this, I conducted a teaching experiment (Steffe & Ulrich, 2020) with two groups of students from public schools in Brazil. In both groups, two sessions were conducted in which the students were asked to explore three inquiry-based tasks. A teaching experiment should not be confused with experimental research, as the former is qualitative in nature and focuses on students' processes (Steffe & Ulrich, 2020). The study was implemented through observing the students' work sessions, and the proceedings were recorded on video. All recordings were viewed multiple times and the data analysis centered on the episodes that highlighted particular ways in which ChatGPT contributed to students' inquiry. It was observed that while ChatGPT provided answers that would ordinarily be the outcome of the students' work on the tasks, it also required them to evaluate or expand upon the answers via the chatbot. As a result, students had to discuss how to phrase the questions to be asked of the AI tool. Thus, it can be said that the presence of ChatGPT underscored the need to evaluate responses and formulate questions that clarify the understanding of the responses. This teaching experiment suggests that the tool might undermine certain aspects of inquiry-based lessons, such as students generating their own answers, since ChatGPT takes on this task. However, the comprehension, evaluation, and expansion of responses need the development of critical competencies by the students. These skills depend on the structure of the pedagogical work and the interaction of the students with the teacher. These findings are preliminary and exploratory, and they underscore the need for further studies to delve deeper into the characteristics of mathematics inquiry-based lessons using ChatGPT and other artificial intelligence tools.

Keywords: Artificial intelligence, Mathematics, ChatGPT.

NOTES

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ONLINE COLLABORATION ON MATH – FOR WHO?

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Keywords: Digital divide, online resources, online collaboration

ONLINE RESOURCES AND SUPPORT IN MATH

The Internet provides new resources for organizing help and support open for all students. But should teachers just start sending links to online support groups and is this really equalizing the playing field? On one hand, there is a promise of equal access to new materials and networks. On the other hand, research on digital divides finds huge differences between how people spend their time online and what value they get from it (van Deursen & Helsper, 2015). While a lot of this research is conducted with a focus on the *users'* choices, a prominent, nearly 20-year-old call is to focus on the structures (and thus platforms) as well (Dijk, 2005). This is important to investigate, before asking students to take part in this promising new room for collaboration as a potential part of future math education.

Despite online platforms providing new communities open to all, there are evident differences in what users gain from participating in them. Evidence from the developer community “StackOverflow” shows that masculine users are more likely to receive answers to their questions and be socially recognized for the answers provided (Brooke, 2021). In the studies, this is thought to reflect a masculine-dominated culture, making it difficult for feminine users to participate. This might be different in a forum targeting math for students due to a larger share of girls engaging with math in schools than the share of women in developing.

GENDERED DIFFERENCES IN ANSWERS ON A MATH SITE FOR HIGH SCHOOL

A plurality of platforms has been formed over the last two decades to enable students to ask for help from peers and co-work on problems. These are often structured as Q&A, with students asking questions and then waiting for others to help them, as exemplified in figure 1.

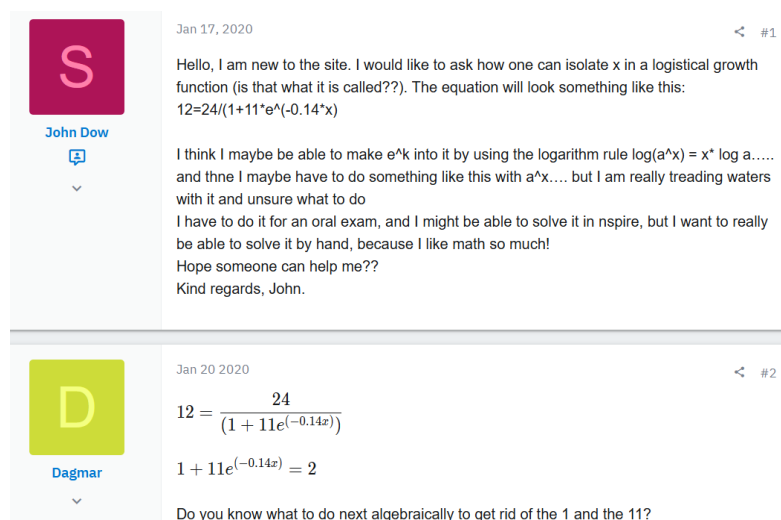


Figure 1: Interaction on math-site (made up example from Math Help Forum)

To investigate gendered differences in answers on an online forum for mathematics similar to the forum for developers, I investigated 29,784 questions on the algebra-site on a high school section on mathhelpforum.com. The platform has existed since 2005 and are allowing students on high

school- and university levels separate discussion fora to communicate about math problems as exemplified in figure 1. 6,929 of the questions could be identified to be asked by a user with a male or female username, found by comparing usernames to lists of global first names through the software `GenderComputer` (as done in above-mentioned studies of the developer-platform). Of these, 37.3 percent were identified as female, while 62.7 percent were identified as male.

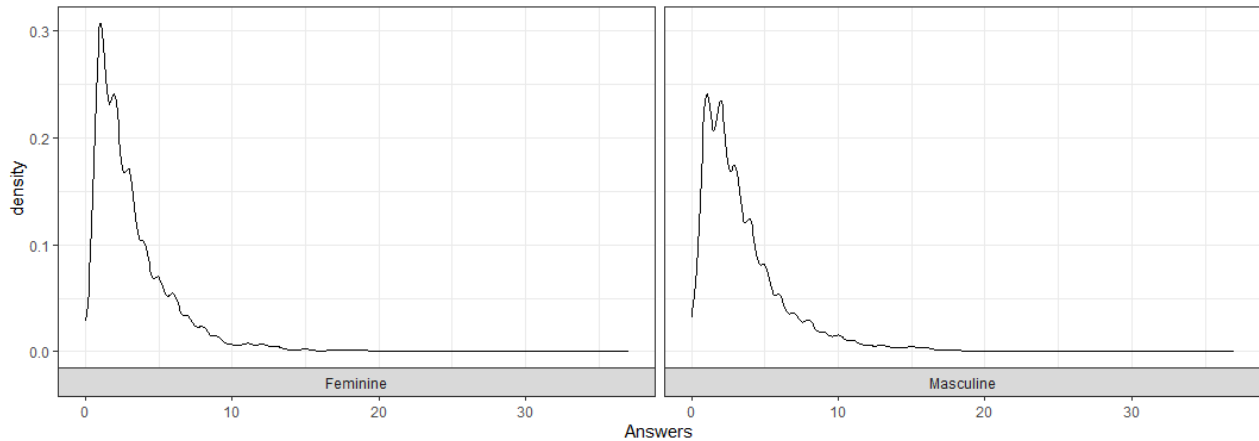


Figure 2: Distribution in answers depending on the asker's masculine or feminine username

Figure 2 shows the distribution in answers depending on their username being feminine or masculine. In a comparison of means, masculine users receive an average of 0.40 more answers than feminine users, a difference significant at a five percent significance level. Meanwhile, masculine users are 0.67 percent less likely to receive any answers at all. Though this effect is minor, it is also statistically significant. Thus, there is mixed evidence for the effect of gender on the use of the platforms. The next step entails looking into whether there are gendered differences in how the questions are posed, in how the answers are provided and whether the students are using the platforms to collaborate with strangers or merely to get the (correct) answers for their homework.

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RECOGNISING MACRO-SPACES USING GOOGLE MAPS: A STUDY WITH EIGHT-YEAR OLD CHILDREN

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Keywords: *Spatial reasoning, macro-spaces, Google Maps, reference systems*

We present results of a study that focuses on developing spatial reasoning in eight-year-old students through a learning trajectory in a teaching experiment. This trajectory includes several tasks involving the analysis of dynamic representations. Some tasks focus on macro-spaces perception using Google Maps and Google Earth, relating the virtual representations with their corresponding physical spaces. The activities were carried out with third grade students from a public primary school in Mexico City.

Based on the research of Galves (1985), who classifies space in three sizes – micro-space, meso-space and macro-space –, the learning trajectory tasks were designed for children to develop an understanding of, and discern, the different spaces in which they live in. In the micro-space one can have a direct perception and manipulation of objects; in a meso-space (e.g., a house), the observation of fixed objects takes place from different reference points without the possibility of manipulating them. Finally, in a macro-space (such as a city or a rural area) perception is neither global nor simultaneous, but through successive local visions. Furthermore, as detailed in the work of Tversky and Hard (2009), there are three reference systems through which objects and subjects can be perceived and represented from different perspectives: (i) the egocentric system, which depends on the observer (it is centred on the observer him/herself); (ii) allocentric (centred on the environment surrounding the object/subject); and (iii) decentred (centred on the perspective of another person).



Figure 1. Tracing the distance between two known points using Google Maps



Figure 2. Vicente shows the spatial movements he remembers of his school-home route and the difference with the one given by Google Maps

In this poster, we present examples of how our study's students used Google Maps for recognizing familiar spaces, making virtual tours and identifying travel routes that are usual for them. In addition to presenting travel routes, Google Maps also provides information on the time and distance needed to travel between two points. In order to make sense of the Google Maps representation, students need to understand how it is linked to the physical space they travel through on a daily basis. This implies recognising that a distance shown onscreen (Fig. 1), which can be measured, corresponds to the distance they travel in reality, but which is difficult to measure directly. As students establish these relationships, they will be able to recognise the scale of the representation in relation to the

real space. This means that they could be able to understand the relationships of proportionality between the two and have a better understanding of the physical world around them.

One task involved students visualizing the route that they usually took between their home and their school. We show how they were able to establish a relationship between the physical space through which they travel on a daily basis and the corresponding dynamic representation of the macro-space that Google Maps offers from a higher or satellite perspective (Fig. 1). To establish this relationship, students needed to construct reference systems, by analysing their location and position in order to recognise the objects, symbols and traces that appear in the Google Maps representation.

For example, one of the children, Vicente (pseudonym), identified whether the route he usually took from home to school, and which was shown on Google Maps, was the correct one. Vicente entered his home address into Google Maps and explained that the given route would not take him home and why he recognized that what Google Maps showed was incorrect. Although the default view on Google Maps is only a graphical representation, Vicente managed to recognise that the route he took home (see transcript below) did not correspond to one given by Google Maps. He identified the points he passes by on his daily route between home and school on Google Maps, and established a relationship between his own bodily movements and the onscreen representation. In this way, he was using an egocentric reference system to imagine his route, and at the same time, he constructed a decentred reference system by comparing (as a third person) his knowledge of the physical space with the representation on the dynamic map.

Teacher: How do you know that this is not your address?

Vicente: Because my address is upwards [...] that is, leaving the school going upwards, this way (Fig. 2).

Teacher: You are here at the school, right? The entrance is this one here [pointing to the corresponding point on Google Maps], right? Which way do you go?

Vicente: By the taxi stand and then up, then by the [shop] to one side, then to the front, then to the other side straight ahead and then up a little bit and then I get to my house.

In another task, students used Google Maps to identify who took the longest route to get to school based on the distance calculated by Google Maps. In this way, students were able to estimate and make connections between distance and time without the use of Google Maps.

In conclusion, these types of tasks can be a useful tool for students to develop notions of macro-spaces by allowing them to explore a place from different perspectives, by identifying such spaces through virtual routes, and even satellite images.

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THE BUTTERFLY PROJECT: MAKING SENSE OF FUNCTIONS THROUGH INTERPOLATION

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Year 2123, butterflies have disappeared from the surface of the Earth. Students are playing the role of scientists working on a butterfly robot to collect pollen from plant to plant.

In an attempt to improve students' understanding of the concept of function as a tool to communicate with a machine, we have designed a half-baked microworld embedded in Scratch, The Butterfly Project. Its originality yields in focusing on resolving an interpolation problem as an answer to a realistic problem, thus creating the need for more complex functions.

Keywords: functions, microworld, algebra

THE BUTTERFLY PROJECT MICROWORLD

4 actions : Plant a flower ; Edit the butterfly's trajectory (see Figure 2) ; Toggle a xy-grid ;



Play/Pause ; Reset.



Figure 1. The graphical interface.

PEDAGOGICAL GOALS

The main goal of this microworld is to provide an environment suitable for students to develop a robust understanding of the concept of function by linking different representations (graph, formula and table) between them.

To help them make sense out of their understanding, we built the environment with a constructionist approach: a half-baked microworld (Kynigos, 2007) paying attention to the 5E's framework (Benton et al., 2017): Explore, Explain, Envisage, Exchange, bridgeE. The Explore and Envisage dimensions are built-in while others are expected to be set up by the teacher.

DESIGN CHOICES

Research in the teaching of Algebra presented in Warren et al. (2016) suggests that for students to grasp the concept of function, tasks which allow them to pass from one representation to the other are helpful.

The Butterfly Project was designed firstly to allow this continuous movement. Furthermore, two choices are embedded in its design: curves are not drawn instantaneously and always represent a trajectory. We believe it fosters the development of the following capacities:

- **Prediction:** although feedback is the strength of computer assisted activities, instantaneous feedback can make prediction unworthy against trial and error. The butterfly's curve *slowly* draws itself.
- **Creativity:** In a simulation context, students' ideas will be stimulated by their experience.

STUDENTS' ACTIVITIES

As a first part, students are expected to explore the microworld on their own through trial and error strategies to solve specific cases of a problem. As a second part, we ask them to generalize their solution so that it lies at the functional level. Two examples are described below.

Intersection problem: Build a program that places a plant on the trajectory. Second part: add a feature to choose the plant's location. The mathematical notions they will encounter are: reading coordinates (first part), image or preimage of a number through a function (second part).

Interpolation problem: Edit the curve so that it passes through given points. Specific cases: 1 point, 2 points, 3 or more aligned. The second part starts with 3 nonaligned points. Mathematical notions: image or preimage of a number by a function (first and second part), piecewise functions (second part).

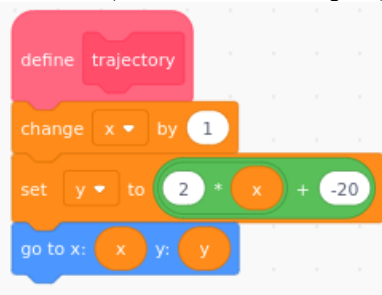


Figure 2. The trajectory's code

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TECHNICAL ADVANCES AND DIDACTIC REASONING IN THE PROJECT AUTHOMATH

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The Erasmus+ project AuthOMath brings together the strengths of GeoGebra and STACK in providing a platform that allows designing randomised digital tasks with interactive and dynamic applets and adaptive feedback. The project also seeks to provide the necessary didactic foundations for authors to design such tasks. In doing so, the project aims to use the specific technical advances of both the STACK and GeoGebra authoring systems to initiate a genuine didactic reflection, being one of the key competencies to develop in teacher education. This poster informs about the present activities within this project to explore the potentials of addressing technical advances for initiating didactic reasoning in teacher education.

Keywords: digital task design, interactivity, multimodality, feedback, teacher education

COMBINING STRENGTHS OF STACK AND GEOGEBRA

Interactivity, dynamization, and multimodality could be considered strengths of DGS such as GeoGebra, while STACK is particularly strong in randomization and adaptive feedback design (Sangwin, 2013). Bringing both together for supporting a transformative change (Fahlgren et al., 2021) in mathematics teaching and learning is one of the aims of the Erasmus+ Project AuthOMath: Authoring Online Material with Multimodal, Dynamic and Interactive Applets and Automated Feedback for Learning Math (2022–2024), with the University of Education Heidelberg (Germany) as coordinator and partners Universidad de Cantabria (Spain), Johannes Kepler Universität Linz (Austria), and University of Edinburgh (United Kingdom).

INITIATING DIDACTIC REASONING

AuthOMath also seeks to provide necessary didactical foundations for authors to design effective digital mathematics tasks. Thus the project aims to use the specific technical features of both the STACK and GeoGebra authoring systems to induce genuine didactic thought, being one of the key competencies to develop in teacher education. Presently, the potentials of initiating didactic reasoning in are being explored in student seminars at the project partners' sites.

At the University of Education Heidelberg (Germany) teacher students in their final bachelor semester are asked to design digital tasks for practising elementary skills. The aim of this seminar is initiate students to recall, research, and apply the didactic foundations that are necessary for designing the tasks. This begins with a thorough mathematical analysis of the chosen concept and research approaches to understanding the concept, e.g. Grundvorstellungen (normative explanatory models, cf. Prediger, 2008). The technical affordances of GeoGebra and STACK then ask for research of ways of representing and interacting with the mathematical concept. Following this, a literature based analysis of possible errors (Oser et al., 1999) and misconceptions provide the material for structure, content and timing of the adaptive feedback (Narciss, 2008).

At the University of Cantabria, we run several seminars to analyse the potentiality of STACK to foster the pedagogical competency of prospective Secondary Education Mathematics teachers when

designing and solving word problems in mathematics. Unlike previous research, centred mostly on procedural tasks (Sangwin & Jones, 2017), we focused on implementing open problems in STACK that can be solved using different solution strategies. The implementation of such problems not only requires having digital skills and experience on STACK, but also displaying a high pedagogical competency in problem posing and solving. The 16 fourth-year Mathematics undergraduate students that participated in the seminars and implemented word problems in STACK received theoretical and practical sessions on the use of the tool as well as on aspects concerning problem posing and solving, including randomization, strategy solution, and feedback (Ríos-San-Nicolás et al., 2022). Despite the limited digital competence of the participants, all reported positive views about STACK as a tool for fostering different pedagogical aspects of problem posing and solving.

Work at the University of Edinburgh has focused on using STACK to prompt students to generate examples of mathematical objects, with several undergraduate mathematics students completing their final-year project. The students have devised example-generation tasks for linear algebra and calculus, drawing on theoretical considerations such as the *dimensions of possible variation* and *range of permissible change* (Watson and Mason, 2005) to inform the design, before implementing the tasks in STACK. Two of the students have published their reports and supporting materials online (see <https://osf.io/a6k2d/>), including analysis of responses from a class of over 500 students.

ACKNOWLEDGEMENT

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HONG KONG ETHNIC MINORITY STUDENTS' MULTIMODAL DISCOURSE ABOUT TRIANGLES IN DYNAMIC GEOMETRY ENVIRONMENTS

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INTRODUCTION

This study focuses on the multimodal geometry discourse when interacting with DGEs for a specific group of young students, namely ethnic minority children in Hong Kong. People of ethnic minority account for 8% of Hong Kong's population. In recent years, equity concerns have grown as a result of ethnic minority students' low academic achievements, especially in mathematics. The research questions are two-fold: (1) What meanings did young ethnic minority children develop when constructing and interacting with triangles in DGEs, and how did their discourse and gestures about triangles evolve during the activities? (2) How did they make sense of triangles through connecting with the real world?

METHODOLOGY

We recruited three third grade students who are ethnic minorities in Hong Kong (only data from Carlos was analysed in this paper) to participate in the study. The study was part of a larger research which aimed to support Hong Kong ethnic minority students through responsive teaching in technology-enhanced environments; it consisted of ten 1.25-hour teaching and learning sessions between the researchers and student participants. The sessions were conducted on consecutive Saturdays at a community center where the students lived nearby. Prior to the study, parental informed consent was obtained.

RESULTS AND DISCUSSION

First, Carlos was tasked with constructing a triangle and then describing his construction in a paper-and-pencil environment. He first gestured the triangle in the air with his index finger, saying "a triangle has three dots and three cords", suggesting that he could define the triangle's properties. Then, he described his drawing as a "pudding" and "ship", both utilizing his knowledge of real-life objects and demonstrating his ability to visualize composite figures in this environment. We note that due to the static environment of paper-and-pencil, Carlos described each of his drawing as a specific object as opposed to multiple objects from real life. This implies that his drawings had elicited the image of one specific object at a time, and that it did not transform into another object.

While in DGE, Carlos was often able to relate multiple objects in real life by exploiting the dragging feature of the DGE to transform the shape of the triangle through dragging its vertices. At first, Carlos did not realize that he could drag the triangle's vertices; upon the researcher asking him to try it and see what would happen, he first dragged one of the vertices such that it became what he called a "straight" triangle (or a triangle with a horizontal base, Figure 1a). Then, he continued to drag it until it became a "cone" (or an upside-down triangle, Figure 1b). Furthermore, he suddenly created a different kind of "cone" (Figure 1c), and made a gesture to show his cone as "standing up" (Figure 1d). It was during this process of making the new triangle (Figure 1c) that a new meaning for the composite shape has developed. When the researcher tried to drag the bottom triangle to ask him a question, he quickly dragged the vertex back to its original position, and said in an excited tone, "If we do this, then this is a cone, and we can eat it. Ice-cream!" This evidenced that he has visualized his composite figure of two inverted triangles with an overlapping base as an ice-cream cone.

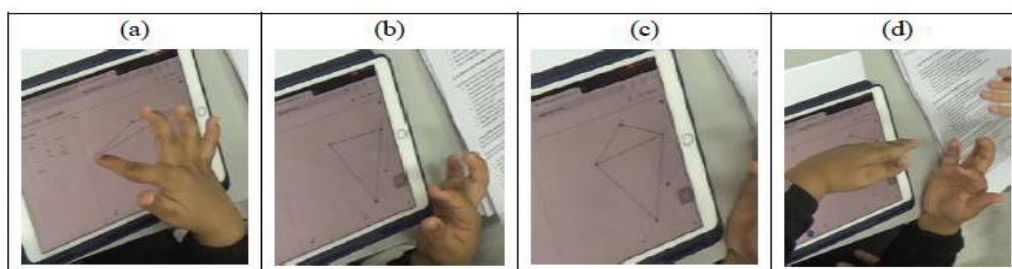


Figure 1. Making and gesturing an ice-cream cone

As he continued to move one of the vertices of his “ice-cream cone”, he made another observation: while maintaining the shape of the two inverted triangles as roughly similar, he dragged the vertex of the lower triangle to coincide with the vertex of the upper triangle, as if he was flipping it up. He then dragged it again, moving it around at random, at one point dragging it such that triangle flipped again, and again. It was this visual effect and dragging-gestural experience of what looked like flipping a triangle that stimulated Carlos to imagine that his composite figure is now an “envelope” in the sense that his triangle could be folded up or down like the triangular part of the envelop to “open” and “close” the envelope. Significantly, this analysis suggests that the dynamic features of the DGE has supported Carlos’s sense-making of triangle as a shape that can be changed and given multiple meanings (e.g., cone with different orientations, ice-cream cone, envelope dynamically opening and closing, etc.). Meanwhile, he showed competence in their geometrical communication by using the everyday register to describe his thinking.

DISCUSSION AND CONCLUSION

This study sheds light on digital mathematical activities as taking on a multimodal form, involving language, gestures and touchscreen-dragging (Ng, 2019). It concurs with previous studies which suggest that DGEs evoke new, mathematically relevant gestures (Edwards et al., 2014), which itself functions as a non-linguistic form of mathematical communication (Ng, 2019). As shown in the results, Carlos communicated about geometrical ideas such as identifying a non-prototypical (an upside-down) triangle, making a composite figure, and flipping a triangle within the DGE. Future research should address both language learners’ communication in DGE-enhanced learning activities and their instrumentalization to cater for learning diversity in the mathematics classroom.

Finally, this study uncovers ethnic minority students’ competences in mathematical activities in DGEs through examining their multimodal discourse (e.g., words, gestures, and touchscreen dragging). It underscores the importance of considering ethnic students’ non-linguistic forms of communication to facilitate their geometry learning in Hong Kong schools and beyond. As such, this study advances a non-deficit, multimodal view of communication and informs future studies on capturing students’ geometry competences through their gestures and touchscreen-dragging actions in DGEs, particularly for language learners from non-dominant communities.

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CHARACTERIZING TOUCHSCREEN ACTIONS FOR ACTION-BASED EMBODIED LEARNING IN K-12 MATHEMATICS INSTRUCTION

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Touchscreen devices have become prevalent in various educational contexts given their hands-on and interactive learning style that textbooks and oral instruction might not always offer, especially for its direct interaction, responsiveness and multitouch affordances. Drawing from the theory of embodiment, mathematical cognition is deeply rooted in the body's interactions with the environment and materials. As such, touchscreen-mediated interactions may contribute to the groundwork for embodied mathematics learning. In view of this, this poster presentation proposes a touchscreen action matrix, which frames all the touchscreen actions retrieved from 33 reviewed journal articles, as a part of systematic review. We coded the operational goals with the matrix and illustrated how mathematical concepts were recast in terms of virtuality (touchscreen actions) to support the design of action-based embodied designs in mathematics instruction.

Keywords: Action-based embodied design, embodied learning, touchscreen actions, gesture

INTRODUCTION

Touchscreen technologies enable users to produce and transform 'screen objects' with their fingers (by means of varied forms of pressure and propulsion) and consequently invoke an intricate interrelationship between the user's fingers and the surface of the device itself. As an important feature of human-touchscreen interactions, touchscreen gestures have been widely discussed in a range of literature (see, for example, Sinclair & de Freitas, 2014), but have rarely been formally categorized, especially in mathematics education. However, we believe this conceptualization is essential as it could potentially inform the design of touchscreen applications, especially for providing constructive mathematical learning experiences in touchscreen environments.

CHARACTERISING TOUCHSCREEN ACTIONS FOR EMBODIED MATH LEARNING

We propose a touchscreen action matrix (Figure 1) which categorizes the actions elicited by the touchscreen applications as part of a systematic review of 33 articles. The column "User's Input" is divided into two subgroups based on the number of fingers used on the touchscreen settings during the action: 1 finger (single touch) vs. 2 or more (multitouch). The row labelled "Temporality" specifies the time that the finger(s) operate on screen during action, with two subgroups, namely, an instantaneous or prolonged action.

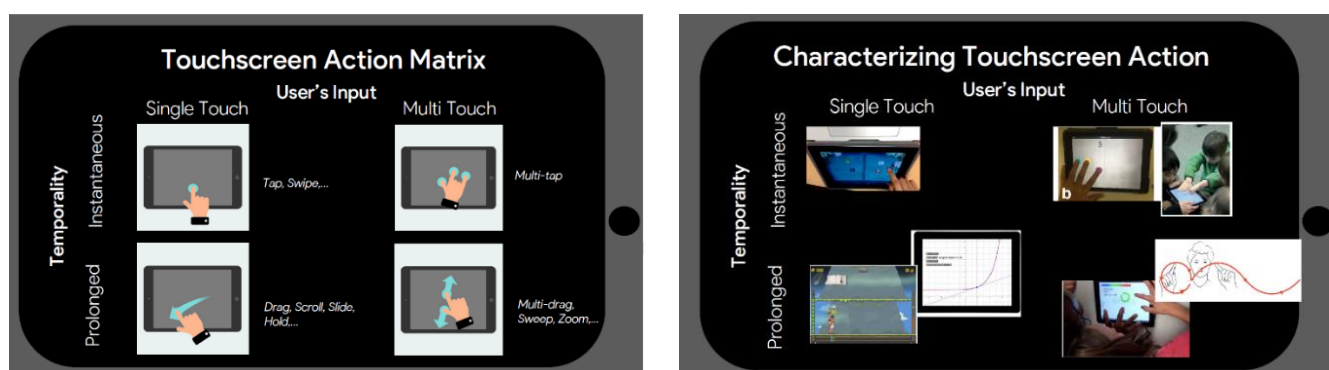


Figure 1. Touchscreen Action Matrix

MATHEMATICAL MEANINGS ELICITED BY TOUCHSCREEN ACTIONS

We coded the operational goals of the action with respect to the touchscreen matrix (Table 1). Following this, we inductively generate the commonalities of mathematical meanings that these actions correspond to. In particular, we find that the temporality of the touchscreen action prompts different nature of mathematical concepts (discrete vs continuous). Multi-touch affordance is also found to influence the number of variables or dimensions in the mathematical tasks involved. Single touch functions, for example, usually support the manipulation of a single quantity or dimension, whereas multi-touch excels at supporting multi-variable operations, such as covariational reasoning.

Mathematical Meaning Being Elicited		
Availability of Variable / Quantity / Dimension		
Mathematical Operation	Single Variable	Multi Variable
	Discrete	Multi Variable
Mathematical Operation	Discrete	Multi Variable
	- Count numbers from 1 to infinitely many (Rodney, 2019) - Create an additive inverse / identity in a linear equation (Tucker & Johnson, 2017; Tucker & Moyer-Packenham, 2021)	Create a number by using the number of corresponding fingers (Güneş, 2021; Bakos & Pimm, 2020)
Mathematical Operation	Continuous	Multi Variable
	- Resize or reorient a vector (Karnam et al., 2021) - Move a vertex of a triangle to conserve its area (Ng & Sinclair, 2015) - Divide a quantity to certain number of equal shares (Janßen et al., 2020; Smith et al., 2015)	- Construct a sine graph with the corresponding value in a unit circle (Shvarts & van Helden, 2022) - Adjust the length and width to vary the area (Panorkou, 2020) - Construct a circle (Hegedus & Otálora, 2022)

Table 1. Coding Results of mathematical meaning being elicited by various touchscreen actions

CONCLUSION AND FUTURE DIRECTION

Our conceptualization contributes to the theoretical and practical groundwork for the design of action-based embodied pedagogy (Abrahamson, 2014). In particular, we expanded the capacity for discussing the action component in user-touchscreen interactions, such as the notion of conceptual congruence. As a concluding remark, we would like to share a new action that we designed throughout the review: employing an instantaneous multi-touch action to create a three-point triangle. This action allows users to instantly create any geometrical figure by multitapping the touchscreen, which complements many existing dynamic geometric software that delays development of geometrical shapes due to the necessity to press all vertices separately. The results of this systematic review will inform K-12 mathematics instructions from the learning science and human-computer interaction perspectives, as well as motivate future research on action-based and touchscreen design for embodied mathematics learning, and the emerging area of multimodality in mathematical thinking.

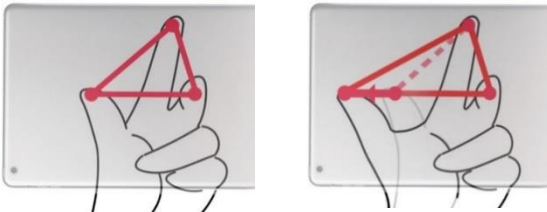


Figure 2. Creating a ‘three-point’ triangle by multitapping on screen and dragging to vary its length

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